USING MATHEMATICS TO SOLVE REAL WORLD PROBLEMS

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Abstract

In this paper, to study the applications of Linear Programming(The BIG M Method) in Business

problems. First we mention construct the Linear Programming Model (LP Model). Secondly,

Slack variables, Surplus Variables, Artificial Variables and Modified Problem. Then by using the

BIG M Method to solve the maximization and minimization of real life Business Problems.

Key Words : Linear Programming (L P), The Big M Method

1. INTRODUCTION

An algebraic method of solving the standard form of a linear programming problem

(l.p.p) which allows the solution of multivariable problems.

1.1 Objectives

This study examined relationships between quality of life in people and cognitive

functioning in both abstract and real-world problem solving.

1.2 A Standard Maximization Problem in Standard Form

A linear programming problem(l p p) is said to be a standard maximization

problem in standard form if its mathematical model is of the following form:

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Maximize the objective function

$$P = c_1x_1 + c_2x_2 + \cdots\cdots\cdots + c_nx_n$$

Subject to the problem constraints of the form

$$a_1x_1 + a_2x_2 + \cdots\cdots\cdots + a_nx_n \leq b \quad , \quad b \geq 0$$

With non-negative constraints

$$x_1, x_2, x_3, \cdots\cdots\cdots, x_n \geq 0$$

1.3   Slack variables

To adopt a linear programming problem to the matrix methods used in the

simplex process, we convert the problem constraint inequalities into a system of linear equations

by using a simple device called a **slack variable**. In particular, to convert the system of problem

constraint inequalities from

$$x_1 + 2x_2 \leq 32$$

$$3x_1 + 4x_2 \leq 84 \qquad \text{--------------- } *$$

into a system of equations, we add variables s_1 and s_2 to the left sides of * to obtain

$$x_1 + 2x_2 + s_1 \qquad = 32$$

$$3x_1 + 4x_2 \qquad + s_2 = 84$$

The variables s_1 and s_2 are called **slack variables** because each makes up the difference (take

up the slack) between the left and right sides of the inequalities in *.

1.4 An Introduction to the BIG M Method

We introduce the big M method through a simple maximization problem with

mixed problem constraints. The key parts of the method will then be summarized and applied to

more complex problems.

Consider the following problem:

$$\text{Maximize} \quad P = 2x_1 + x_2$$

$$\text{Subject to} \quad x_1 + x_2 \leq 10$$

$$-x_1 + x_2 \geq 2 \qquad \text{------------(1)}$$

$$x_1, x_2 \geq 0$$

To form an equation out of the first inequality, we introduce a slack variable s_1, as before, and

write $\qquad x_1 + x_2 + s_1 = 10$

How can we form an equation out of the second inequality ? We introduce a second variable s_2

and subtract it form the left side so that we can write

$$-x_1 + x_2 - s_2 = 2$$

The variable s_2 is called *a surplus variable*, because it is the amount (surplus) by which the left

side of the inequality exceeds the right side.

We now express the linear programming problem (1) as a system of equations

$$x_1 + x_2 + s_1 = 10$$

$$-x_1 + x_2 - s_2 = 2$$

$$-2x_1 - x_2 + P = 0 \qquad \text{------------- (2)}$$

$$x_1, x_2, s_1, s_2 \geq 0$$

It can be shown that a basic solution of (2) is not feasible if any of the variables (excluding P)

are negative. Thus, a surplus variable is required to satisfy the non-negative constraint.

The basic solution found by setting the nonbasic variables x_1 *and* x_2 equal to 0 is

$$x_1 = 0, \ x_2 = 0, \ s_1 = 10, \ s_2 = -2, \ P = 0$$

But this basic solution is not feasible, since the surplus variable s_2 is negative (which is a violation of the nonnegative requirements of all variables except P). The simplex method works only when the basic solution for a tableau is feasible, so we cannot solve this problem simply by writing the tableau for (2) and starting pivot operations.

In order to use the simplex method on problems with mixed constraints, we turn to an ingenious device called an *artificial variable.* This variable has no physical meaning in the original problem (which explains the use of the word " *artificial* ") and is introduced solely for the purpose of obtaining a basic feasible solution so that we can apply the simplex method. An *artificial variable* is a variable introduced into each equation that has a surplus variable. As before, to ensure that we consider only feasible basic solutions, *an artificial variable is required to satisfy the nonnegative constraint.*

Returning to the problem at hand, we introduce an artificial variable a_1 into the equation involving the surplus variable s_2 :

$$-x_1 + x_2 - s_2 + a_1 = 2$$

To prevent an artificial variable from becoming part of an optimal solution to the original problem, a very large " penalty " is introduced into the objective function. This penalty is created by choosing a positive constant M so large that the artificial variable is forced to be 0 in any final optimal solution of the original problem. We then add the term $-Ma_1$ to the objective function :

$$P = 2x_1 + x_2 - Ma_1$$

We now have a new problem , which we call the *modified problem*:

$$\text{Maximize} \quad P = 2x_1 + x_2 - Ma_1$$

$$\text{Subject to} \quad x_1 + x_2 + s_1 \quad\quad\quad\quad = 10$$

$$-x_1 + x_2 \quad\quad - s_2 + a_1 = 2 \quad\quad\text{----------------- (3)}$$

$$x_1, \ x_2, \ s_1, \ s_2, \ a_1 \ \geq \ 0$$

The initial system for the modified problem (3) is

$$x_1 + x_2 + s_1 \quad\quad\quad\quad\quad\quad = 10$$

$$-x_1 + x_2 \quad\quad - s_2 + a_1 \quad\quad = 2$$

$$-2x_1 - x_2 \quad\quad\quad\quad + Ma_1 + P \quad = 0 \quad\text{----------- (4)}$$

$$x_1, \ x_2, \ s_1, \ s_2, \ a_1 \ \geq \ 0$$

We next write the augmented coefficient matrix for (4), which we call the **preliminary simplex tableau** for the modified problem .

