Title	Structure Analysis of Light [I] - Hypernuclei
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Issue Date	2016

Structure Analysis of Light Ξ^- Hypernuclei

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Abstract

In this research work, the binding energies of light Ξ^- hypernuclei namely Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O have been calculated within the frame work of Ξ^- single-particle model by solving nonrelativistic Schrödinger equation. In this calculation, the Gaussian basis wave function and phenomenological Woods-Saxon central potential including coulomb interaction are used. By using the strength of Woods-Saxon potential -14.0 MeV, the calculated bound state energy of Ξ^--^{14}N is 3.07 MeV over binding than the recent experimental results, 4.38 ± 0.25 MeV which is observed from E373 experiment, KEK, Japan. Therefore, the binding energies of Ξ^--^{14}N have investigated by varying potential strength. At the potential depth -8.5 MeV, the calculated binding energy of Ξ^--^{14}N is in good agreement with the experimental result. Therefore, this potential strength is applied to study the structure analysis of other two light Ξ^- hypernuclei. The observed binding energies of 1S state for Ξ^--^{12}C and Ξ^--^{16}O are 3.64 MeV and 5.09 MeV respectively. In addition, the root-mean-square radii of these light Ξ^- hypernuclei have also investigated. The calculated results are 3.24 fm for Ξ^--^{12}C, 3.11 fm for Ξ^--^{14}N and 3.02 fm for Ξ^--^{16}O respectively. It is also found that 3D and 4F states are only pure Ξ^-- atomic states while the other S and P states are Coulomb-assisted nuclear Ξ^--bound states called hybrid states.

Key words: Ξ^- hypernuclei, Woods-Saxon potential, root-mean-square radii, hybrid states.

Introduction

In order to study hyperon-nucleon and hyperon-hyperon interactions for the understanding of high-density nuclear matter inside neutron star, it is difficult to perform the scattering experiments due to the short-life hyperons. Therefore, the investigation of many hypernuclear structure analysis such as emulsion experiment and reaction theory etc., can supply the useful information about hyperon-nucleon and hyperon-hyperon interactions. From the theoretical point of view, the study of hyperon-nucleon and hyperon-hyperon interactions have been discussed in various description [1,2,3,4,5,6,7]. Yamaguchi et al., calculated the bound states of Ξ^-Nuclei, Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O with the use of Ehime potential including coulomb interaction [8]. In E176 experiment at KEK, Japan, the (K^-,K^+) reaction with the emulsion target was carried out to search S=-2 hypernuclei [9]. Moreover, Ξ^-hypernuclei have already studied in the (K^-,K^+) reaction on a scintillation fiber target [10]. They presented that the potential well depth for Ξ^- hyperon should be less than 20 MeV. Furthermore, E885 experiment was carried out by P. Khaustov and his collaboration [11] to analyze Ξ^- hypernucleai with the use of (K^-,K^+) reaction on carbon. In this experiment, Ξ^- hypernuclear state are expected to be produced through the reaction $K^- + {}^{12}C \rightarrow K^+ + {}^{12}_{\Xi^-}Be$. Reasonable agreement between their data and theory

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was achieved by assuming a Ξ-nucleus potential depth of about -14 MeV within the Woods-Saxon prescription. From E373 experiment, the first evidence of a deeply bound state of the Ξ^--^{14}N system is now observed with the binding energy $\mathbf{B}_{\Xi^-} = 4.38 \pm 0.25$ MeV [12]. This Ξ^--^{14}N decayed into twin single-hypernucleai in nuclear emulsion. Experiments with Ξ^- atoms are proposed in order to study the nuclear interaction of Ξ^- hyperons [13]. However, the experimental information about Ξ-N interaction is very limited to study not only the structure analysis of Ξ^- hypernucleus and Ξ-atoms but also double-Λ hypernucleus. Therefore, we would like to study the structure analysis of light Ξ^- hypernuclei namely Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O.

Mathematical Formulation

Derivation of Kinetic Energy and Centrifugal Potential Energy Matrix Elements

In this section, we will derive the kinetic energy, centrifugal potential energy and norm matrix elements from the one-body Schrödinger equation.

