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Study on Some Properties of Matrix and their Applications

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Abstract

In this paper, the basic concepts of matrices that are described with notations, operations, and properties are expressed. Then matrix addition, matrix subtraction, and matrix multiplication are presented. Finally, some problems are modeled with matrices and presented as applications to matrices.
Keywords:matrix addition and subtraction, matrix multiplication, applications

1. Introduction

Matrix is a modeling and solving tool for different types of fields such as Geometry, Economics, Engineering, Statistics, Chemistry, Sociology, etc. There is no doubt that the matrix takes part as an important role in Mathematics. Properties and operations on the matrix are expressed as preliminary studies. The basic concepts of matrix as Row matrix, Column matrix, Square matrix, Diagonal matrix, and Identity matrix are fairly expressed as some types of matrix. In the final section, the applications of some properties of matrices are mainly presented.

1.1 Objectives

The objectives of this research paper are as follows:

1. to emphasize some matrices such as matrix addition, matrix subtraction, and matrix multiplication with notations, operations, and properties.
2. to extend knowledge on matrix for thinking and the calculation of matrix.

2. Matrices

Matrix is a tool for solving problems in different fields of science, business, technology, etc. To understand the term “Matrix,” the following preliminary model is studied:

Let us consider the marks obtained by three students Mg Mg, KoKo, and Su Su in Myanmar, English, and Mathematics as follows:

1. Mg Mg secured (65) marks in Myanmar, (80) marks in English and, (70) marks in Mathematics.
2. KoKo secured (75) marks in Myanmar, (58) marks in English and, (60) marks in Mathematics.
3. Su Su secured (50) marks in Myanmar, (70) marks in English and, (55) marks in Mathematics.

The above information can be summarized in tabular form as follows:

Table1. Summarized Tabular Form

Subjects / Students	Myanmar	English	Mathematics
Mg Mg	65	80	70
KoKo	75	58	60
Su Su	50	70	55

Source:own compilation

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The numbers represented in the above data are said to form a rectangular array or arrangement if they are expressed as below:

$$\begin{bmatrix} 65 & 80 & 70 \\ 75 & 58 & 60 \\ 50 & 70 & 55 \end{bmatrix}$$

In the above array, the horizontal lines are called rows, and the vertical lines are called columns. The first row gives the marks of Mg Mg obtained in all three subjects; the second and third rows represent the marks of KoKo and Su Su secured in all three subjects.

Similarly, the first column represents the marks obtained by each student in Myanmar. The second and third columns give the marks of each student in English and Mathematics, respectively. Hence, we observe that this representation of data gives a clear view of the data, and such a rectangular array is called a matrix.

2.1 Definition

An ordered rectangular arrangement of mn numbers having m horizontal lines (rows) and n vertical lines (columns) is called a matrix of order m by n and is written as m x n. Such an arrangement is enclosed in a bracket [ ] or parenthesis ( ). For example,

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}, \begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}, \begin{bmatrix} 3 & 1 & 2 \\ 9 & 4 & 5 \\ 6 & 3 & 1 \end{bmatrix}$$

are all matrices. The numbers constituting a matrix are called elements or entries of the matrix.

2.2 A General Form of a Matrix

An m x n matrix A consisting of mn elements is usually written as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & & & & . \\ . & . & & & & . \\ . & . & & & & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{bmatrix}$$

The above representation contains m rows and n columns. The order of the matrix is denoted by m x n, where m represents the number of rows and n denotes the number of columns of the matrix. Matrices are generally represented by the capital letters A,B,C, etc, and their elements are denoted by the small letters a,b,c, etc.

We can also denote matrices by their general elements. Thus, matrix A can also be written as A = $\left[a_{ij}\right]_{m\times n}$, where a_{ij} is the element in i^{th} row and j^{th} column. We have stated above if a matrix contains m rows and n columns, then its order is m x n.

The i^{th} row of A is

$$\begin{bmatrix} a_{i1} & a_{i2} & . & . & . & a_{in} \end{bmatrix} \qquad (1 \le i \le m)$$

The j^{th} column of A is

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ . \\ . \\ . \\ a_{mj} \end{bmatrix} (1 \le j \le n)$$

2.3 Position of an Element in a Matrix (Order of a Matrix)

$$\begin{array}{ccc} \text{Column 1} & \text{column 2} & \\ \downarrow & \downarrow & \\ \left[\begin{array}{c} 5 \\ 1 \\ -3 \end{array}\right. & \left.\begin{array}{c} -6 \\ 7 \\ 0 \end{array}\right] & \begin{array}{l} \longrightarrow \text{Row 1} \\ \longrightarrow \text{Row 2} \\ \longrightarrow \text{Row 3} \end{array} \end{array}$$

The order of the matrix is 3 x 2 because it has 3 rows and 2 columns.

