APPLICATION OF BUSINESS MATHEMATICS IN REAL – LIFE

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ABSTRACT

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Applied mathematics in this paper is direct application to commerce and real – life problems. Business mathematics includes mathematics course taken at an undergraduate level by business students. Examples of business mathematics in this paper are matrix algebra, exponential function and natural logarithm function. This paper emphasizes on the formulas derivation for financial mathematics and continuous growth.

Key Words : Matrix Algebra, Exponential Function and Natural Logarithm.

1.Introductio

Mathematics is an important subject and knowledge of it enhances a person’s reasoning, problem – solving skills, and in general, the ability to think. This paper introduces undergraduate level mathematics which is related and useful to solve business problems and other real – life problems. This paper wants to know using exponential function and natural logarithm function solved for real – life problems.

1.1Aim and Objectives

The aim of the study is all of the teachers in our university to know business mathematics which has been taught to our students. This foundation course is useful to real – life.

The objectives of the research are

(1) Formulate multi-variable economic models in matrix format

(2) Compute simple interest and compound interest

(3) Use compound interest formula to derive the irrational number e.

(4) Use the exponential function and natural logarithm to derive the final sum and the length of time when continuous growth takes place.

1.2 Scope of the study

This paper emphasizes only on the undergraduate level mathematics for second year students at Co-operative University, Thanlyin . It concerns with second year business mathematics.

2. Applied Matrix Algebra for Business Problems

2.1 Definition

Matrix is set of numbers which is bounded by two brackets. Vertical lines are called columns and horizontal lines are called rows. Matrix is always in m x n order. Here m is number of rows and n is numbers of columns. Each entry is usually known as element.

Example (1)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}, \quad B = \begin{bmatrix} 2 & 3 & 5 \\ 6 & 7 & 8 \end{bmatrix}_{2\times 3}$$

$$C = \begin{bmatrix} 2 & 3 & 6 & 7 & 8 \end{bmatrix}_{1\times 5},$$

$$D = \begin{bmatrix} 2 \\ 3 \\ 6 \\ 7 \end{bmatrix}_{4\times 1}$$

2.2 Matrix addition and subtraction

Matrices that have the same order can be added together, or subtracted. The addition, or subtraction , is performed on each of the corresponding elements.

Example (2)

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 3 & 8 \end{bmatrix}$, find (i) A + B , (ii) A – B

<u>Solution</u>

(i) $$A + B = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 \\ 7 & 15 \end{bmatrix}$$

(i) $$A - B = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 5 & 1 \\ 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$$

Example (2)

Matrix A	
2	3
4	7

Matrix B	
5	1
3	8

Matrix A+B	
7	4
7	15

Matrix A-B	
-3	2
1	-1

2.3 Basics Principles of Matrix Multiplication

If one matrix is multiplied by another matrix , the basic rule is to multiply elements along the rows of the first matrix by the corresponding elements down the columns of the second matrix. The basic principle of matrix multiplication involves the elements across a row multiplying the elements down the columns of the matrix being multiplied , and then summing all the products obtained.

2.4 Matrix Multiplication

Let A = $[aij]$, i = 1, 2, ---, m

j = 1, 2, ---, n

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots a_{1n} \\ a_{21} & a_{22} & a_{23} \dots a_{2n} \\ \vdots & \vdots & \vdots \quad \vdots \\ a_{m1} & a_{m2} & a_{m3} \dots a_{mn} \end{bmatrix}_{m \times n}$$

Let B = $[b_{jk}]$, j = 1, 2, 3,---, n,

k = 1, 2, 3,---,p,

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \dots b_{1P} \\ b_{21} & b_{22} & b_{23} \dots b_{2P} \\ \vdots & \vdots & \vdots \quad \vdots \\ b_{n1} & b_{n2} & b_{n3} \dots b_{nP} \end{bmatrix}_{n \times p}$$

The product matrix AB will have m rows and p columns, i . e, if A is m x n matrix and B is n x p matrix then AB is an m x p matrix.

