Determination of Crop Growing Profit in Wundwin Township by Using Linear Programming (2020-2021 Year Data)

Khin Hla Thi[1], Khin Wai San[2], Wai Phyoe May[3]
khinhlathi19@gmail.com

Abstract

In this paper, firstly, linear programming concepts, linear programming models are expressed. Then, crop growing profits of Wundwin Township, Mandalay Division are determined.

Keywords: objective function, constraint, slack variable, surplus variable, simplex method.

Introduction

Linear programming is one of the most remarkable (and useful) mathematical techniques developed in the last 78 years. It is used to deal with a variety of issues faced by businesses, financial planners, medical personnel, sport leagues, and others. Typical applications include maximizing company profits by adjusting production schedules, minimizing shipping costs by locating warehouses efficiently, and maximizing pension income by choosing the best mix of financial products.

Linear Programming Concepts

A linear programming is a mathematical program in which the objective function is linear in the unknowns and constraints consists of linear equalities and linear inequalities. The exact form of these constraints may differ from one problems to another, any linear program can be transformed into the following standard form:

$$\begin{array}{ll} \text{minimize} & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ & a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ & \vdots \qquad\qquad\qquad \vdots \qquad\quad \vdots \\ & a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \\ \text{and} & x_1 \ge 0, x_2 \ge 0, \ldots, x_n \ge 0, \end{array}$$

where the b_i's, c_i's and a_{ij}'s are fixed real constants, x_i's are real number to be determined. We always assume that each equation has been multiplied by minus unity if necessary, so that each $b_i \ge 0$. In more compact vector notation, this standard problem becomes

[1] Professor, Dr., Department of Mathematics, Kyaukse University, Myanmar

[2] Associate Professor, Department of Mathematics, Kyaukse University, Myanmar

[3] Associate Professor, Department of Mathematics, Kyaukse University, Myanmar

minimize $c^T x$

subject to $A\underline{x} = b$ and $x \geq 0$,

where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$, $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$, $c = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$.

A. Slack Variables

Consider the problem

minimize $c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$

subject to
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leq b_2 \\ \vdots \qquad\qquad\qquad\quad \vdots \qquad &\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq b_m \end{aligned}$$

and $x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$.

The problem may be alternately expressed as

minimize $c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$

subject to
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 &= b_2 \\ \vdots \qquad\qquad\qquad\quad \vdots \qquad &\quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m &= b_m \end{aligned}$$

and $x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$,

$y_1 \geq 0, y_2 \geq 0, \ldots, y_m \geq 0$.

The new positive variables $y_1, y_2, \ldots, y_m$ are called slack variables.

B. Surplus Variables

Consider the problem

$$\begin{aligned}
\text{minimize}\quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
\text{subject to}\quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ge b_1 \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ge b_2 \\
& \quad\vdots \qquad\qquad\qquad \vdots \qquad \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ge b_m \\
\text{and}\quad & x_1 \ge 0, x_2 \ge 0, \ldots, x_n \ge 0 .
\end{aligned}$$

The problem may be alternatively expressed as

$$\begin{aligned}
\text{minimize}\quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\
\text{subject to}\quad & a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n - y_1 = b_1 \\
& a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n - y_2 = b_2 \\
& \vdots \qquad\qquad\qquad \vdots \qquad \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n - y_m = b_m \\
\text{and}\quad & x_1 \ge 0, x_2 \ge 0, \ldots, x_n \ge 0 , \\
& y_1 \ge 0, y_2 \ge 0, \ldots, y_m \ge 0 .
\end{aligned}$$

The new positive variables $y_1, y_2, \ldots, y_m$ are called surplus variables.

Linear Programming Models

To illustrate the mathematical description of the linear programming models, we shall discuss the following problems:

(a) The diet problem

(b) The manufacture problem

(c) The transportation problem

The procedure for mathematical formulation of a linear programming problem consists of the following steps:

Step 1. To write down the decision variables of the problem.

Step 2. To formulate the objective function to be optimized (minimized or maximized) as a linear function of the decision variables.

Step 3. To formulate the other conditions of the problem.