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+\frac{\hbar^2}{2M}\frac{\ell(\ell+1)}{r^2}+V(r)\right]u(r)=Eu(r)\,. \tag{1}$$

In order to solve the above radial wave equation, we will use the Gaussian basis wave function which is described as follow.

$$u(r)=r^{(\ell+1)}\sum_j c_j e^{-(r/b_j)^2} \tag{2}$$

In this equation, c_j 's are expansion coefficients and b_j 's are range parameters. The Schrödinger equation, therefore, becomes,

$$\left\{-\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+\frac{\hbar^2}{2M}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}=E\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}\,. \tag{3}$$

By multiplying both sides of the equation by $r^{(\ell+1)}e^{-(r/b_j)^2}$ from the left and solving it, we can obtain as

$$\int r^{(\ell+1)}e^{-(r/b_i)^2}\left\{-\frac{\hbar^2}{2M}\frac{d^2}{dr^2}+\frac{\hbar^2}{2M}\frac{\ell(\ell+1)}{r^2}+V(r)\right\}\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}dr$$
$$=E\int r^{(\ell+1)}e^{-(r/b_i)^2}\sum_j c_j r^{(\ell+1)}e^{-(r/b_j)^2}dr\,. \tag{4}$$

The above equation (4) can be expressed as the following equation including the kinetic energy matrix element (T_{ij}^{ℓ}), centrifugal potential energy matrix element (F_{ij}^{ℓ}), the potential energy matrix element (V_{ij}^{ℓ}) and the norm matrix element (N_{ij}^{ℓ}).

$$\sum_j\left(T_{ij}^{\ell}+F_{ij}^{\ell}+V_{ij}^{\ell}\right)c_j=E\sum_j c_j N_{ij}^{\ell}$$

where,

$$T_{ij}^{\ell} = \int r^{(\ell+1)} e^{-(r/b_i)^2} \left\{ -\frac{\hbar^2}{2M} \frac{d^2}{dr^2} \right\} r^{(\ell+1)} e^{-(r/b_j)^2} dr \tag{5}$$

$$F_{ij}^{\ell} = \frac{\hbar^2}{2M} \int r^{(\ell+1)} e^{-(r/b_i)^2} \frac{\ell(\ell+1)}{r^2} r^{(\ell+1)} e^{-(r/b_j)^2} dr \tag{6}$$

$$V_{ij}^{\ell} = \int r^{(\ell+1)} e^{-(r/b_i)^2} V(r) r^{(\ell+1)} e^{-(r/b_j)^2} dr \tag{7}$$

$$N_{ij}^{\ell} = \int r^{2(\ell+1)} e^{-(r/b_i)^2} e^{-(r/b_j)^2} dr . \tag{8}$$

These kinetic energy matrix element, centrifugal potential energy matrix element and norm matrix element are solved analytically with the use of standard integral, $\int_0^{\infty} x^{2n} e^{-a^2x^2} dx = \frac{(2n-1)!!}{2^{(n+1)}} \frac{\sqrt{\pi}}{a^{(2n+1)}}$. Therefore, the kinetic energy matrix element becomes as

$$T_{ij}^{\ell} = -\frac{\hbar^2}{2M} \left\{ \frac{4}{b_j^4} \frac{(2\ell+3)!!}{2^{(\ell+3)}} \frac{\sqrt{\pi}}{B^{(\ell+\frac{5}{2})}} - \frac{(4\ell+6)}{b_j^2} \frac{(2\ell+1)!!}{2^{(\ell+2)}} \frac{\sqrt{\pi}}{B^{(\ell+\frac{3}{2})}} + \ell(\ell+1) \frac{(2\ell-1)!!}{2^{(\ell+1)}} \frac{\sqrt{\pi}}{B^{(\ell+\frac{1}{2})}} \right\} \tag{9}$$

where $B = \frac{1}{b_i^2} + \frac{1}{b_j^2}$.