For example,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3} \quad B = \begin{bmatrix} 2 & 3 & 5 \\ 6 & 7 & 8 \end{bmatrix}_{2\times 3} \quad C = \begin{bmatrix} 2 & 3 & 6 & 7 & 8 \end{bmatrix}_{1\times 5} \quad D = \begin{bmatrix} 4 \\ 2 \\ 9 \\ 1 \end{bmatrix}_{4\times 1}$$

3. Types of Matrix

3.1 Row Matrix

A row matrix is a type of matrix that has a single row.

$$\begin{bmatrix} 5 & 8 & 2 \end{bmatrix}_{1\times 3}$$

3.2 Column Matrix

A column matrix is a type of matrix that has only one column.

$$\begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}_{3\times 1}$$

3.3 Square Matrix

A matrix in which the number of rows is equal to the number of columns is called a square matrix.

$$\begin{bmatrix} 1 & 4 & 5 \\ 6 & 2 & 3 \\ 8 & 1 & 4 \end{bmatrix}_{3\times 3}$$

3.4 Diagonal Matrix

A square matrix is called a diagonal matrix if all its non-diagonal elements are zero.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$$

3.5 Unit or Identity Matrix

A square matrix is called a unit matrix if all the diagonal elements are units and the non-diagonal elements are zero.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

4. Operations on Matrices

The next step in the development of matrix algebra is defining various operations in matrices. Matrices generally behave like numbers and follow all the fundamental operations of arithmetic. Two matrices can be added, subtracted, and multiplied together like two numbers. The three elementary operations in matrices are addition, subtraction, and multiplication.

4.1 Addition and Subtraction Matrices

Addition and Subtraction matrices are very simple. Matrices can be added or subtracted if they have the same order. The following example is studied.

4.2 Example

If $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 5 & 2 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 4 & 6 & 8 \\ 9 & 6 & 5 \end{bmatrix}_{2\times 3}$, Find (i) A + B (ii) A – B

(i) $$A + B = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 5 & 2 \end{bmatrix}_{2\times 3} + \begin{bmatrix} 4 & 6 & 8 \\ 9 & 6 & 5 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 3+4 & 2+6 & 5+8 \\ 1+9 & 5+6 & 2+5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 8 & 13 \\ 10 & 11 & 7 \end{bmatrix}_{2\times 3}$$

(ii) $A - B = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 5 & 2 \end{bmatrix}_{2\times 3} - \begin{bmatrix} 4 & 6 & 8 \\ 9 & 6 & 5 \end{bmatrix}_{2\times 3}$

$$= \begin{bmatrix} 3-4 & 2-6 & 5-8 \\ 1-9 & 5-6 & 2-5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 & -3 \\ -8 & -1 & -3 \end{bmatrix}_{2\times 3}$$

4.3 Example

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 8 & 9 \end{bmatrix}_{3\times 2}$ and $B = \begin{bmatrix} 2 & 7 \\ 6 & 3 \end{bmatrix}_{2\times 2}$, Find (i) A + B (ii) A – B

These two matrices cannot be added or subtracted because they are not in the same order.

4.4 Matrix Multiplication

The product of two matrices will be defined if the number of columns in the first matrix is equal to the number of rows in the second matrix. If the product is defined, the resulting matrix will have the same number of rows as the first matrix and the same number of columns as the second matrix.

Suppose we have a matrix A of m x n dimensions and a matrix B of n x k dimensions. The resultant matrix of the product of matrix A and matrix B will be m x k matrix.

(m x n) . (n x k) = (m x k)

equal

result matrix

4.5 Example

If $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}_{2\times 3}$ and $B = \begin{bmatrix} 4 & 2 \\ 5 & 1 \\ 2 & 3 \end{bmatrix}_{3\times 2}$ then what is AB?

$$AB = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 2 & 1 \end{bmatrix}_{2\times 3} \begin{bmatrix} 4 & 2 \\ 5 & 1 \\ 2 & 3 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 8+20+4 & 4+4+6 \\ 12+10+2 & 6+2+3 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 14 \\ 24 & 11 \end{bmatrix}_{2\times 2}$$

Here A is 2 x 3 matrix, and B is 3 x 2 matrix, therefore, the matrix AB is defined as the number of columns of A is 3 and the number of rows of B is also 3. AB is of order 2 x 2.

4.6 Example

If $A = \begin{bmatrix} 3 & 7 \\ 6 & 5 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} 4 & 1 \\ 5 & 8 \\ 7 & 3 \end{bmatrix}_{3\times 2}$ then what is AB?

The product AB is not impossible because the number of columns in A is not equal to the number of rows in B.

5. Application of Matrix Addition, Subtraction, Multiplication and its Modeled Problems

5.1 Application of Addition and Subtraction of Matrix

Operations on matrices such as addition and subtraction are studied in examples in the previous section. In this section, some problems that are modeled with a matrix are studied as applications of matrix operations.