Step 4. To add the non-negative constraint from the consideration.

A. The Diet Problem

A mother wishes her children to obtain certain amounts of nutrients from their breakfast. The children have the choice of eating sandwiches or hamburger. From their breakfast, they should obtain at least 180 gm of protein, 30 gm of starch and 2,400 calories per week. One ounce of sandwiches contains 8 gm of protein, 2 gm of starch and 120 calories. One ounce of hamburger contains 10 gm of protein, 1 gm of starch and 100 calories. One ounce of sandwiches costs 500 kyats and one ounce of hamburger costs 700 kyats.

Let x_1, x_2 be ounce of sandwiches and hamburger.

	1 ounce of sandwiches	1 ounce of hamburger	minimum requirement
Protein	8 gm	10 gm	180 gm
Starch	2 gm	1 gm	30 gm
Calories	120 cal	100 cal	2,400 cal
Costs	500 kyats	700 kyats	

minimize $500x_1 + 700x_2$

subject to $8x_1 + 10x_2 \geq 180$

$2x_1 + x_2 \geq 30$

$120x_1 + 100x_2 \geq 2,400$

and $x_1 \geq 0, x_2 \geq 0$.

B. The Manufacture Problem

A baker starts the day with a certain supply of flour, eggs, sugar, milk and yeast. He specializes in making bread, cakes and cookies. He wishes to determine how much of each product and he should make so as to maximize his profit. The recipes are given in the following table.

	Bread	Cake	Cookies	Available resources
Flour	12	3	1.5	1,800 oz
Eggs	0	2	1	180 eggs
Sugar	0.25	1.5	1	150 oz
Milk	2	0.75	0.25	100 cups
Yeast	0.25	0	0	50 cakes
Profit	300 kyats	1,000 kyats	100 kyats	

Let x_1, x_2, x_3 be the number of Bread, Cakes and Cookies to be produced. The given manufacture problem can be written in a linear programming problem as follow:

maximize $300x_1 + 1,000x_2 + 100x_3$

subject to
$$12x_1 + 3x_2 + 1.5x_3 \le 1,800$$
$$2x_2 + x_3 \le 180$$
$$0.25x_1 + 1.5x_2 + x_3 \le 150$$
$$2x_1 + 0.75x_2 + 0.25x \le 100$$
$$0.25x_1 \le 50$$

and $x_1 \ge 0, x_2 \ge 0, \ldots, x_n \ge 0$.

C. The Transportation Problem

A manufacturer has distribution centers located at Yangon and Mandalay. These centers have available 500 units and 300 units of an item of product respectively. His retail outlets require the following number of units: Mawlamyine 300, Sittwe 300, Tauggyi 200. The shopping cost per unit in kyats between each center and outlet is given in the following table.

Outlets

Centers	Mawlamyine	Sittwe	Taunggyi
Yangon	20	45	55
Mandalay	75	60	35

Let x_{ij} be the amount of the product shipped from the i^{th} center to j^{th} outlet and $x_{ij} \ge 0, \quad i = 1,2, \ j = 1,2,3$.

	Mawlamyine	Sittwe	Taunggyi	Supplies
Yangon	x_{11}	x_{12}	x_{13}	500
Mandalay	x_{21}	x_{22}	x_{23}	300
Demand	300	300	200	

The given transportation problem can be written in a linear programming problem as follow:

minimize $20x_{11} + 45x_{12} + 55x_{13} + 75x_{21} + 60x_{22} + 35x_{23}$

subject to
$$x_{11} + x_{12} + x_{13} = 500$$
$$x_{21} + x_{22} + x_{23} = 300$$
$$x_{11} + x_{21} = 300$$
$$x_{12} + x_{22} = 300$$
$$x_{13} + x_{23} = 200$$

and $x_{ij} \ge 0, \quad i = 1,2, \ j = 1,2,3$.