2. THE BIG M METHOD

The Big M Method – Introducing Slack, Surplus and Artificial Variables to Form the

Modified Problem

Step 1 - If any problem constraints have negative constants on the right side, multiply

both sides by (-1) to obtain a constraint with a nonnegative constant. (If the

constraint is an inequality , this will reverse the direction of the inequality.)

Step 2 - Introduce a slack variable in each $\leq$ constraint.

Step 3 - Introduce a surplus variable and an artificial variable in each $\geq$ constraint.

Step 4 - Introduce an artificial variable in each $=$ constraint.

Step 5 - For each artificial variable a_i , add $-Ma_i$ to the objective function .

Use the same constant M for all artificial variables.

2.1 Example Find the modified problem for the following linear programming problem.

(Do not attempt to solve the problem)

Maximize $P = 2x_1 + 5x_2 + 3x_3$

Subject to $\quad x_1 + 2x_2 - x_3 \leq 7$

$$-x_1 + x_2 - 2x_3 \leq -5$$

$$x_1 + 4x_2 + 3x_3 \geq 1$$

$$2x_1 - x_2 + 4x_3 = 6$$

$$x_1, x_3, x_3 \geq 0$$

Solution :

First, we multiply the second constraint by -1 to change -5 to 5.

Next,

We introduce the slack, surplus, and artificial variables according to the rules stated in the box

$$x_1 + 2x_2 - x_3 + s_1 \qquad\qquad\qquad = 7$$

$$x_1 - x_2 + 2x_3 \quad - s_2 + a_1 \qquad\qquad = 5$$

$$x_1 + 4x_2 + 3x_3 \qquad\qquad - s_3 + a_2 \quad = 1$$

$$2x_1 - x_2 + 4x_3 \qquad\qquad\qquad + a_3 = 6$$

$$x_1, x_2, x_3, s_1, s_2, s_3, a_1, a_2, a_3 \quad \geq 0$$

Finally,

We add $-Ma_1, -Ma_2 \ and \ -Ma_3$ to the objective function :

$$P = 2x_1 + 5x_2 + 3x_3 - Ma_1 - Ma_2 - Ma_3$$

The modified problem is

Maximize $P = 2x_1 + 5x_2 + 3x_3 - Ma_1 - Ma_2 - Ma_3$

Subject to $\quad x_1 + 2x_2 - x_3 + s_1 \qquad\qquad\qquad\qquad = 7$

$\qquad\qquad\qquad x_1 - x_2 + 2x_3 \quad - s_2 + a_1 \qquad\qquad = 5$

$\qquad\qquad\qquad x_1 + 4x_2 + 3x_3 \qquad\qquad - s_3 + a_2 \quad = 1$

$\qquad\qquad 2x_1 - x_2 + 4x_3 \qquad\qquad\qquad\qquad + a_3 = 6$

$\qquad\qquad\qquad x_1, x_2, x_3, s_1, s_2, s_3, a_1, a_2, a_3 \quad \geq 0$

2.2 The Big M Method − Solving the Problem

STEP – 1 From the preliminary simplex tableau for the modified problem.

STEP – 2 Use row operations to eliminate the M 's in the bottom row of the preliminary

simplex tableau in the columns corresponding to the artificial variables. The

resulting tableau is the initial simplex tableau.

STEP – 3 Solve the modified problem by applying the simplex method to the initial simplex

tableau found in STEP – 2.

STEP – 4 Relat the solution of the modified problem to the original problem.

(A) If the modified problem has no solution, then the original problem has no solution.

(B) If all artificial variables are 0 in the solution to the modified problem, the delete the

artificial variables to find a solution to the original problem.

(C) If any artificial variables are nonzero in the solution to the modified problem, then the
original problem has no solution.

2.3 Petroleum Blending

A refinery produces two grades of gasoline, regular and premium, by blending together two

components, A and B. Component A has an octane rating of 90 and costs $28 a barrel.

Component B has an octane rating of 110 and costs $32 a barrel . The octane rating for regular

gasoline must be at least 95, and the octane rating for premium must be at least 105. Regular

gasoline sells for $34 a barrel and premium sells for $40 a barrel. Currently, The company has

30,000 barrels of component A and 20,000 barrels of component B. It also has orders for 20,000

barrel s of regular and 10,000 barrels of premium that must be filled. Assuming that all the

gasoline produced can be sold, determine the maximum possible profit .