The centrifugal potential energy matrix element can be expressed as

$$F_{ij}^{\ell} = \frac{\hbar^2}{2M} \ell(\ell+1) \left(\frac{(2\ell-1)!!\sqrt{\pi}}{2^{(\ell+1)} \left(\frac{1}{b_i^2} + \frac{1}{b_j^2} \right)^{(\ell+\frac{1}{2})}} \right) . \tag{10}$$

and the norm matrix element can be written by

$$N_{ij}^{\ell} = \frac{(2\ell+1)!!}{2^{(\ell+2)}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2} \right)^{(\ell+\frac{3}{2})}} . \tag{11}$$

The combination of the kinetic energy matrix element and the centrifugal potential energy matrix element can be expressed as,

$$T_{ij}^{\ell} + F_{ij}^{\ell} = \frac{\hbar^2}{2M} N_{ij}^{\ell} \frac{(4\ell+6)}{b_i^2 + b_j^2} . \tag{12}$$

Interaction for Ξ –Nucleus System

In our present work, we will assume that a Ξ− hyperon stays independently in an average potential well for our consideration nucleus, ^{12}C, ^{14}N and ^{16}O. We use the phenomenological Woods-Saxon central potential [14] which is described as follows.

$$V_{WS}(r) = V_0 f(r) \qquad (13)$$

In this equation, V0 are the strength of the Woods-Saxon potential and f(r) is the nuclear density which can be expressed as $f(r) = \frac{1}{1+e^{r-R/a}}$, where, $R = r_0 A^{1/3}$ with r_0 =1.1 fm and the diffuseness parameter a=0.65fm.

Generally, the potential strength (V_0) for Ξ^- hypernucleus is used $V_0 = -24 \pm 4$ MeV. However, the potential strength is used -14.0 MeV in our calculation [5, 11]. The Woods-Saxon potential energy matrix element is

$$V_{ij}^{\ell} = \int r^{(\ell+1)} e^{-(r/b_i)^2} V_{WS}(r) r^{(\ell+1)} e^{-(r/b_j)^2} dr \, .$$

After solving the above equation,

$$V_{ij}^{WS} = \int r^{(2\ell+2)} e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2} \frac{V_0}{1+e^{(r-R)/a}} dr \, . \qquad (14)$$

In addition, it is also necessary to consider the coulomb contribution between Ξ^- hyperon and the core nucleus ^{12}C, ^{14}N and ^{16}O. Due to internal structure of core nucleus, ^{12}C, ^{14}N and ^{16}O, it is insufficient the coulomb point charge potential. Thus, the Coulomb potential will be used as

$$V_{Coulomb}(r) = Z_1 Z_2 \alpha\hbar c \frac{1}{2R}\left[3 - \frac{r^2}{R^2}\right] \qquad (\text{for } r \langle R \text{) and}$$

$$V_{Coulomb}(r) = Z_1 Z_2 \alpha\hbar c \frac{1}{r} \qquad (\text{for } r \geq R \text{)}$$

where, Z_1 and Z_2 are charge of the Ξ^- hyperon and core nucleus, α is the fine structure constant. The coulomb attractive potential energy matrix element, $V_{ij}^{coul} = \int r^{(\ell+1)} e^{-(r/b_i)^2} V_{Coulomb}(r)\, r^{(\ell+1)} e^{-(r/b_j)^2} dr$, becomes

$$V_{ij}^{coul} = V_{ij}^{coul}(r \langle R) + V_{ij}^{coul}(r \geq R) \qquad (15)$$

where, $V_{ij}^{coul}(r \langle R) = \int r^{(2\ell+2)} e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2} Z_1 Z_2 \alpha\hbar c \frac{1}{2R}\left[3 - \frac{r^2}{R^2}\right] dr$

$$V_{ij}^{coul}(r \geq R) = \int r^{(2\ell+2)} e^{-\left(\frac{1}{b_i^2}+\frac{1}{b_j^2}\right)r^2} Z_1 Z_2 \alpha\hbar c \frac{1}{r} dr$$

By using the above equations (14) and (15), the potential energy matrix element can be solved numerically. To obtain the energy eigen value of our consideration system, the Schrödinger equation will be solved by using power inverse iteration method.