In the product AB, A is known as pre – factor and B as post – factor. Thus we notice that the product AB is defined if and only if the number of columns of the pre – factor is equal to the number of rows of the post – factor. Two matrices A and B are said to be conformable for multiplication if the number of columns of A is equal to the number of rows of B .

Example (3)

(i) If $A = \begin{bmatrix} 3 & 4 & 7 \\ 4 & 6 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 6 \\ 4 & 7 \\ 5 & 8 \\ 9 & 10 \end{bmatrix}$, then what is AB ?

$$A = \begin{bmatrix} 3 & 4 & 7 \\ 4 & 6 & 9 \end{bmatrix}_{2\times 3}, \quad B = \begin{bmatrix} 3 & 6 \\ 4 & 7 \\ 5 & 8 \\ 9 & 10 \end{bmatrix}_{4\times 2}$$

The product AB is not impossible because the number of columns of A is not equal to the number of rows of B

(ii) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$, find AB

<u>Solution ,</u>

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}_{3\times 3}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2}$$

Here A is a 3 x 3 matrix , B is a 3 x 2 matrix , therefore, the matrix A B is defined as the number of columns of A is 3 and number of row of B is also 3 . Obviously , A B is of order 3 x 2 .

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+6 & 0+2+3 \\ 0+0+2 & 0+2+1 \\ 1+0+8 & 0+2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 \\ 2 & 3 \\ 9 & 6 \end{bmatrix}_{3\times 2}$$

Example (3)

Matrix A	
3	4
4	6

Matrix B	
3	6
4	7
9	10

The Product AB is not impossible.

Matrix A	
1	2
0	2
1	2

Matrix B	
1	0
0	1
2	1

Matrix AB	
7	5
2	3
9	6

Example (4)

There are 3 Co-operative Colleges and 2 Co-operative Universities in our Ministry. Each college and university has 11 peons , 20 clerks and 1 cashier. Each university, in addition , has 1 section officer and 1 librarian. Their salary are :

Peons – Ks 120000 , Clerk – Ks 160000, Cashier – Ks 250000, Section officer – Ks 310000 and Librarion – Ks 280000.

Using matrix notation, find

(i) Total number of posts of each kind in colleges and universities taken together.

(ii) The total monthly salary bill of all the colleges and Universities taken together.

Solution

Let A = $\begin{bmatrix} 11 & 11 \\ 20 & 20 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$

Let B = $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Let C = [120000 160000 250000 310000 280000]

Where

The matrix A represents the number of posts of each kind in colleges and universities.

The matrix B represents the number of Co – operative Colleges and Co-operative Universities in our Ministry.

The matrix C represents the salary for each post.

(i) AB $= \begin{bmatrix} 11 & 11 \\ 20 & 20 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}_{5\times 2} \begin{bmatrix} 3 \\ 2 \end{bmatrix}_{2\times 1}$

$$= \begin{bmatrix} 33+22 \\ 60+40 \\ 3+2 \\ 0+2 \\ 0+2 \end{bmatrix} = \begin{bmatrix} 55 \\ 100 \\ 5 \\ 2 \\ 2 \end{bmatrix}_{5\times 1}$$

Total number of posts of each kind in colleges and universities taken together:

Peons = 55; Clerks = 100; Cashiers = 5;Section officers = 2; Librarians = 2

(ii) C (AB) $= \begin{bmatrix} 120000 & 160000 & 250000 & 310000 & 280000 \end{bmatrix}_{1\times 5} \begin{bmatrix} 55 \\ 100 \\ 5 \\ 2 \\ 2 \end{bmatrix}_{5\times 1}$

$$= [6600000 + 16000000 + 1250000 + 620000 + 560000]$$

$$= [25030000]_{1\times 1}$$

Total monthly salary bill of all colleges and universities taken together =Ks 25030000

Example (4)

Matrix A			
11	11	-	Peons
20	20	-	Clerks
1	1	-	Cashiers
0	1	-	Section officer
0	1	-	Librarian

Matrix B		
3	-	College
2	-	University

Matrix C				
120000	160000	250000	310000	280000

Matrix AB		
55		
100		
5		
2		
2		

Matrix C				
120000	160000	250000	310000	280000

Matrix (AB)	
55	
100	
5	
2	
2	

Matrix C(AB)	
25030000	

Example (5)

A courier has to deliver snacks to 3 shops in 3 wards every week. Kind of snacks are potato-chips, biscuits and Shan toh-Phuu.

