Study of Phase Shift of N-N Scattering Using Green Function Method

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Abstract

In order to know the information concerning two-body interaction, the scattering parameters related to it such as phase shift, scattering differential cross section, total cross section and scattering amplitude were stuied. By using the Green function method to find the phase shift and the respective partial cross-section for N-N scattering process. Then the time independent Schrodinger equation is transformed into the Lippmann Schwinger equation. And it is solved numerically with Exponential potential within the energy range of 0 to 330MeV.

Keywords: Phase Sift, Scattering

Introduction

Scattering Process

Scatteringin physics is a change in the direction of motion of a particle because of a collision with another particle. A collision can occur between particles that repel one another, such as two positive (or negative) ions, and need not involve direct physical contact of the particles. There are two types of scattering. They are elastic scattering and inelastic scattering.

N - N scattering process

In principle there are four types of scattering measurements involving two nucleons that can be carried out. They are p-p scattering, p-n scattering,n-n scattering and n-p scattering. The scattering of an incident proton off a proton (p-pscattering) is the simplest one of the four to carry out experimentally, since it is relatively easy to accelerate protons and to construct targets containing hydrogen (proton). For neutron scattering, there are two major sources of incident beams. At low energies, neutrons from nuclear reactor may be used. At higher energies, neutrons produced by proton bombardment, for instance, through a (p, n) reaction on a ^{7}Li target, are often used. However, both the intensity and the energy resolution of neutron beams obtained in this way are much more limited than those of proton beams. As a result, neutron scattering is, in general, a more difficult experiment than proton scattering. The scattering of neutrons off proton targets (n p-scattering) is an important source of information on two-nucleon systems.

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Phase Shift

Phase shiftis any change that occurs in the phase of one quantity, or in the phase difference between two or more quantities. At very low energy the incoming particle does not see any structure, therefore to lowest order one has only a spherical symmetric outgoing wave, the so-called s-wave scattering (angular momentum). At higher energies one also needs to consider p and d-wave scattering. The phase shift represents the movement of the waves based on the amplitude and periods. The phase shift of a sine curve is how much the curve shifts from zero. If the phase shift is zero, the curve starts at the origin, but it can move left or right depending on the phase shift. In the diagram below, the second wave is shifted by the specified amount from the original wave.

Phase shift can also determine the behavior of potential. A repulsive potential makes negative phase shift. A negative phase shift means that the radial wave function is pushed out in comparison with the force-free wave function. A negative potential makes positive phase shift. This means that the radial wave function is pulled in by the attractive potential.

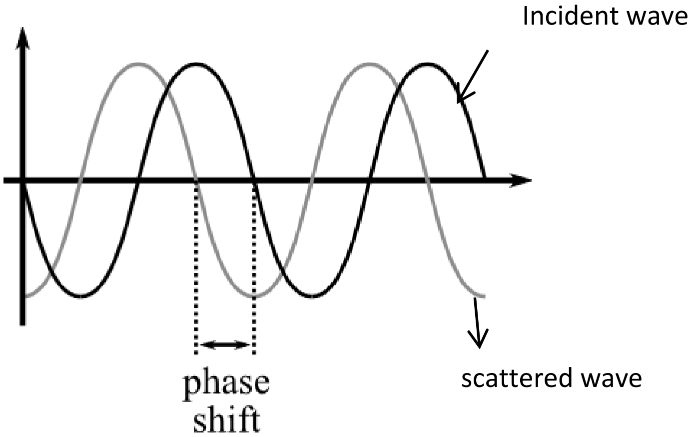

Figure (1) Phase shift

Calculation of Phase Shift

The total wave-function

$$u(\vec{r}) = \phi_{inc}(\vec{r}) + \phi_{sc}(\vec{r}) \tag{1}$$

Where $\phi_{inc}(\vec{r})$ = incident wave function

$\phi_{sc}(\vec{r})$ = scattering wave function

The wavefunction at a large distance r is a linear combination of a plane wave, made of the incident beam and particles no scattered by potential, and a spherical wave, made of scattered particles. So the wave function becomes,

$$u(\vec{r}) \xrightarrow[r \to \infty]{} e^{ikz} + f(\theta,\phi)\frac{e^{ikr}}{r} \tag{2}$$

The wave function $u(\vec{r})$ is satisfied the time independent Schrodinger equation,

$$\frac{-\hbar^2}{2\mu}\nabla^2 \quad U(\vec{r})+V(\vec{r}) \quad U(\vec{r})=E \quad U(\vec{r}) \tag{3}$$

So the equation can be written as

$$\nabla^2 u(\vec{r})+\frac{2\mu}{\hbar^2}\left[E-V(\vec{r})\right]u(\vec{r})=0 \tag{4}$$