Calculation of Root-mean-square Radius

After solving the Schrödinger equation by using power inverse iteration method, the energy eigen value and the corresponding eigen vector (c_j) have obtained. We use the Gaussian basis wave function in order to investigate the binding energy and root-mean-square radius. The normalized Gaussian basis wave function is defined as follows;

$$u(r) = Ar^{(\ell+1)} \sum_j c_j e^{-(r/b_j)^2}$$

The normalization constant can be obtained by using normalization condition; $\int u^*(r)u(r)dr = 1$.

After inserting the wave function in the above equation, we can obtain as follows,

$$\int A^* \sum_i r^{(\ell+1)} c_i e^{-\left(\frac{r}{b_i}\right)^2} A \sum_j r^{(\ell+1)} c_j e^{-\left(\frac{r}{b_j}\right)^2} dr = 1$$

$$|A|^2 \sum_i \sum_j c_i c_j N_{ij} = 1$$

and $N^{\ell}_{ij} = \int r^{2(\ell+1)} e^{-(r/b_i)^2} e^{-(r/b_j)^2} dr$

Finally, we can obtain the normalization constant;

$$A = \frac{1}{\sqrt{\sum_i \sum_j c_i c_j N_{ij}}} \quad . \tag{16}$$

After calculating normalization constant, we can find root-mean-square radius $\langle rms \rangle$. The root-mean-square radius can be obtained as;

$$\langle rms \rangle = \int u(r)^* r^2 \; u(r) dr \; .$$

Since, $u(r) = Ar^{(\ell+1)} \sum_j c_j e^{-(r/b_j)^2}$

$$\langle rms \rangle = \int A^* \sum_i c_i^* r^{\ell+1} e^{-\left(\frac{r}{b_i}\right)^2} r^2 A \sum_j c_j r^{\ell+1} e^{-\left(\frac{r}{b_j}\right)^2} dr \tag{17}$$

$$\langle r^2 \rangle = |A|^2 \sum_i \sum_j c_i^* c_j \frac{(2\ell+2)!!}{2^{(\ell+3)}} \frac{\sqrt{\pi}}{\left(\frac{1}{b_i^2} + \frac{1}{b_j^2}\right)^{(\ell+\frac{5}{2})}} \; . \tag{18}$$

We use the initial range parameter $b_1 = 0.1$, the final range parameter $b_N = 20.0$ and the number of basis N=40 for our calculation.

Results and Discussions

In order to understand the interaction between a Ξ^- hyperon and the remaining nucleons, we have plotted the Woods-Saxon central potentials, which are displayed in Fig. 1 (a). The potential strength is used -14.0 MeV in our calculation. In this figure, we can see that the interaction range is about 6 fm. Moreover, we also studied the characteristic of Coulomb interaction between Ξ^- hyperon and core nuclei; ^{12}C, ^{14}N and ^{16}O. In our calculation, the core nuclei are considered as uniformly charge distribution. Fig. 1 (b) represents this Coulomb interaction which is a long range forceFrom this figure, we can clearly see the difference between the finite size Coulomb interaction and point charge Coulomb interaction. According to our graph, while the potential strength for point charge Coulomb interaction is infinitely large, that for finite size Coulomb interaction is only -5.7 MeV for ^{14}N.

Fig. 1 (c) represents the finite size Coulomb interaction for ^{12}C, ^{14}N and ^{16}O. In this figure, the potential strengths are -5.2 MeV for ^{12}C, -5.7 MeV for ^{14}N and -6.2 MeV for ^{16}O respectively.