Solution:

First we organize the information given in the problem in table

Component	Octane Rating	Cost	Available Supply
A	90	$28	30,000 barrels
B	110	$32	20,000 barrels

Grade	Minimum Octane Rating	Selling Price	Existing orders
Regular	95	$34	20,000 barrels
Premium	105	$40	10,000 barrels

Let x_1 = Number of barrels of component A used in regular gasoline

x_2 = Number of barrels of component A used in premium gasoline

x_3 = Number of barrels of component B used in regular gasoline

x_4 = Number of barrels of component B used in premium gasoline

The total amount of component A used is $x_1 + x_2$.

$$x_1 + x_2 \ \leq \ 30,000$$

The total amount of component B used is $x_3 + x_4$.

$$x_3 + x_4 \leq 20{,}000$$

The amount of regular and premium gasoline produced must be sufficient to meet the existing

orders

$$x_1 + x_3 \geq 20{,}000 \qquad \text{Regular}$$

$$x_2 + x_4 \geq 10{,}000 \qquad \text{Premium}$$

Now consider the octane ratings.

The octane rating of regular gasoline must be at least 95 ; thus

$$90 \frac{x_1}{x_1 + x_3} + 110 \frac{x_3}{x_1 + x_3} \geq 95$$

$$90x_1 + 110 x_3 \geq 95(x_1 + x_3)$$

$$90x_1 + 110 x_3 \geq 95x_1 + 95x_3$$

$$-5x_1 + 15x_3 \geq 0$$

The octane rating of premium gasoline must be at least 105; thus

$$90 \frac{x_2}{x_2 + x_4} + 110 \frac{x_4}{x_2 + x_4} \geq 105$$

$$90x_2 + 110 x_4 \geq 105(x_2 + x_4)$$

$$90x_2 + 110 x_4 \geq 105x_2 + 105x_4$$

$$-15x_2 + 5x_4 \geq 0$$

The cost of the components used is

$$C = 28(x_1 + x_2) + 32 (x_3 + x_4)$$

The revenue from selling all the gasoline is

$$R = 34(x_1 + x_3) + 40 (x_2 + x_4)$$

The profit is

$$P = R - C$$

$$= 34(x_1 + x_3) + 40 (x_2 + x_4) - 28(x_1 + x_2) - 32 (x_3 + x_4)$$

$$= (34 - 28)x_1 + (40 - 28)x_2 + (34 - 32)x_3 + (40 - 32)x_4$$

$$= 6x_1 + 12x_2 + 2x_3 + 8x_4$$

To find the maximum profit , we must solve the following linear programming problem

Maximize $P = 6x_1 + 12x_2 + 2x_3 + 8x_4$

Subject to $x_1 + x_2$ $\leq 30,000$

$x_3 + x_4$ $\leq 20,000$

$x_1 + x_3$ $\geq 20,000$

$x_2 + x_4$ $\geq 10,000$

$-5x_1 + 15x_3$ ≥ 0

$-15x_2 + 5x_4$ ≥ 0

and $x_1, x_2, x_3, x_4 \geq 0$

Maximize $P = 6x_1 + 12x_2 + 2x_3 + 8x_4 - Ma_1 - Ma_2 - Ma_3 - Ma_4$

Subject to $x_1 + x_2 \qquad\quad + s_1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad = 30{,}000$

$\qquad\qquad\quad x_3 + x_4 \quad + s_2 \qquad\qquad\qquad\qquad\qquad\qquad = 20{,}000$

$\quad x_1 \quad + x_3 \qquad\qquad - s_3 + a_1 \qquad\qquad\qquad\qquad\quad = 20{,}000$

$\qquad\quad x_2 \quad + x_4 \qquad\qquad\qquad - s_4 + a_2 \qquad\qquad\quad = 10{,}000$

$-5x_1 \quad + 15x_3 \qquad\qquad\qquad\qquad\qquad - s_5 + a_3 \qquad = 0$

$\qquad -15x_2 \quad + 5x_4 \qquad\qquad\qquad\qquad\qquad\qquad - s_6 + a_4 = 0$

$x_1,\ x_2,\ x_3,\ x_4\ ,a_1,\ a_2,\ a_3,\ a_4,\ s_1,\ s_2,\ s_3,\ s_4,\ s_5,\ s_6\ \geq\ 0$

$x_1 + x_2 \qquad\qquad\qquad + s_1 \qquad\qquad\qquad\qquad\qquad\qquad = 30{,}000$

$\qquad\qquad x_3 + x_4 \qquad + s_2 \qquad\qquad\qquad\qquad\qquad\qquad = 20{,}000$

$\quad x_1 \qquad + x_3 \qquad\qquad - s_3 + a_1 \qquad\qquad\qquad\qquad = 20{,}000$

$\qquad x_2 \qquad + x_4 \qquad\qquad\qquad - s_4 + a_2 \qquad\qquad\quad = 10{,}000$

$-5x_1 \quad + 15x_3 \qquad\qquad\qquad\qquad\qquad - s_5 + a_3 \qquad = 0$

$\qquad -15x_2 \quad + 5x_4 \qquad\qquad\qquad\qquad\qquad\qquad - s_6 + a_4 = 0$

$-6x_1 - 12x_2 - 2x_3 - 8x_4 + Ma_1 + Ma_2 + Ma_3 + Ma_4 + P \qquad = 0$

$x_1,\ x_2,\ x_3,\ x_4\ ,a_1,\ a_2,\ a_3,\ a_4,\ s_1,\ s_2,\ s_3,\ s_4,\ s_5,\ s_6\ \geq\ 0$