Before delivery, the remaining packets are 2 potato – chips, 8 biscuits and 14 Shan toh – phuu at the first shop. At the second shop, 4 potato-chips, 10 biscuits and 16 shan toh - phuu are left. Then, 6 potato - chips, 12 biscuits and 18 shan toh - phuu are remaining at the third shop.

At the beginning of the first week, the courier delivered 4 potato-chips, 10 biscuits and 16 Shan toh - phuu to the first shop, 6 potato- chips, 12 biscuits and 18 Shan toh - phuu to the second shop and 8 potato-chips, 14 biscuits and 20 Shan toh-phuu to the third shop.

The selling rate in the first week is : 2 potato- chips, 8 biscuits and 10 Shan toh - phuu at the first shop, 10 potato – chips, 12 biscuits and 24 Shan toh – phuu at the second shop and 8 potato – chips, 16 biscuits and 38 Shan ton – phuu at the third shop.

Using matrix algebra, find

(i) The total amount for the each snack in each shop after delivery.

(ii) The amount for each item which is left after selling at the end of the week.

Solution

$$\text{Let } A = \begin{matrix} A_1 & A_2 & A_3 \\ \end{matrix} \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 6 & 8 \\ 10 & 12 & 14 \\ 16 & 18 & 20 \end{bmatrix}$$

$$\text{and } C = \begin{bmatrix} 2 & 10 & 8 \\ 8 & 12 & 16 \\ 10 & 24 & 38 \end{bmatrix}$$

Where

The matrix A shows the snacks of 3 types of items in three shops A_1, A_2 and A_3 before delivery.

The matrix B shows the number of items delivered to the three shops at the beginning of a week.

The matrix C shows the number of items sold during that week.

$$\text{(i) } A+B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 8 \\ 10 & 12 & 14 \\ 16 & 18 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 10 & 14 \\ 18 & 22 & 26 \\ 30 & 34 & 38 \end{bmatrix}$$

(iii) $(A+B)- C = \begin{bmatrix} 6 & 10 & 14 \\ 18 & 22 & 26 \\ 30 & 34 & 38 \end{bmatrix} - \begin{bmatrix} 2 & 10 & 8 \\ 8 & 12 & 16 \\ 10 & 24 & 38 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 6 \\ 10 & 10 & 10 \\ 20 & 10 & 0 \end{bmatrix}$

Example (5)

Matrix A		
2	4	6
8	10	12
14	16	18

Matrix B		
4	6	8
10	12	14
16	18	20

Matrix C		
2	10	8
8	12	16
10	24	38

Matrix A+B		
6	10	14
18	22	26
30	34	38

Matrix A+B		
6	10	14
18	22	26
30	34	38

Matrix C		
2	10	8
8	12	16
10	24	38

Matrix (A+B)-C		
4	0	6
10	10	10
20	10	0

Example (6)

The following table shows the price of each kind of snacks at the snack – counter of a Mini Mart. It shows for three weeks in detail for the amount of one week respectively, find the total revenue in each week using matrix algebra.

No	Description	Price Kyats	UOM	First week	Second week	Third week
1.	Gery Cheese Crackers (150 g)	1550	sku	250	300	2700
2.	Gery Cheese Crackers (80 g)	850	sku	112	99	109
3.	Hup SEMG Cream Crackers (650 g)	5800	sku	100	125	150
4.	Hup SEMG Cream Crackers (165 g)	1450	sku	1500	900	200
5.	Good morning Cheese Cake (200 g)	1800	sku	300	370	420

Solution

The sale matrix S and the price matrix P are given by.