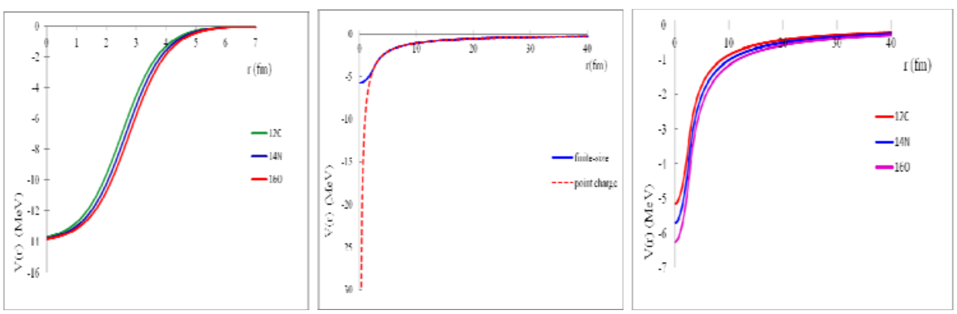
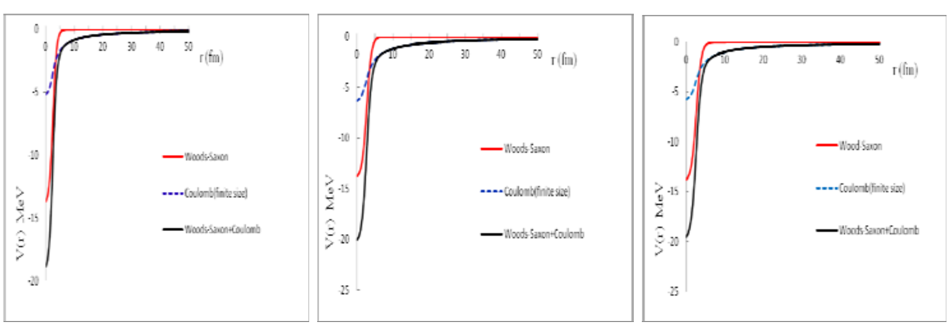


Fig. 1 (a) Woods-Saxon potential (b) Coulomb potential for finite size and point Charge (c) Coulomb potential for finite size

The Woods-Saxon potentials including Coulomb interaction for each Ξ^- hypernucleus are described in Fig. 2(a), (b) and (c). From our graph, we found that the Coulomb interaction is also a significant role for our consideration system.



(a) for ^{12}C (a) for ^{14}N (a) for ^{16}O

Fig. 2 Woods-Saxon potential including Coulomb interaction

The binding energies of light Ξ^- hypernuclei; Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O have calculated by using Coulomb potential and Woods-Saxon potential including Coulomb interaction. The interaction strength of Woods-Saxon potential -14.0 MeV is used for our calculation. The calculated results are described in Table (1). According to our results, the greater the atomic number of the core nucleus, the larger the binding energy of Ξ^- hypernuclei has occurred.

Table (1) Binding energy of light Ξ-hypernuclei by using $V_0 = -14.0$ MeV

Ξ^- Hypernuclei	Binding Energy(MeV) (only Coulomb Potential)	Binding Energy(MeV) (Woods-Saxon + Coulomb)
Ξ^--^{12}C	1.06	6.48
Ξ^--^{14}N	1.37	7.45
Ξ^--^{16}O	1.69	8.36

However, an event namely KISO which can recently determine the binding energy of Ξ^--^{14}N was observed from the emulsion E-373 experiment. This event is proposed by K.Nakazawa and his collaborators. It is the first evidence of bound state of Ξ^--^{14}N and its binding energy is 4.38 ± 0.25 MeV. Our calculated bound state energy of Ξ^--^{14}N is 3.07 MeV over binding than the experimental results. So, we have checked the value of binding energy of Ξ^--^{14}N by changing the potential strength of Woods-Saxon potential. The calculated results are described in Table (2). In the case of Coulomb potential, the binding energy of Ξ^--^{14}N is 1.37 MeV which is not negligible in the study of light Ξ^- hypernuclei.

Table (2) Binding energy of Ξ^--^{14}N with various potential strength of Woods-Saxon potential

Potential Strength (MeV)	Binding Energy (MeV) (Woods-Saxon + Coulomb Pot.)	Binding Energy (MeV) (only Coulomb Pot.)
-14	7.45	
-13	6.86	
-12	6.27	
-11	5.71	1.37
-10	5.16	
-9	4.64	
-8.5	4.38	

In addition, we have also found that while the potential strength is decreased by 1.0 MeV, the binding energy is also decreased by about 0.6 MeV. Furthermore, the binding energy of Ξ^--^{14}N becomes 4.38 MeV which is consistent with the experimental results at the potential strength -8.5 MeV. Therefore, binding energies of the other two Ξ^- hypernuclei, Ξ^--^{12}C and Ξ^--^{16}O, have been studied by using this potential strength -8.5 MeV.