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
s_1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	Eliminate M
s_2	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	20,000	from a_1 column
a_1	1	0	1	0	0	0	-1	1	0	0	0	0	0	0	0	20,000	
a_2	0	1	0	1	0	0	0	0	-1	1	0	0	0	0	0	10,000	
a_3	-5	0	15	0	0	0	0	0	0	0	-1	1	0	0	0	0	
a_4	0	-15	0	5	0	0	0	0	0	0	0	0	-1	1	0	0	
P	-6	-12	-2	- 8	0	0	0	M	0	M	0	M	0	M	1	0	$R_7 + (-M)R_3 \to R_7$

$\sim$

x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	Eliminate M
0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	20,000	from a_2 column
1	0	1	0	0	0	-1	1	0	0	0	0	0	0	0	20,000	
0	1	0	1	0	0	0	0	-1	1	0	0	0	0	0	10,000	
- 5	0	15	0	0	0	0	0	0	0	-1	1	0	0	0	0	
0	-15	0	5	0	0	0	0	0	0	0	0	-1	1	0	0	
-M-6	-12	-M-2	-8	0	0	M	0	0	M	0	M	0	M	1	-20,000M	$R_7 + (-M)R_4 \to R_7$

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
s_1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	Eliminate M
s_2	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	20,000	from a_3 column
a_1	1	0	1	0	0	0	-1	1	0	0	0	0	0	0	0	20,000	
a_2	0	1	0	1	0	0	0	0	-1	1	0	0	0	0	0	10,000	
a_3	-5	0	15	0	0	0	0	0	0	0	-1	1	0	0	0	0	
a_4	0	-15	0	5	0	0	0	0	0	0	0	0	-1	1	0	0	
P	-M-6	-M-12	-M-2	-M-8	0	0	M	0	M	0	0	M	0	M	1	-30,000M	$R_7 + (-M)R_5 \to R_7$

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	Eliminate M
	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	20,000	from a_4 column
$\sim$	1	0	1	0	0	0	-1	1	0	0	0	0	0	0	0	20,000	
	0	1	0	1	0	0	0	0	-1	1	0	0	0	0	0	10,000	
	-5	0	15	0	0	0	0	0	0	0	-1	1	0	0	0	0	
	0	-15	0	5	0	0	0	0	0	0	0	0	-1	1	0	0	
	4M-6	-M-12	-16M-2	-M-8	0	0	M	0	M	0	M	0	0	M	1	-30,000M	$R_7 + (-M)R_6 \to R_7$

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	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
s_1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	
s_2	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	20,000	
a_1	1	0	1	0	0	0	−1	1	0	0	0	0	0	0	0	20,000	
a_2	0	1	0	1	0	0	0	0	−1	1	0	0	0	0	0	10,000	
Exit → a_3	−5	0	(15)	0	0	0	0	0	0	0	−1	1	0	0	0	0	$\frac{1}{15}R_5 \to R_5$
a_4	0	−15	0	5	0	0	0	0	0	0	0	0	−1	1	0	0	
P	4M-6	14M-12	-16M-2	-6M-8	0	0	M	0	M	0	M	0	M	0	1	-30,000M	

x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	
0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	20,000	$R_2 - R_5 \to R_2$
1	0	1	0	0	0	−1	1	0	0	0	0	0	0	0	20,000	$R_3 - R_5 \to R_3$
0	1	0	1	0	0	0	0	−1	1	0	0	0	0	0	10,000	
$-\frac{1}{3}$	0	(1)	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0	
0	-15	0	5	0	0	0	0	0	0	0	0	−1	1	0	0	
4M-6	14M-12	-16M-2	-6M-8	0	0	M	0	M	0	M	0	M	0	1	-30,000M	$R_7 + (16M + 2)R_5 \to R_7$

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First tableau:

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P	
s_1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000
s_2	$\frac{1}{3}$	0	0	1	0	1	0	0	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000
a_1	$\frac{4}{3}$	0	0	0	0	0	-1	1	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000
a_2	0	1	0	1	0	0	0	0	-1	1	0	0	0	0	0	10,000
x_3	$-\frac{1}{3}$	0	1	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0
Exit → a_4	0	-15	0	(5)	0	0	0	0	0	0	0	0	-1	1	0	0
P	$\frac{-4M-20}{3}$	14M-12	0	-6M-8	0	0	M	0	M	0	$\frac{-M-2}{15}$	$\frac{16M+2}{15}$	M	0	1	$-30,000M$

$\frac{1}{5}R_6 \to R_6$

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Second tableau:

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x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	
$\frac{1}{3}$	0	0	1	0	1	0	0	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000	$R_2 - R_6 \to R_2$
$\frac{4}{3}$	0	0	0	0	0	-1	1	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000	
0	1	0	1	0	0	0	0	-1	1	0	0	0	0	0	10,000	$R_4 - R_6 \to R_4$
$-\frac{1}{3}$	0	1	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0	
0	-3	0	(1)	0	0	0	0	0	0	0	0	$-\frac{1}{5}$	$\frac{1}{5}$	0	0	
$\frac{-4M-20}{3}$	14M-12	0	-6M-8	0	0	M	0	M	0	$\frac{-M-2}{15}$	$\frac{16M+2}{15}$	M	0	1	$-30,000M$	$R_7 + (6M + 8)R_6 \to R_7$