Calculation of Linear Programming Problem

Consider the system in canonical form:

$$\begin{aligned} x_1 \qquad\quad + x_4 + x_5 - x_6 &= 5 \\ x_2 \quad + 2x_4 - 3x_5 + x_6 &= 3 \\ x_3 - x_4 + 2x_5 - x_6 &= -1 \end{aligned}$$

We can find the basic solution having basic variables x_4, x_5, x_6.

x_1	x_2	x_3	x_4	x_5	x_6		
1	0	0	**1**	1	–1	5	
0	1	0	2	–3	1	3	$R_2 \to R_2 - 2R_1$
0	0	1	–1	2	–1	–1	$R_3 \to R_3 + R_1$

The circle indicated is our first pivot element and corresponds to the replacement of x_1 by x_4 as a basic variable. After pivoting we obtain the array

x_1	x_2	x_3	x_4	x_5	x_6		
1	0	0	1	1	–1	5	
–2	1	0	0	**–5**	3	–7	$R_2 \to -\frac{1}{5}R_2$
1	0	1	0	3	–2	4	

and again we have circled the next pivot element indicating our intention to replace x_2 by x_5. We then obtain

x_1	x_2	x_3	x_4	x_5	x_6		
1	0	0	1	1	–1	5	$R_1 \to R_1 - R_2$
$\frac{2}{5}$	$-\frac{1}{5}$	0	0	**1**	$-\frac{3}{5}$	$\frac{7}{5}$	
1	0	1	0	3	–2	4	$R_3 \to R_3 - 3R_2$

x_1	x_2	x_3	x_4	x_5	x_6		
$\frac{3}{5}$	$\frac{1}{5}$	0	1	0	$-\frac{2}{5}$	$\frac{18}{5}$	
$\frac{2}{5}$	$-\frac{1}{5}$	0	0	1	$-\frac{3}{5}$	$\frac{7}{5}$	
$-\frac{1}{5}$	$\frac{3}{5}$	1	0	0	$-\frac{1}{5}$	$-\frac{1}{5}$	$R_3 \to -5R_3$

x_1	x_2	x_3	x_4	x_5	x_6		
$\frac{3}{5}$	$\frac{1}{5}$	0	1	0	$-\frac{2}{5}$	$\frac{18}{5}$	$R_1 \to R_1 + \frac{2}{5}R_3$
$\frac{2}{5}$	$-\frac{1}{5}$	0	0	1	$-\frac{3}{5}$	$\frac{7}{5}$	$R_2 \to R_2 + \frac{3}{5}R_3$
1	–3	–5	0	0	1	1	

Continuing the results,

x_1	x_2	x_3	x_4	x_5	x_6	
1	–1	–2	1	0	0	4
1	–2	–3	0	1	0	2
1	–3	–5	0	0	1	1

From this last canonical form we obtain the new basic solution is $x_4 = 4,\ x_5 = 2, x_6 = 1$.

Maximizing Application Concepts

The steps involved in solving a standard maximum linear programming problem by the simplex method are the followings:

Step 1. Determine the objective function.

Step 2. Write down all necessary constraints.

Step 3. Convert each constraint into an equation by adding a slack variable.

Step 4. Set up the initial simplex tableau.

Step 5. Locate the most negative indicator. If there are two such indicators, choose one. This indicator determines the pivot column.

Step 6. Use the positive entries in the pivot column to form the quotients necessary for determining the pivot. If there are no positive entries in the pivot column, no maximum solution exists. If two quotients are equally the smallest, let either determine the pivot.

Step 7. Multiply every entry in the pivot row by the reciprocal of the pivot to change the pivot to 1. Then use row operations to change all other entries in the pivot column to 0 by adding suitable multiples of the pivot row to the other rows.

Step 8. If the indicators are all positive or 0, we have found the final tableau. If not, go back to step 5 and repeat the process until a tableau with no negative indicators is obtained.

Step 9. In the final tableau, the basic variables correspond to the columns that have one entry of 1 and the rest 0. The non-basic variables correspond to the other columns. Set each non-basic variable equal to 0 and solve the system for the basic variables. The maximum value of the objective function is the number in the lower right-hand corner of the final tableau.