Table (3), (4) and (5) give the binding energy for each possible state of light Ξ^- hypernuclei; Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O by using only Woods-Saxon potential, only Coulomb potential and Woods-Saxon potential including Coulomb interaction. Our calculated results are compared with the results calculated by M. Yamaguchi and his group. According to Table (3), the binding energies of each possible state of light Ξ^- hypernuclei by using Coulomb potential are in good agreement with their results. However, our calculated binding energy of each state by using Woods-Saxon including Coulomb potential is always smaller than that calculated by M. Yamaguchi's group with the use of Ehime potential except 3D and 2S state for ^{12}C and ^{14}N.

Table (3) The Binding energy of light Ξ^- hypernucleus (Ξ^--^{12}C)

State	Binding Energies of Ξ^--^{12}C (MeV) Our calculated results			Binding Energies of Ξ^--^{12}C(MeV) M. Yamaguchi's Results	
	V_W	V_C	$(V_W + V_C)$	V_C	$(V_C + V_{Ehime})$
1S	0.61	1.06	3.64	0.94	4.77
2P	-	0.32	0.37	0.28	0.58
3D	-	0.1302	0.1303	0.126	0.126
4F	-	0.03	0.03	-	-
2S	-	0.29	0.43	0.26	0.40
3P	-	0.11	0.14	0.13	0.19

Table (4) The Binding energy of light Ξ^- hypernucleus (Ξ^--^{14}N)

State	Binding Energies of Ξ^--^{14}N (MeV) Our calculated results			Binding Energies of Ξ^--^{14}N (MeV) M. Yamaguchi's Results	
	V_W	V_C	$(V_W + V_C)$	V_C	$(V_C + V_{Ehime})$
1S	0.82	1.37	4.38	1.22	5.93
2P	-	0.43	0.57	0.39	1.14
3D	-	0.1860	0.1863	0.174	0.174
4F	-	0.07	0.07	-	-
2S	-	0.38	0.55	0.34	0.54
3P	-	0.18	0.23	0.17	0.28

Table (5) The Binding energy of light Ξ^- hypernucleus (Ξ^--^{16}O)

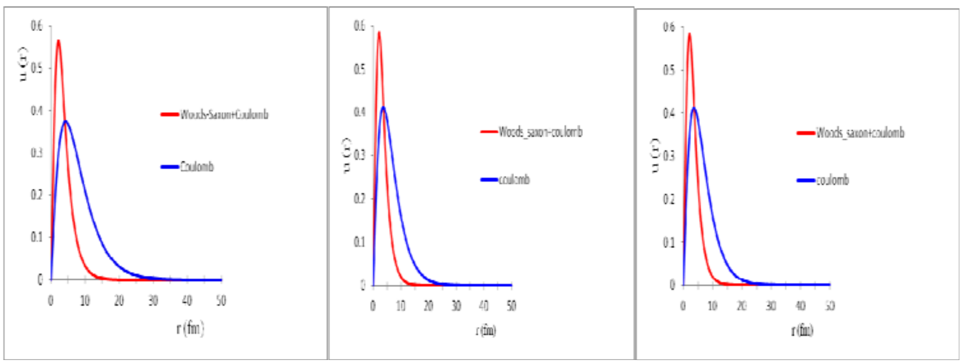
State	Binding Energies of Ξ^--^{16}O (MeV) Our calculated results			Binding Energies of Ξ^--^{16}O (MeV) M. Yamaguchi's Results	
	V_W	V_C	$(V_W + V_C)$	V_C	$(V_C + V_{Ehime})$
1S	1.02	1.69	5.09	1.51	7.07
2P	-	0.56	0.85	0.51	1.77
3D	-	0.2475	0.2483	0.230	0.232
4F	-	0.11	0.11	-	-
2S	-	0.49	0.69	0.44	0.72
3P	-	0.24	0.32	0.23	0.37

