Normal Distribution That Changed the World

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Abstract—Normal distribution is one of the sectors of probability potion. It is applied in statistics field and nowadays it is a very famous equation that changed the world. These idea and results were came from my own test such as examination condition, health problems in public area, incomes for urban families , IQ tests for some students levels , phone selling promotion in Hlaing Tharyar township and so on.

Keywords— Normal distribution, mean, variance, standard deviation.

Introduction

Normal distribution is found by De Moivre (Statistician) and later Gauss derived the established normal distribution equation by systemically. Normal probability distribution is used in Physics, Biology and social sciences to model various properties. The application of normal distribution can be found in statistics and many unexpected survey operations.

In 2013, Mathematics and science writer Ian Steward looked at 17 equations that shaped our standing of the world [3]. They changed human history. In those 17 equations, normal distribution is one of the most important and famous equations which can promote people's thinking and living standard.

I. Mathematics Concepts and 17 Equations

Mathematics is the mother of science. Before the Christ, people are thinking with fundamental concepts and calculations to create innovations for human beings. Then they derived many equations not only for mathematics but also for science fields. Hence mathematics equations play a vital role in each sector.

All of us are following in mathematics concepts. We can't live away from numbers that are basic layer of mathematics. We based on the mathematics equations and theorem and later we apply it for physical improvement. So some of laws, equations, theorem and principles are very important not only for science fields but also for all human beings. There are many theorems and equations in related fields. Some of them are very useful and favourite for us.

The famous mathematician Thales (BC 582-500) started mathematical history and created equations. After that many wisdom and scientists form related fields derived useful equations. At first, there have 10 equations that changed the world [2]. They are

(1) Logarithm

$$\log xy = \log x + \log y$$

(2) Pythagoras Theorem

$$a^2 + b^2 = c^2$$

(3) Law of Gravity

$$F = G\,\frac{m_1\ m_2}{r^2}$$

(4) Euler's Formula for Polyhedron

$$V - E + F = 2$$

(5) Fourier Transform

$$f(w) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ixw}\ dx$$

(6)Maxwell's Equations

$$\nabla . E = L$$

$$\nabla . H = 0$$

$$\nabla \times E = \frac{1}{e}\frac{\partial H}{\partial t}\ ,$$

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(7) Second Law of Thermodynamics

$$dS \geq 0$$

(8) Relativity Theory

$$E = mc^2$$

(9) Schrodinger's Equation

$$ih\frac{\partial}{\partial t}\Psi = H\Psi$$

(10) Chaos Theory

$$x_{t+1} = kx_t(1 - x_t)$$

Later, in 2013, it increased form10 equations to 17 equations. They are very useful and famous in the world [2].

(11) Calculus

$$\frac{df}{dt} = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h}$$

(12) The Square Root minus One

$$i^2 = -1$$

(13) Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right),$$

(14) Wave Equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(15) Navier- Stokes equation

$$\rho\left(\frac{\partial v}{\partial t} + v.\nabla v\right) = -\nabla \rho + \nabla.T + f$$

(16) Information Theory

$$H = -\sum p(x) log p(x)$$

(17) Black – Scholes Equation

$$\frac{1}{2}a^2 s^2 \frac{\partial^2 v}{\partial s^2} + rs\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} - rv = 0$$

From these famous equations, I would like to express the analysis of my favourite equations. Especially I' m really interested in normal distribution. Chaos theory, Navier- Stokes equations, Pythagoras theorem, calculus and Law of gravity can attract me to study the details of their history and applications. Similarly Logarithm, Fourier Transforms and the Square Root minus One can persuade me to analyse their applications.

II. Normal Distribution

Normal distribution is the most important continuous distributions because in applications many random variables are normal random variables, or they are approximately normal or it can be transformed into normal random variables in a relatively simple fashion. Furthermore the normal distribution is a useful application of more complicated distributions and it also occurs in the proofs of various statically tests [1].

It is known as Gauss distribution and it defined as the distribution with the density.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), \qquad \infty < x < \infty \qquad (\sigma > 0)$$

In here, μ is the mean and σ is the standard deviation.

The curve of $f(x)$is, with symmetric with respect to $x = \mu$ because the exponent is quadratic. Hence for $= 0$, it is symmetric with respect to the y axis ,we have bell shaped curve.

The exponential function goes to zero very fast, the faster the smaller, the standard deviation is.

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(v)dv$$

where $F(x)$ is distribution function. The distribution function of the standardized normal random variable $Z = (\frac{X-\mu}{\sigma})$ with mean 0 and variance 1, is

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{\frac{-u^2}{2}} du$$

Later we have

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

III. Theory Test (1)

I would like to apply the normal distribution in a problem. In an exam, the question is very difficult and most of the students can't answer very well. For giving 100 marks question, 50 marks are their passed marks. If the students get under 50 marks, they will fail in this exam.

(a) Let's their passed mark is 45. The mean is 50 and standard deviation is 25, what percent of the student will pass the exam?

Solution, μ=50, σ=25

$P(X \geq 45) = 1 - P(X < 45)$

$= 1 - P(X \leq 45)$

$= 1 - F(45)$

$= 1 - \Phi(\frac{45-\mu}{\sigma})$

$= 1 - \Phi(\frac{45-50}{25})$

= 0.5793 = 57%

(b) Let's their passed mark is 45. The mean is 50 and standard deviation is 25, what percent of the student will fail the exam?