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	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
s_1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	
s_2	$\frac{1}{3}$	3	0	0	0	1	0	0	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{5}$	$-\frac{1}{5}$	0	20,000	
a_1	$\frac{4}{3}$	0	0	0	0	0	-1	1	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000	
Exit → a_2	0	④	0	0	0	0	0	0	-1	1	0	0	$\frac{1}{5}$	$-\frac{1}{5}$	0	10,000	$\frac{1}{4}R_4 \to R_4$
x_3	$-\frac{1}{3}$	0	1	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0	
x_4	0	-3	0	1	0	0	0	0	0	0	0	0	$-\frac{1}{5}$	$\frac{1}{5}$	0	0	
P	$\frac{-4M-20}{3}$	-4M-36	0	0	0	0	M	0	M	0	$\frac{-M-2}{15}$	$\frac{16M+2}{15}$	$\frac{-M-8}{5}$	$\frac{6M+8}{5}$	1	$-30{,}000M$	

~

x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	30,000	$R_1 - R_4 \to R_1$
$\frac{1}{3}$	3	0	0	0	1	0	0	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{5}$	$-\frac{1}{5}$	0	20,000	$R_2 - 3R_4 \to R_2$
$\frac{4}{3}$	0	0	0	0	0	-1	1	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000	
0	①	0	0	0	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	2,500	
$-\frac{1}{3}$	0	1	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0	
0	-3	0	1	0	0	0	0	0	0	0	0	$-\frac{1}{5}$	$\frac{1}{5}$	0	0	$R_6 + 3R_4 \to R_6$
$\frac{-4M-20}{3}$	- 4M-36	0	0	0	0	M	0	M	0	$\frac{-M-2}{15}$	$\frac{16M+2}{15}$	$\frac{-M-8}{5}$	$\frac{6M+8}{5}$	1	$-30{,}000M$	$R_7 + (4M + 36)R_4 \to R_7$

↓ Enter

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P	
s_1	1	0	0	0	1	0	0	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	27,500
s_2	$\frac{1}{3}$	0	0	0	0	1	0	0	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{20}$	$-\frac{1}{20}$	0	12,500
Exit → a_1	(4/3)	0	0	0	0	0	-1	1	0	0	$\frac{1}{15}$	$-\frac{1}{15}$	0	0	0	20,000
x_2	0	1	0	0	0	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	2,500
x_3	$-\frac{1}{3}$	0	1	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0
x_4	0	0	0	1	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	7,500
P	$\frac{-4M-20}{3}$	0	0	0	0	0	M	0	-9	M+9	$\frac{-M-2}{15}$	$\frac{16M+2}{15}$	$\frac{1}{5}$	$\frac{5M-1}{5}$	1	$-20{,}000M+90{,}000$

$\frac{3}{4}R_3 \rightarrow R_3$

~

x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
1	0	0	0	1	0	0	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	27,500	$R_1 - R_3 \rightarrow R_1$
$\frac{1}{3}$	0	0	0	0	1	0	0	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{20}$	$-\frac{1}{20}$	0	12,500	$R_2 + (-\frac{1}{3})R_3 \rightarrow R_2$
(1)	0	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0	15,000	
0	1	0	0	0	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	2,500	
$-\frac{1}{3}$	0	1	0	0	0	0	0	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	0	0	0	0	$R_5 + \frac{1}{3}R_3 \rightarrow R_5$
0	0	0	1	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	7,500	
$\frac{-4M-20}{3}$	0	0	0	0	0	M	0	-9	M+9	$\frac{-M-2}{15}$	$\frac{16M+2}{15}$	$\frac{1}{5}$	$\frac{5M-1}{5}$	1	$-20{,}000M+90{,}000$	$R_7 + \frac{4M+20}{3}R_3 \rightarrow R_7$

↓ Enter

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
s_1	0	0	0	0	1	0	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{20}$	$\frac{1}{20}$	$-\frac{1}{20}$	$\frac{1}{20}$	0		12,500
Exit → s_2	0	0	0	0	0	1	$\frac{1}{4}$	$-\frac{1}{4}$	$\boxed{3/4}$	$-\frac{3}{4}$	$\frac{1}{20}$	$-\frac{1}{20}$	$\frac{1}{20}$	$-\frac{1}{20}$	0		7,500	$\frac{4}{3}R_2 \to R_2$
x_1	1	0	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0		15,000
x_2	0	1	0	0	0	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0		25,000
x_3	0	0	1	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	0	0		5,000
x_4	0	0	0	1	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0		7,500
P	0	0	0	0	0	0	-5	M+5	-9	M+9	$\frac{1}{5}$	$\frac{5M-3}{15}$	$\frac{1}{5}$	$\frac{5M-1}{5}$	1		190,000