So, we have analyzed the interaction strength of Woods-Saxon potential including Coulomb term in order to compare Ehime potential which is included the Coulomb term. It is found that their interaction strength is about 1.3 times greater than ours for Ξ^--^{12}C and Ξ^--^{14}N and 1.4 times for Ξ^--^{16}O respectively. Moreover, binding energies of 3D and 4F states for light Ξ^- hypernuclei are determined almost entirely by the Coulomb potential. Therefore, 3D and 4F states are only pure Ξ^-- atomic states while the other S and P states are Coulomb-assisted nuclear Ξ^--bound states which is called hybrid state. Furthermore, the root-mean-squared radii of these light Ξ^- hypernuclei have also investigated for various states. The results are shown in Table (6). From our results, we can understand that the root-mean-squared radii calculated by using only Coulomb potential are represented by that for atomic state which captures Ξ^- hyperon by the corresponding atomic orbits.

Table (6) The root- mean- squared radii of Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O

State	Root-mean- square radii of Ξ^--^{12}C (fm) Our calculated results		Root- mean-square radii of Ξ^--^{14}N (fm) Our calculated results		Root-mean-square radii of Ξ^--^{16}O (fm) Our calculated results	
	V_C	$(V_W + V_C)$	V_C	$(V_W + V_C)$	V_C	$(V_W + V_C)$
1S	6.97	3.24	6.27	3.11	5.76	3.02
2P	18.58	15.06	16.04	11.22	14.09	8.61
3D	31.56	31.53	29.31	29.25	27.00	26.89
4F	38.06	38.06	36.79	36.79	35.48	35.48
2S	23.72	16.61	21.14	14.96	19.08	13.68
3P	34.82	32.93	33.22	29.44	31.26	36.53

In order to understand the behaviour of wave function of light Ξ-hypernuclei, we have plotted the wave functions by using Coulomb potential and Woods-Saxon potential including Coulomb interaction. Fig. 3 (a), (b) and (c) represent the behaviour of wave function of Ξ^--^{12}C, Ξ^--^{14}N and Ξ^--^{16}O. It is clearly seen that the wave function with the use of former potential is extended to the long range while the wave function rapidly decrease at the Woods-Saxon potential including Coulomb interaction. This behavior of the extended wave function shows a loosely bound system and it could be interpreted that it is the Ξ-atomic states. The wave function with the use of the latter can be understood that for the Ξ-nuclear states



(a) for Ξ- ^{12}C (a) for Ξ- ^{14}N (a) for Ξ- ^{16}O

Fig. 3 Behaviors of wave functions

Conclusion

We have calculated the binding energies and root-mean-square radii of the Ξ^- hypernuclei. We have solved nonrelativistic Schrödinger equation by using Gaussian basis wave function and phenomenological Woods-Saxon central potential including Coulomb interaction. In our calculation, the potential strength -14.0 MeV, r_0=1.1 fm and diffuseness parameter a=0.65 fm are used. Moreover, we have also studied the characteristic of Coulomb interaction between Ξ^- hyperon and core nuclei; ^{12}C, ^{14}N and ^{16}O. In this calculation, the core nuclei are considered as uniformly charge distribution. The calculated bound state energy of Ξ^--^{14}N is 3.07 MeV over binding than the experimental results, 4.38 ± 0.25 MeV. Therefore, the binding energies of Ξ^--^{14}N have checked by changing potential strength. At the potential depth -8.5 MeV, the calculated binding energy of Ξ^--^{14}N is in good agreement with the experimental result. The calculated binding energies for light Ξ^- hypernuclei with the use of Coulomb potential are in good agreement with the results presented by M. Yamaguchi's group. Moreover, the characteristics of wave function of light Ξ^- hypernuclei have investigated by using Coulomb potential and Woods-Saxon potential including Coulomb interaction. The behaviour of wave function represented by Ξ-atomic states is extended to the long range while that by Ξ-nuclear states rapidly decreases.

Acknowledgements

The authors would like to acknowledge Dr Thida Win, Acting Rector, University of Mandalay, for her encouragement. The author wish to express my thanks to Professor Dr Lei Lei Win, Head of Department of Physics, University of Mandalay, for permission and also would like to extend my thanks to Prof. K. Nakazawa for his useful discussion.

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