$$S = \begin{bmatrix} 250 & 300 & 2700 \\ 112 & 99 & 109 \\ 100 & 125 & 150 \\ 1500 & 900 & 200 \\ 300 & 370 & 420 \end{bmatrix}_{5\times 3}$$

$$P = [1550 \;\; 850 \;\; 5800 \;\; 1450 \;\; 1800]_{1\times 5}$$

$$PS = [1550 \;\; 850 \;\; 5800 \;\; 1450 \;\; 1800]_{1\times 5} \begin{bmatrix} 250 & 300 & 2700 \\ 112 & 99 & 109 \\ 100 & 125 & 150 \\ 1500 & 900 & 200 \\ 300 & 370 & 420 \end{bmatrix}_{5\times 3}$$

$$= [3{,}777{,}700 \quad 3{,}245{,}150 \quad 6{,}193{,}650 \,]$$

Example (6)

No	Description	Price	UOM	1 week	2 week	3 week
1	gary cheese (150g)	1550	SKU	250	300	2700
2	gary cheese (80g)	850	SKU	112	99	109
3	husen cracker (650g)	5800	SKU	100	125	150
4	husen cracker (165g)	1450	SKU	1500	900	200
5	goodmorning cheese cake (200g)	1800	SKU	300	370	420

Matrix P					Matrix S		
1550	850	5800	1450	1800	250	300	2700
					112	99	109
					100	125	150
					1500	900	200
					300	370	420

Matrix PS		
3777700	3245150	6193650

3. Application to financial Mathematics

3.1 Interest

When money is borrowed , the lender expects to pay back the amount of the loan plus an additional charge for the use of the money. This additional charge is called interest. when money is deposited in a band , the band pays the depositor for the use of the money. The money the deposit earns is also called interest.

Interest can be computed in two ways : either as simple interest or as compound interest.

3.2 Simple Interest

Simple interest is the interest that accrues on a given sum in a set time period. It is not reinvested along with the start – up capital. The amount of interest earned on a given investment each time period will be the same (if interest rates are fixed) as the total amount of capital invested remains unaltered.

Simple interest is computed by finding the product of the principal (the amount of money on deposit) , the rate of interest (usually written as a decimal), and the time (usually expressed in years)

Simple Interest = Principal . rate . time

$$I = P\,r\,t$$

Example (1)

Find the simple interest earned on a deposit of $ 5,750 that is left on deposit for $3\frac{1}{2}$ years and earns an annual interest rate of $4\frac{1}{2}\%$.

<u>Solution</u>

$P = \$\ 5750$

$t = 3\frac{1}{2}$ years = 3.5 years

$r = 4\frac{1}{2}\% = 0.045$

$I = P.\ r\ .t$

$= (5750)\ (0.045)\ (3.5)$

= 9.05.625

In $3\frac{1}{2}$ years, the account will earn \$ 905.63 in simple interest.

3.3. Compound Interest

Compound interest is the interest which is added to the original investment every time it accrues. The interest added in one time period will itself earn interest in the following time period. The total value of an investment will therefore grow over time.

Example (2)

If \$ 600 is invested for 3 years at 8% interest compounded annually at the end of each year, what will the final value of the investment be?

	$
Initial sum invested	600.0
Interest at end of year 1 = 0.08 x 600	48.0
Total sum invested for years 2	648.00
Interest at end of years 2 = 0.08 x 648	51.84
Total sum invested for years 3	699.84
Interest at end of years 3 = 0.08 x 699.84	55.99
Final value of investment	755.83

3.4 Calculating the final value of an investment

Suppose that the start – up deposit in the account is P dollars , that interest is paid an annual rate r, and that the accumulated amount or the future value in the account at the end of the first year is A_1. Then the interest earned that year is P r and

The amount after one year	equals	The start – up deposit	plus	The interest earned on the original deposit

$$A_1 = P + Pr$$

$$A_1 = P\ (1 + r)$$ Factor out the common factor , P

The amount , A_1 , at the end of the first year is the balance in the account at the beginning of the second year . So, the amount at the end of the second year, A_2 , is

The amount after two years		The amount after one year		The interest earned on the amount after one year

equals plus

$A_2 = A_1 + A_1$ r

$= P (1+r) + P (1+r) r$ Substitute $P(1 + r)$ for A_1

$= P (1+r) . (1+r)$ Factor out the common P (1+r)

$= P (1 + r)^2$ Simplify

By the end of the third year, the amount will be

$A_3 = A_2 + A_2$ r

$= P (1 + r)^2 + P (1 + r)^2 . r$ Substitute $P (1 + r)^2$ for A_2

$= P (1 + r)^2 (1+r)$ Factor out the common $P(1 + r)^2$

$= P (1 + r)^3$ Simplify

We continue in this matter , establishing the formula for the amount after n years.