Where μ=reduced mass

E=kinetic energy of particles

V(r)=potential energy of particles

The scattering wave function is convenient to produce the radial and angular parts so the general equation of wave function is

$$u(\vec{r})=\sum_{l=0}^{\infty} R_l(r)P_l(\cos\theta) \tag{5}$$

The radial wavefunction for partial wave l satisfies the equation

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dR_l}{dr}\right]+\frac{2\mu}{\hbar^2}\left[E-V-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]R_l=0 \tag{6}$$

$$\frac{d^2}{dr^2}(\chi_l)+\left[k^2-U(\vec{r}')-\frac{l(l+1)}{r^2}\right]\chi_l=0 \tag{7}$$

Where,

$$rR_l(r)=\chi_l, k^2=\frac{2\mu E}{\hbar^2}, U(r)=\frac{2\mu V(r)}{\hbar^2}$$

In the asymptotic region r→∞, u and l terms can be neglected. So we get χ_l approximation method that is shown below;

$$\chi_l(r)\ \alpha\ e^{\pm ikr}$$

$$\chi_l(r)=A_l(r)e^{\pm ikr} \tag{8}$$

$$\chi_l(r)\xrightarrow[r\to\infty]{} A_l'\sin(kr+\Delta_l) \tag{9}$$

Where A_l' and Δ_l are constants.

$$\delta_l=\Delta_l+\frac{l\pi}{2} \tag{10}$$

Where δ_l is called the phase shift in the l^{th} partial wave .

The Scattering cross section and phase shift

The radial wave function is

$$R_l = \frac{1}{r}\chi_l \tag{11}$$

$$R_l(r) \xrightarrow[r\to\infty]{} (kr)^{-1} A_l' \sin(kr - \frac{l\pi}{2} + \delta_l) \tag{12}$$

$$u(\vec{r}) = \sum_{l=0}^{\infty} A_l (kr)^{-1} \sin(kr - \frac{l\pi}{2} + \delta_l) P_l(\cos\theta) \tag{13}$$

By using the Bessel function expansion,

$$e^{ikz} = e^{ikr\cos\theta} = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr) p_l(\cos\theta) \tag{14}$$

$$u(r) = \sum_{l=0}^{\infty} \frac{(2l+1)i^l}{kr} C_l(r) p_l(\cos\theta) \tag{15}$$

$$\tan\delta_l = \frac{\mathrm{Im}\{C_l(r)\}}{\mathrm{Re}\{C_l(r)\}} \tag{16}$$

If we know the function $C_l(r)$,the phase shift can be calculated from the above equation.

By according equation(2), equation(13) and equation(14), get the following equation,

$$f(\theta) = (k)^{-1} \sum_{l=0}^{\infty} \left[(2l+1)(e^{i\delta_l}) \sin\delta_l\right] p_l(\cos\theta) \tag{17}$$

$$\left(\frac{e^{i\delta_l} - e^{-i\delta_l}}{2i} = \sin\delta_l\right)$$

Thus the differential cross section is

$$\sigma = |f(\theta)|^2$$

$$\sigma = 2\pi \int_0^{\pi} \sigma(\theta) \sin\theta \, d\theta = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l \tag{18}$$

Is called the partial cross section of this l partial wave.

Green function method is

$$G(\vec{r}, \vec{r}') = \frac{e^{i|r-r'|\rho}}{|\vec{r} - \vec{r}'|} \tag{19}$$

To solve the wave function using the Green Function method

$$u(\vec{r}) = \vec{u}_0(\vec{r}) - \frac{1}{4\pi} \int G(\vec{r}, \vec{r}') \, U(\vec{r}) \, u(\vec{r}) \, d\vec{r}' \tag{20}$$

$$u(\vec{r}) = \vec{u}_0(\vec{r}) - \frac{1}{4\pi}\int \frac{e^{i\vec{k}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} U(\vec{r})u(\vec{r})\, d\vec{r}' \tag{21}$$

This equation is called the integral equation for the wave function of scattering process.

We used the following expansion;

$$u(r) = \sum_{l=0}^{\infty} \frac{(2l+1)i^l}{kr} C_l(r) p_l(\cos\theta) \tag{22}$$

$$u_0(r) = e^{ikz} = \sum_{l=0}^{\infty} (2l+1)\ i^l j_l(kr) p_l(\cos\theta) \tag{23}$$

$$\frac{e^{ik|r-r'|}}{|r-r'|} = k\sum_{l=0}^{\infty} (2l+1)\ j_l(kr_\langle) h_l^+(kr_\rangle) p_l(\cos\theta) \tag{24}$$

By using equation (22),equation(23) and equation(24),so the equation(21)becomes,

$$\sum_{l=0}^{\infty} \frac{(2l+1)}{kr} i^l C_l(r) p_l(\cos\theta) = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr)\, p_l(\cos\theta) - \sum_{l=0} (2l+1) i^l p_l(\cos\theta) \frac{2\mu}{\hbar^2} \int_0^{\infty} r' j_l\ (kr_\langle) h_l^+(kr_\rangle) V(r') C_l(r')\, dr' \tag{25}$$

For L partial wave

$$C_l(r) = krj_l(kr) - \frac{2\mu k}{\hbar^2} \int_0^{\infty} r' j_l\ (kr_\langle) h_l^+(kr_\rangle) V(r') C_l(r') dr' \tag{26}$$

where, σ = the angle between r and r'

$h_l(kr)$ = The Hankel function

$r_\langle$ = the smaller of the length r and r'

$r_\rangle$ = the larger of the length of r and r'

This relation is often called Lippmann Schrodinger integral equation for scattering process.