Let matrix S represent the sales for the T-shirt company in 2021 in various cities, and let matrix P represent the sales for the same company in 2022 in the same cities.

$$S = \begin{array}{c} \begin{array}{ccc} \text{Anchor} & \text{Piano} & \text{Hello} \end{array} \\ \begin{bmatrix} 500 & 330 & 900 \\ 450 & 400 & 200 \end{bmatrix} \end{array} \begin{array}{l} \text{wholesale} \\ \text{Retail} \end{array} \quad \text{in 2021 and}$$

$$P = \begin{array}{c} \begin{array}{ccc} \text{Anchor} & \text{Piano} & \text{Hello} \end{array} \\ \begin{bmatrix} 425 & 350 & 760 \\ 460 & 350 & 250 \end{bmatrix} \end{array} \begin{array}{l} \text{wholesale} \\ \text{Retail} \end{array} \quad \text{in 2022}$$

It is to solve the following question in matrix form.

(a) Write the matrix that represents the total sales by T-shirts for both years in the samecity.

(b) Write the matrix that represents the change in sales by T-shirt from 2021 to 2022 in the same city.

(a) By using the properties of matrix addition, total sales for both years can be solved as-

$$S + P = \begin{bmatrix} 500 & 330 & 900 \\ 450 & 400 & 200 \end{bmatrix}_{2\times 3} + \begin{bmatrix} 425 & 350 & 760 \\ 460 & 350 & 250 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 500+425 & 330+350 & 900+760 \\ 450+460 & 400+350 & 200+250 \end{bmatrix}$$

$$= \begin{bmatrix} 925 & 680 & 1660 \\ 910 & 750 & 450 \end{bmatrix}_{2\times 3}$$

∴The matrix represents the total sales of T-shirts for wholesale, retail, for both years in the same city.

(b) By using the properties of matrix subtraction, the change in sales can be solved as follows:

$$P - S = \begin{bmatrix} 425 & 350 & 760 \\ 460 & 350 & 250 \end{bmatrix}_{2\times 3} - \begin{bmatrix} 500 & 330 & 900 \\ 450 & 400 & 200 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 425-500 & 350-330 & 760-900 \\ 460-450 & 350-400 & 250-200 \end{bmatrix}$$

$$= \begin{bmatrix} -75 & 20 & -140 \\ 10 & -50 & 50 \end{bmatrix}_{2\times 3}$$

∴The matrix represents the change in sales by T-shirt from 2021 to 2022 in the same city.

5.2 Application of Matrix Multiplication

The operations on matrix multiplication are presented in the previous section. In this section, the application of matrix multiplication will be studied. The following modeled problem helps this section.

In the Ministry of Cooperatives and Rural Development, there are two universities. Office furniture needed for each university will be purchased.

The necessary furniture items are as follows:

Table 2. Necessary Furniture Items

University	Table	Chair	Book-shelf
University of Co-operative and Management,Sagaing(UCMS)	30	40	18
University of Co-operative and Management, Thanlyin(UCMT)	32	35	25

Source:own compilation

The furniture will be purchased from the following companies: The charges for each item from these companies are listed below.

Table 3. Purchased Furniture List from Companies

	Rose Co.Ltd	Jasmine Co.Ltd
Table	$ 310	$ 300
Chair	$ 110	$ 100
Book-shelf	$ 155	$ 190

Source:own compilation

Using matrix multiplication,

(a) To find a company that can import cost-effectively for UCMS.

(b) To find a company that can import cost-effectively for UCMT.

(c) To find a company that can import cost-effectively for both universities.

First, the required items for both universities will be modeled as matrix A.

$$\text{Let} \quad A = \begin{array}{c} \begin{bmatrix} 30 & 40 & 18 \\ 32 & 35 & 25 \end{bmatrix} \end{array} \begin{array}{l} \text{UCMS} \\ \text{UCMT} \end{array}$$

Second, the charges for each item can be modeled as matrix B.

$$B = \begin{array}{cc} \text{Rose} & \text{Jasmine} \\ \end{array}$$

$$B = \begin{bmatrix} 310 & 300 \\ 110 & 100 \\ 155 & 190 \end{bmatrix}$$

Then, matrix multiplication will be used.

$$AB = \begin{bmatrix} 30 & 40 & 18 \\ 32 & 35 & 25 \end{bmatrix}_{2\times 3} \begin{bmatrix} 310 & 300 \\ 110 & 100 \\ 155 & 190 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 9300+4400+2790 & 9000+4000+3420 \\ 9920+3850+3875 & 9600+3500+4750 \end{bmatrix}$$

$$= \begin{array}{c} \begin{array}{cc} \text{Rose} & \text{Jasmine} \end{array} \\ \begin{bmatrix} 16490 & 16420 \\ 17645 & 17850 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{UCMS} \\ \text{UCMT} \end{array}$$

(a) Jasmine company can import cost-effectively for UCMS.
(b) Rose company can import cost-effectively for UCMT.
(c) Rose company can import cost-effectively for both universities.

6. Conclusion

Amatrix is one of the essential mathematical tools. All mathematicians will not be perfect as researchers without the use of matrices. In this paper, the operations and properties of matrices have been studied. It shares the basic concepts of a matrix. Moreover, the relation between theory and application of matrices is presented with modeled and applied problems. This study shows researchers in applied mathematics that the matrix is a problem-solving tool. It points out the main finding that the matrix helps researchers in applied business science to be able to model and solve different types of problems easily.

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