A. Data Collection

The data for this paper is collected by farmers, U Kyaw Min, Daw Tin Tin Mya and U Soe Lwin of Daing Kaung Gone Village, Wundwin Township, Mandalay Division (2020-2021 year).

In cultivation of crops, the activities budgets and the cost of required materials per acre are

No	The costs for cultivation of crops	Corns	Sesames	Groundnuts
1	Cost of seeds	40,000	5,000	80,000
2	Costs of power tiller	70,000	65,000	70,000
3	Cost of labour used in this whole process	150,000	20,000	90,000
4	Cost of fertilizers and urea	90,000	25,000	35,000
5	Cost of transportation	40,000	25,000	15,000
6	Other general costs	10,000	10,000	10,000
Total cost		400,000	150,000	300,000

The profits of the crops per acre are

No	Type of crops	Total income per acre	Total cost per acre	Profit per acre
1	Corns	700,000	400,000	300,000
2	Sesames	210,000	150,000	60,000
3	Groundnuts	380,000	300,000	80,000

B. Analysis For Crops Growing Profit

We have 10 acres of available land we wish to plant with a mixture of corn, sesame and groundnut. We have the total limitation of costs cultivation are 2,000,000 kyats. We want to know (i) how many acres of each crop we should plant to maximize our profit (ii) if we maximize our profit, how much land will remain unplanted. The followings are explanation for this.

Let the number of acres allotted to each of corn, sesame, and groundnut be x_1, x_2 and x_3 respectively. Then summarize the given information as follows:

Table 1 (2020-2021 year data)

Crops	Number of acres	Cost per acre	Profit per acre
Corn	x_1	400,000 kyats	300,000 kyats
Sesame	x_2	150,000 kyats	60,000 kyats
Groundnut	x_3	300,000 kyats	80,000 kyats
Maximum available	10	2,000,000 kyats	

maximize $x_1 + x_2 + x_3 \le 10$

subject to $400,000x_1 + 150,000x_2 + 300,000x_3 \le 2,000,000$

$z =$ profit of corn + profit of sesame + profit of groundnut.

$z = 300,000x_1 + 60,000x_2 + 80,000x_3$

with $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

The linear programming problem can be started as follows

maximize $x_1 + x_2 + x_3 + s_1 = 10$

subject to $400,000x_1 + 150,000x_2 + 300,000x_3 + s_2 = 2,000,000$

$z = 300,000x_1 + 60,000x_2 + 80,000x_3$

with $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, s_1 \ge 0, s_2 \ge 0$

x_1	x_2	x_3	s_1	s_2		
1	1	1	1	0	10	
400,000	150,000	300,000	0	1	2,000,000	$R_2 \to \frac{R_2}{400,000}$
–300,000	–60,000	–80,000	0	0	0	

x_1	x_2	x_3	s_1	s_2		
1	1	1	1	0	10	$R_1 \to R_1 - R_2$
1	0.375	0.75	0	0.0000025	5	
–300,000	–60,000	–80,000	0	0	0	$R_3 \to 300{,}000R_2 + R_3$

x_1	x_2	x_3	s_1	s_2	
0	0.625	0.25	1	–0.0000025	5
1	0.375	0.75	0	0.0000025	5
0	52,500	145,000	0	0.75	1,500,000

Non-basic variables x_2, x_3 and s_2 equal to zero.

$x_1 = 5,\ x_2 = 0,\ x_3 = 0,\ s_1 = 5,\ s_2 = 0,\ z = 1{,}500{,}000$.

z (our maximum profit) is in the lower right-hand corner.

$s_1 =$ unplanted acres.

By planting 5 acres of corn, no sesame and no groundnut, 5 of 10 acres are planted, 5 acres will remain unplanted.

$s_2 =$ the amount of unused cash.

But $s_2 = 0$, all the available money has been used.

Therefore, we have used 2,000,000 kyats most efficiently.

Thus, if we have more cash, we would plant more crop and make a larger profit.

Conclusion

In this paper, firstly, linear programming concepts, linear programming models are expressed. Then, simplex method and maximizing application concept are presented. And finally, crop growing profits of Wundwin Township, Mandalay Division are determined.

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