Results

The purpose of this research work is to inversitage the phase shift for N-N scattering process. Scattering process is very important because a lot of knowledge about the forces and interactions in atoms and nuclei have been learned from scattering process. This scattering process is sufficient way to calculate the phase-shifts and partial cross section in the energy range 0 to 330MeV.

To adjust the depth parameter and range parameter for exponential potential, then introduce the experimental data for s wave at energy range from 0 to 330MeV.These values are shown in Table(1). By employing the phase shift values in Table (1), then compare between the experimental curve and the theoretical curves that are shown in Figure (2).

This Figure (2) shows that the experimental values and calculated values are agree at energy ranges from 0 to 100 MeV. But above 100MeV energies, the set1, the set2 and experimental curve are slightly shift to each other. But they are in fairly good agreement with the experimental curve.

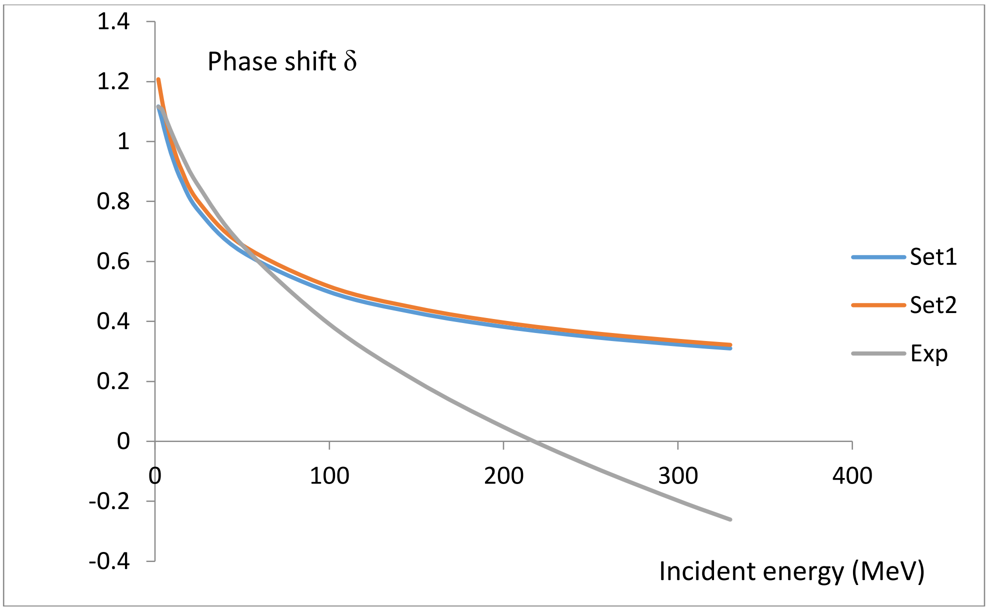

Figure (2) Phase shifts Vs Energy for Exponential potential of s -state

Table (1) Energy and Phase shift for Exponential potential of s-state

Exponential Potential			
Incident Energy (MeV)	V_0=55.9MeV α=1fm δ(rad)	V_0=58MeV α=1fm δ(rad)	Experimental Value δ(rad)
2	1.1168	1.208	1.1162
4	1.0745	1.1404	1.1077
5	1.0509	1.1103	1.0952
6	1.0281	1.0827	1.0812
8	0.986	1.0339	1.0524
10	0.94877	0.99216	1.0243
15	0.8727	0.9094	0.9598
25	0.76954	0.79951	0.8524
50	0.63188	0.65527	0.6553
100	0.49831	0.51641	0.3912
150	0.42874	0.44434	0.2009
200	0.38211	0.39603	0.0475
250	0.34847	0.36121	-0.0831
300	0.32305	0.33488	-0.1918
330	0.31007	0.32143	-0.2614

Conclusion

We use Green function method for the determination of s-wave phase shifts of N-N scattering process. The method finds a solution for the scattering amplitude by transforming the time independent Schrodinger equation of scattering process into the Lippmann Schwinger integral equation.

With the help of computer program we calculate numerically the phase shift for s-wave using Exponential potential. In general, these values are not in agreement with the experimental values. Though the fitting is good in high energy regions, it is not good in low energy regions. We need more than two parameters to be adjusted to reproduce the experimental values in all energy regions.

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