$\sim$

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P			
	0	0	0	0	1	0	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{20}$	$\frac{1}{20}$	$-\frac{1}{20}$	$\frac{1}{20}$	0		12,500	$R_1 - \frac{1}{4}R_2 \to R_1$
	0	0	0	0	0	$\frac{4}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\boxed{1}$	-1	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{15}$	$-\frac{1}{15}$	0		10,000	
	1	0	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0		15,000	
	0	1	0	0	0	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0		25,000	$R_4 + \frac{1}{4}R_2 \to R_4$
	0	0	1	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	0	0		5,000	
	0	0	0	1	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0		7,500	$R_6 + \frac{3}{4}R_2 \to R_6$
	0	0	0	0	0	0	-5	M+5	-9	M+9	$\frac{1}{5}$	$\frac{5M-3}{15}$	$\frac{1}{5}$	$\frac{5M-1}{5}$	1		190,000	$R_7 + 9R_2 \to R_7$

↓ Enter

Exit → s_1, enter s_3. First simplex tableau:

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
Exit → s_1	0	0	0	0	1	$-\frac{1}{3}$	($2/3$)	$-\frac{2}{3}$	0	0	$-\frac{1}{15}$	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{15}$	0	10,000	$\frac{3}{2}R_1 \to R_1$
s_4	0	0	0	0	0	$\frac{4}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	1	-1	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{15}$	$-\frac{1}{15}$	0	10,000	
x_1	1	0	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0	15,000	
x_2	0	1	0	0	0	$\frac{1}{3}$	$\frac{1}{12}$	$-\frac{1}{12}$	0	0	$\frac{1}{60}$	$-\frac{1}{60}$	$\frac{1}{15}$	$-\frac{1}{15}$	0	5,000	
x_3	0	0	1	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	0	0	5,000	
x_4	0	0	0	1	0	1	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0	15,000	
P	0	0	0	0	0	12	-2	M+2	0	M	$\frac{4}{5}$	$\frac{5M-12}{15}$	$\frac{4}{5}$	$\frac{5M-4}{5}$	1	280,000	

$\sim$

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	a_1	s_4	a_2	s_5	a_3	s_6	a_4	P		
	0	0	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	(1)	-1	0	0	$-\frac{1}{10}$	$\frac{1}{10}$	$-\frac{1}{10}$	$\frac{1}{10}$	0	15,000	
	0	0	0	0	0	$\frac{4}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	1	-1	$\frac{1}{15}$	$-\frac{1}{15}$	$\frac{1}{15}$	$-\frac{1}{15}$	0	10,000	$R_2 - \frac{1}{3}R_1 \to R_2$
	1	0	0	0	0	0	$-\frac{3}{4}$	$\frac{3}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0	15,000	$R_3 + \frac{3}{4}R_1 \to R_3$
	0	1	0	0	0	$\frac{1}{3}$	$\frac{1}{12}$	$-\frac{1}{12}$	0	0	$\frac{1}{60}$	$-\frac{1}{60}$	$\frac{1}{15}$	$-\frac{1}{15}$	0	5,000	$R_4 - \frac{1}{12}R_1 \to R_4$
	0	0	1	0	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$-\frac{1}{20}$	$\frac{1}{20}$	0	0	0	5,000	$R_5 + \frac{1}{4}R_1 \to R_5$
	0	0	0	1	0	1	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{1}{20}$	$-\frac{1}{20}$	0	0	0	15,000	$R_6 - \frac{1}{4}R_1 \to R_6$
	0	0	0	0	0	12	-2	M+2	0	M	$\frac{4}{5}$	$\frac{5M-12}{15}$	$\frac{4}{5}$	$\frac{5M-4}{5}$	1	280,000	$R_7 + 2R_1 \to R_7$

$$
\begin{array}{c}
\begin{array}{c} s_3 \\ s_4 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ \\ P \end{array}
\left(
\begin{array}{cccccccccccccc|c}
0 & 0 & 0 & 0 & \frac{3}{2} & -\frac{1}{2} & 1 & -1 & 0 & 0 & -\frac{1}{10} & \frac{1}{10} & -\frac{1}{10} & \frac{1}{10} & 0 & 15{,}000 \\[6pt]
0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{3}{2} & 0 & 0 & 1 & -1 & \frac{1}{10} & -\frac{1}{10} & \frac{1}{10} & -\frac{1}{10} & 0 & 5{,}000 \\[6pt]
1 & 0 & 0 & 0 & \frac{9}{8} & -\frac{3}{8} & 0 & 0 & 0 & 0 & -\frac{1}{40} & \frac{1}{40} & -\frac{3}{40} & \frac{3}{40} & 0 & 26{,}250 \\[6pt]
0 & 1 & 0 & 0 & -\frac{1}{8} & \frac{3}{8} & 0 & 0 & 0 & 0 & \frac{1}{40} & -\frac{1}{40} & \frac{3}{40} & -\frac{3}{40} & 0 & 3{,}750 \\[6pt]
0 & 0 & 1 & 0 & \frac{3}{8} & -\frac{1}{8} & 0 & 0 & 0 & 0 & -\frac{3}{40} & \frac{3}{40} & -\frac{1}{40} & \frac{1}{40} & 0 & 8{,}750 \\[6pt]
0 & 0 & 0 & 1 & -\frac{3}{8} & \frac{9}{8} & 0 & 0 & 0 & 0 & \frac{3}{40} & -\frac{3}{40} & \frac{1}{40} & -\frac{1}{40} & 0 & 11{,}250 \\[6pt]
\hline
0 & 0 & 0 & 0 & 3 & 11 & 0 & M & 0 & M & \frac{3}{5} & \frac{5M-9}{15} & \frac{3}{5} & \frac{5M-3}{5} & 1 & 310{,}000
\end{array}
\right)
\end{array}
$$

$$\therefore \; x_1 = 26{,}250, \quad x_2 = 3{,}750, \quad x_3 = 8{,}750, \quad x_4 = 11{,}250, \quad s_3 = 15{,}000, \; s_4 = 5{,}000$$

$$s_1 = s_2 = s_5 = s_6 = a_1 = a_2 = a_3 = a_4 = 0 \quad , \quad P = 310{,}000$$