The formula for the final value A_n of an investment P for n years at an annual rate r, is therefore

$$A_n = P (1 + r)^n$$

Let us rework example using this formula just to check that we get the same answer.

Example (reworked)

If $ 600 is invested for 3 years at 8% interest

P = $ 600 , n = 3 years , r = 8% = 0.08

$A_n = P (1 + r)^n$

$A_3 = 600 (1 + 0.08)^3$

$= 600 (1.08)^3$

$= 600 (1.259712)$

$= 755.83$

Thus the final sum will be $ 755.83

3.5 Periodic Rate

$$\text{Periodic rate} = \frac{\text{annual rate}}{\text{number of periods per yaer}}$$

This formula is often written as

$$i = \frac{r}{k}$$

Where i is the periodic interest rate , r is the annual rate, and k is the number of times interest is paid each year. If interest is calculated k times each year, in n years there will be nk conversions. Each conversion is at the periodic rate i . This leads to another form of the compound interest formula.

3.6 Compound interest Formula

An amount P , earning interest compounded k times a year for n years at an annual rate r, will grow to the future value A_n, according to the formula.

$$A_n = P\ (1 + \frac{r}{k})^{nk}$$

$$A = (1 + i)^t$$

Where

$i = \frac{r}{k}$ = the interest rate per period

r = annual interest rate

k = number of times compounded per year

A = amount (final value) at the end of k compound periods.

t = nk = the number of periods (t= n k where n is the number of years)

P = Principal (present value)

Remark

Interest paid twice each year is called semiannual compounding, four times each year quarterly compounding, twelve times each year monthly compounding, and 360 or 365 times each year daily compounding

Example (3)

\$ 800 is invested at 12% for two years. Find the amount at the end of two years if the interest is compounded.

(a) annually (b) semiannually (c) quarterly

Solution

P = $ 800, r = 12% = 0.12, n = 2 years

(a) annually

k = 1 , i = $\frac{r}{k}$= $\frac{0.12}{1}$ = 0.12, n = 2 years, t = n k= 1.2 = 2

A = P $(1 + i)^t$

= 800 $(1 + 0.12)^2$

A = 800 $(1 + 0.12)^2$

= 800 (1.2544)

= 1003.52

The future value = $1003.52

(b) semiannually

k = 2, i = $\frac{r}{k}$ = $\frac{0.12}{2}$ = 0.06, n = 2 years, t = n k = 2.2. = 4

A = P $(1 + i)^t$

= 800 $(1 + 0.06)^4$

= 800 $(1.06)^4$

= 800 (1.26248)

= 1009.98

The future value = $ 1009.98

(c) quarterly

k = 4, i = $\frac{0.12}{4}$ = 0.03, n = 2 years, t = n k = 4.2 = 8

A = P $(1 + i)^t$

= 800 $(1 + 0.08)^8$

= 800 $(1.08)^8$

= 800 (1.26677)

= 1013.42

The future value = $1013.42

4. Application of Exponential and Logarithm Functions

4.1 Exponential Function

Functions with a variable exponent are called exponential function .

Example

Y = A^x

Where A is a constant and A $>$ 1

This is known as an exponential function to base A .

$$A^0 = 1 \text{ and } A^1 = A$$

4.2 The Natural Exponential Function

The specific function y = e^x should be know as the "natural exponential function" .

In economic , exponential functions to the base e are particularly useful for analyzing growth rates.

How calculate value for e is derived, we return to use the compound interest formula.

$$A_n = P\,(1+i)^{kn}$$

$$A = P\,(1+i)^t$$

$$A = P\left(1+\frac{r}{k}\right)^t$$

Where

$i = \frac{r}{k}$ = the interest rate per period

r = annual interest rate

k = number of times compounded per year

t = the number of periods (t = kn where n is the number of years)

A = amount (future value) at the end of k compound periods.

P = principal (present value)

(i) Annually

k = 1, r = 100% = 1, n = 1 year , P = 1 , t = n k = 1.1 = 1

$$A = P\,(1+\frac{r}{k})^t$$

$$A = P\,(1+\frac{r}{k})^{nk}$$

$$= 1\,(1+1)^{1.1}$$

$$= 2^1$$

$$= 2$$

(ii) Monthly

k = 12, r = 100% = 1, i = $\frac{r}{k}$ = $\frac{1}{12}$, n = 1year, P = 1, t = n k = 1.12 = 12

A = 1 $(1+\frac{1}{12})^{12}$

= 2.6130353

(iii) Daily

k = 365, r = 100% = 1 , i = $\frac{r}{k}$ = $\frac{1}{365}$, n = 1year,P = 1, t = n k = 1. 365 = 365

A = 1 $(1+\frac{1}{365})^{365}$

= 2.7145677

(iv) Hourly

k = 8760, r = 100% = 1, i = $\frac{r}{k}$ = $\frac{1}{8760}$, n = 1year ,p = 1,

t = n k = (1) (8760) = 8760

A = 1 $(1+\frac{1}{8760})^{8760}$

= 2.7181267

From the above calculations we can see that the more frequently that interest is credited the closer the value of the final sum accumulated gets to 2.7182818, the value of e .

When interest at a nominal annual rate of 100% is credited at infinitesimally small time intervals then growth is continuous and e is equal to the final sum credited. Thus

e = $\left(1+\frac{1}{k}\right)^{k}$ where $k \to \infty$

= 2.7182818

This result means that a sum P invested for one year at a nominal annual interest rate of 100% credited continuously will accumulate to the final sum of

A = e P = 207182818 P

Continuous growth also accurse in other variables relevant to economics, e.g , population , the amount of natural materials mined.

4.3 Accumulated final Values after Continuous Growth

To derive a formula that will give the final sum accumulated after a period of continuous growth, we first assume that growth occurs at serval discrete time intervals throughout a year.

We return to use the final sum formula

$$A = P\left(1 + \frac{r}{k}\right)^{nk}$$

Where $i = \frac{r}{k}$ = the interest rate per period

r = annual interest rate

k = number of times compounded per year

A = amount (final vale) at the end of k compound periods.

P = Principal (present value)

To reduce this to a simpler formulation, multiply top and bottom of the exponent by r so that

$$A = P\left(1 + \frac{r}{k}\right)^{\frac{k\,(rn)}{r}} \qquad \text{———— (1)}$$

If we let $m = \frac{k}{r}$ then $\frac{1}{m} = \frac{r}{k}$ and so (1) can be written as

$$A = P\left(1 + \frac{1}{m}\right)^{m\,r\,n}$$

$$= P\left(1 + \frac{1}{m}\right)^{m(rn)} \qquad \text{———— (2)}$$

Growth becomes continuous as the number of times per year that increments in growth are accumulated increases towards infinity.

When $k \rightarrow \infty$ then $\frac{k}{r} = m \rightarrow \infty$

Therefore, using the result derived in section (3.2) above,

$$\left(1 + \frac{1}{m}\right)^{m} \rightarrow e \text{ as } m \rightarrow \infty$$

Substituting this result back into (2) above gives

$$A = P\,\boldsymbol{e}^{\boldsymbol{rn}}$$

This formula can be used to find the final value of any variable growing continuously at a known rate from a given original value.

4.4 Formula for continuous compounding

If P dollars is invested at an annual interest rate r compounded continuously, then after t years the amount A is given by

$$A = P\, e^{rt}$$

4.5 Application to Continuous Compounding

Example (1)

If \$ 1000 is invested at 5% compounded continuously, how much is the investment worth after four years?