The refinery should blend 26,250 barrels of component A and 8,750 barrels of component B to produce 35,000 barrels of regular.

They should blend 3,750 barrels of component A and 11,250 barrels of component B to produce 15,000 barrels of premium.

They will result in a maximum profit of $310,000.

3. Minimization By The BIG M Method

The Big M method can be used to solve minimization problems. To minimize an objective function, we have only to maximize its negative.   Figure illustrates the fact that the minimum value of a function f occurs at the same point as the maximum value of the function $-f$. Furthermore, if m is the minimum value of f , then $-m$ is the maximum value of $-f$, and conversely. Thus, we can find the minimum value of a function f by finding the maximum value of $-f$ and then changing the sign of the maximum value.
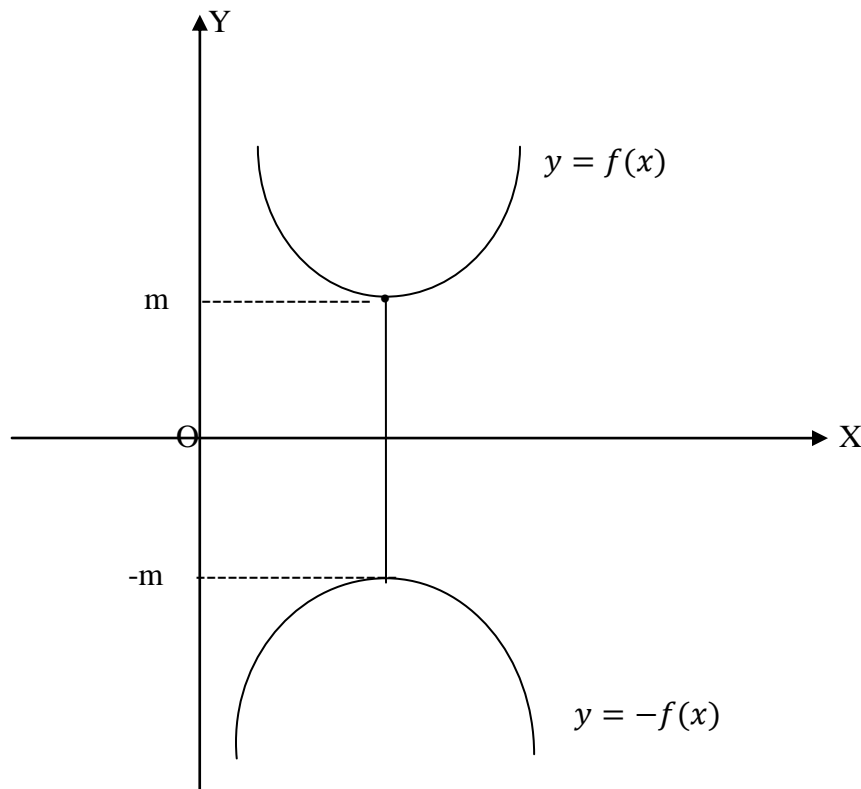
3.1 Manufacturing – Production Scheduling

A tire company has plants in Chicago and Detroit . The Chicago plant can make 600 radials and 100 standard tires per day . The Detroit plant can make 300 radials and 100 standard tires per day. Its costs $20,000 per day to operate the Chicago plant and $15,000 per day to operate the

Detroit plant. The company has a contract to make at least 24,000 radials and 5,000 standard

tires. How many days should each plant be scheduled to minimize operating costs?

solution

Let x = number of days the Chicago plant operate

y = number of days the Detroit plant operate

The number of radial tires produced is $600\,x + 300\,y$

The number of standard tires produced is $100\,x + 100\,y$

The operating expenses are $20,000\,x + 1,500\,y$

The linear programming problem is as follows:

$$\text{Minimize } C = 20,000\,x + 15,000\,y$$

$$\text{Subject to} \quad 600x + 300y \ge 24,000$$

$$100x + 100y \ge 5,000$$

$$x, y \ge 0$$

$$\text{Maximize } P = -C = -20,000\,x - 15,000\,y$$

$$\text{Subject to} \quad 600x + 300y \ge 24,000$$

$$100x + 100y \ge 5,000$$

$$x, y \ge 0$$

State the modified problem

$$\text{Maximize } P = -20,000\,x - 15,000\,y - Ma_1 - Ma_2$$

$$\text{Subject to} \quad 600x + 300y - s_1 + a_1 \qquad\qquad = 24,000$$

$$100x + 100y \qquad\qquad - s_2 + a_2 = 5,000$$

$$x,\ y,\ s_1,\ s_2,\ a_1,\ a_2 \ge 0$$

$$600x + 300y - s_1 + a_1 \qquad\qquad = 24{,}000$$

$$100x + 100y \qquad\quad - s_2 + a_2 \quad = 5{,}000$$

$$20{,}000\,x + 15{,}000\,y + Ma_1 + Ma_2 + P \quad = 0$$

	x	y	s_1	a_1	s_2	a_2	p		
a_1	600	300	-1	1	0	0	0	24,000	Eliminate M from
a_2	100	100	0	0	-1	1	0	5,000	a_1 column
	-----	-----	-----	-----	-----	-----	-----	-----	
P	20,000	15,000	0	M	0	M	1	0	$R_3 + (-M)R_1 \to R_3$

$$\sim$$

	x	y	s_1	a_1	s_2	a_2	p		
	600	300	-1	1	0	0	0	24,000	Eliminate M from
	100	100	0	0	-1	1	0	5,000	a_2 column
	-----	-----	-----	-----	-----	-----	-----	-----	
	$-600M$ $+20{,}000$	$-300M$ $+15{,}000$	M	0	0	M	1	$-24{,}000M$	$R_3 + (-M)R_2 \to R_3$

$\downarrow$ Enter

	x	y	s_1	a_1	s_2	a_2	p		
Exit $\to$ a_1	600	300	-1	1	0	0	0	24,000	$\frac{1}{600}R_1 \to R_1$
a_2	100	100	0	0	-1	1	0	5,000	
	-----	-----	-----	-----	-----	-----	-----	-----	
P	$-700M$ $+20{,}000$	$-400M$ $+15{,}000$	M	0	M	0	1	$-29{,}000M$	

$$\sim$$

	x	y	s_1	a_1	s_2	a_2	p		
	1	$\frac{1}{2}$	$-\frac{1}{600}$	$\frac{1}{600}$	0	0	0	40	
	100	100	0	0	-1	1	0	5,000	$R_2 + (-100)R_1 \to R_2$
	-----	-----	-----	-----	-----	-----	-----	-----	
	$-700M$ $+20{,}000$	$-400M$ $+15{,}000$	M	0	M	0	1	$-29{,}000M$	$R_3 + (700M - 20{,}000)R_1$

$$\downarrow \text{Enter}$$

$$
\begin{array}{c}
 \\
x \\
\text{Exit} \to a_2 \\
\\
\\
P
\end{array}
\begin{array}{ccccccc|c}
x & y & s_1 & a_1 & s_2 & a_2 & p & \\
1 & \frac{1}{2} & -\frac{1}{600} & \frac{1}{600} & 0 & 0 & 0 & 40 \\
0 & \boxed{50} & \frac{1}{6} & -\frac{1}{6} & -1 & 1 & 0 & 1{,}000 \\
\hline
 & -50M & & & & & & -1{,}000M \\
0 & +5{,}000 & \frac{-M+200}{6} & \frac{7M-200}{6} & M & 0 & 1 & -800{,}000M
\end{array}
$$

$\frac{1}{50}R_2 \to R_2$

$$
\sim
\begin{array}{ccccccc|c}
1 & \frac{1}{2} & -\frac{1}{600} & \frac{1}{600} & 0 & 0 & 0 & 40 \\
0 & \boxed{1} & \frac{1}{300} & -\frac{1}{300} & -\frac{1}{50} & \frac{1}{50} & 0 & 20 \\
\hline
 & -50M & & & & & & -1{,}000M \\
0 & +5{,}000 & \frac{-M+200}{6} & \frac{7M-200}{6} & M & 0 & 1 & -800{,}000M
\end{array}
$$

$R_1 + (-\tfrac{1}{2})R_2 \to R_1$

$R_3 + (50M - 5000)R_2$

$$
\begin{array}{c}
x \\
y \\
\\
\\
P
\end{array}
\begin{array}{ccccccc|c}
1 & 0 & -\frac{1}{300} & \frac{1}{300} & \frac{1}{100} & -\frac{1}{100} & 0 & 30 \\
0 & 1 & \frac{1}{300} & -\frac{1}{300} & -\frac{1}{50} & \frac{1}{50} & 0 & 20 \\
\hline
0 & 0 & \frac{50}{3} & \frac{4M-50}{3} & 100 & M-100 & 1 & -900{,}000
\end{array}
$$

Since the bottom row has no negative indicator the optimal solution for the modified problem is

$$x = 30, y = 20, \ s_1 = 0, s_2 = 0, a_1 = 0, a_2 = 0, P = -900{,}000$$

Thus

$$\text{Min C} = -Max\ P = -(-900{,}000) = 900{,}000$$

The minimum operating costs are $900,000 when the Chicago plant operates 30 days and the Detroit plant operates 20 days.

4. Conclusion

This paper is to teach the various tools and techniques so that we will be able to solve the various real life business problems.

References

1. Raymond A Barnet & Michael R. Ziegler, *Finite Mathematics*, **For Business, Economics, Life Sciences, and Social Sciences,** Fifth Edition, 1990.

2. Howard L.Rolf, Raymond J.Cannon, Jr. & Gaerth Willams, *Mathematics for Management*, *Social & Life Sciences,* Wm.C.Brown Publishers, 1991.