Solution

Let P = \$ 1000, r = 5% = 0.05 , t = 4 years

$A = P\, e^{rt}$

$A = 1000 \,.\, e^{0.05\,(4)}$

$= 1000 \,.\, e^{0.2}$

= 1000 (1.2214)

= 1221.40

The investment worth after four years = \$ 1221.40

Example (2)

Suppose \$ 5000 is invested in an account earing 6.5% interest find the balance in the account after 10 years under the following options.

a . Compounded annually

b . Compounded quarterly

c . Compounded monthly

d . Compounded daily

e . Compounded continuously

Solution

Compounding Option	n Value	Formula	Result
Annually	n = 1	$A = 5000\,(1+\frac{0.065}{1})^{(1)(10)}$	\$ 9385.69

Quarterly	n = 4	A = $5000(1+\frac{0.065}{4})^{(4)(10)}$	\$ 9527.79
Monthly	n = 12	A = $5000(1+\frac{0.065}{1})^{(12)(10)}$	\$ 9560.92
Daily	n = 365	A = $5000(1+\frac{0.065}{1})^{(365)(10)}$	\$ 9577.15
Continuously	Not applicable	A = $5000\ e^{(0.065)(10)}$	\$ 9577.70

Notice that there is a \$ 191.46 difference in the account balance between annual compounding and daily compounding. However, the difference between compounding daily and compounding continuously is small - \$ 0.55

Example (3)

Population in a developing country is growing continuously at an annual rate of 3%. If the population is now 4.5 millions , what will it be in 15 years' time?

<u>Solution</u>

initial value P = 4.5 millions ,

rate of growth r = 3% = 0.03

number of time period t = 15 years

$$A = P\ \boldsymbol{e^{rt}}$$

$$= (4.5)\ e^{(0.03)\ (15)}$$

$$= (4.5)\ e^{0.45}$$

$$= (4.5)\ (1.5683122)$$

$$= 7.0574048$$

Thus the predicted final population is 7,057,405 millions

Example (4)

A river flows through a hydroelectric dam is 18 million gallons a day and shrinking continuously at an annual rate of 4% what will the flow be in 6 years' time?

<u>Solution</u>

The 4% rate of decline becomes the negative growth rate r = - 4% = - 0.04 We also know the initial values P = 18 million gallons and t = 6 years . Thus the final value is

$A = P\, \boldsymbol{e^{rt}}$

$= 18\ e^{-0.04\,(6)}$

$= 18\ e^{-0.24}$

$= 18\ (\ 0.78662\ )$

$= 14.159$

$= 14.16$

Therefore, the river flow will shrink to 14.16 million gallons per day.

4.6 The natural logarithm

The logarithm to the base e is an important function. If it is also known as the natural logarithm. It is defined for all x> 0.

$y = \log_e x \Leftrightarrow x = e^y$

Remark

Logarithm function and exponential function inverse of each other.

$ln e^x = e^{lnx} = x$

Proof

(i) $ln e^x = \log_e e^x = x\log_e e = x.\,1 = x$

(ii) $e^{lnx} = ?$

Let $y = ln\, x$

$x = e^y$

$e^{lnx} = e^y = x$ substitute $ln\, x$ for y

By (i) and (ii)

$ln e^x = e^{lnx} = x$

This fact is used under the example.

If we want to know the length of time taking the double investment, we have to use the equation $e^x = 2$. If we want the triple investment, we have to use the equation $e^x = 3$ and so on.

Example

How long will it take an investment to double if the interest is 6% compounded continuously?

Let P be the initial amount invested.

$r = 6\% = 0.06$

$A = P\,e^{rt}$

$2p = P\,e^{rt}$

$2 = e^{rt}$

$2 = e^{(0.06)t}$

$ln\,2 = ln\,e^{(0.06)t}$

$ln\,2 = (0.06)t$

$t = \frac{ln2}{0.06}$

$= \frac{100 \text{ x } ln2}{6}$

$= \frac{100 \text{ x } 0.6937}{6}$

$= \frac{69.31}{6}$

$= 11.55$

The investment will double in a little more than $11\frac{1}{2}$ years.

5. Conclusion

Mathematics is useful in business and real - life. We study the problems in reference books which are real data from business work. We found that business and economic problems are solved by means of business mathematics in developed countries. The weakness of this paper is that real data from business work are not used for mathematics models. As the economics is improving, our own real data must try to be used for solving real – life business problems.

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