Solution, μ =50, σ=25

P $(X < 45) = F(45)$

$= \Phi\left(\frac{45-\mu}{\sigma}\right)$

$= \Phi\left(\frac{45-50}{25}\right)$

=1-0.5793

= 0.4207 = 42%

(c) Let's their passed mark is 40. The mean is 50 and standard deviation is 25, what percent of the student will pass the exam?

Solution, μ=50, σ=25

P (X ≥40) = $1 - P(X < 40)$

$= 1 - P(X \leq 40)$

$= 1 - F(40)$

$= 1 - \Phi(\frac{40-\mu}{\sigma})$

$= 1 - \Phi(\frac{40-50}{25})$

= 0.6554 = 65%

(d) Let's their passed mark is 45. The mean is 50 and standard deviation is 25, what percent of the student will fail the exam?

Solution, μ =50, σ=25

P (X <40) = $F(40)$

$= \Phi\left(\frac{40-\mu}{\sigma}\right)$

$=\Phi\left(\frac{40-50}{25}\right)$

=1-0.6554

= 0.3446= 34%

(e) Let's their passed mark is 30. The mean is 50 and standard deviation is 25, what percent of the student will pass the exam?

Solution, μ=50, σ=25

P (X ≥30) = $1 - P(X < 30)$

$= 1 - P(X \leq 30)$

$= 1 - F(30)$

$= 1 - \Phi(\frac{30-\mu}{\sigma})$

$= 1 - \Phi(\frac{30-50}{25})$

= 0.7880= 78%

(f) Let's their passed mark is 30. The mean is 50 and standard deviation is 25, what percent of the student will fail the exam?

Solution, μ =50, σ=25

P (X < 30) = $F(30)$

$= \Phi\left(\frac{30-\mu}{\sigma}\right)$

$= \Phi\left(\frac{30-50}{25}\right)$

=1- 0.7880

= 0.2119 = 21%

(g) How should we set the distinction mark for these exams to denote for 80% as usual?

Solution, μ =50, σ= 25

P (X ≤ c) = 80%

$F(c)$ =80%

$\Phi\left(\frac{c-\mu}{\sigma}\right)$ =80%

$\Phi\left(\frac{c-50}{25}\right)$ =80%

$\frac{c-50}{25}$ =0.842

c =71.05

IV. Result of Theory Test (1)

In this exam test, passed mark is not constant and it is alive and it can be changed any condition what you want. According to their getting marks, graph them from top to bottom whatever you want to set their passed rate. *x* is their getting marks and function *f(x)* calculate the value of variable *x.* For example, if you want to know the quantity of students who get 50 marks, you must set *x=50* or *f(50).* At the same time, you can integrate this function with limits, it will become probability density function.

In brief, according to these results, if you like to promote your students' chance or ability, you must set their passed mark be 30. In my opinion, the margin mark 40 is suitable and reasonable for them.

Moreover, if the student gets 71 marks in this exam, his marks will reach the distinction level due to the calculation of normal distribution method.

So normal distribution is fair for all students at any condition.

V. Theory Test (2)

In urban, I would like to survey the salary of every family. Some family get 3 lakh per month and some have 3.5 lakh per month and others get 4 lakh. Then the average income for each family is 3.5 and standard deviation is 0.5.

(a) How many percent of the family in this region that their at least income is 3 lakh?<

Solution, μ =3.5, σ= 0.5

P (X ≥3) = $1 - P(X < 3)$

= $1 - P(X \le 3)$

= $1 - F(3)$

= $1 - \Phi\left(\frac{3-3.5}{0.5}\right)$

= 0.8413 = 84 %

(b) How many percent of the family in this region that their at most income is 3 lakh?

Solution, μ =3.5, σ= 0.5

P (X ≤ 3) = $F(3)$

= $\Phi\left(\frac{3-\mu}{\sigma}\right)$

= $\Phi\left(\frac{3-3.5}{0.5}\right)$

=1- 0.8413

= 0.2119 = 21%

VI. Theory Test (3)

Another test for business is the most favourite test or my interesting attempt. I arrived Hlaing Tharyar township two month ago and I met many workers from garments. All of these are very young and most of them come from Ayarwaddy division such as Pathein, Myaung Mya, Phyar Pone, Eainme, Hinthada and especially from undeveloped areas. They stay hostel and their income is around one lakh in kyats and nearly two lakh. As they are adult, they are very interesting in handsets. They collect their salary and they spend by buying phone handsets.

So there are many handset shops around their hostels and their garment. Famous handsets, such as Samsung, OPPO, Vivo, Huawai, Sony, Mi. i Phone and other Chinese handsets are persuading them. These handset companies are trying to sell their products to these workers and all of them are surveying before starting their marketing.

First of all, they collect data of workers' ranks and then they investigate their salary and interesting. Later they use normal distribution method for their statistics. They realized that standard deviation one is the best selling condition for these workers especially for medium level. The more, the better for them.

Theory can prove these conditions.

$$P(\mu - \sigma < X < \mu + \sigma) \approx 68\%$$

$$P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 95.5\%$$

$$P(\mu - 3\sigma < X < \mu + 3\sigma) \approx 99.7\%$$

According to these result, normal distribution is very useful for company's benefits. They can estimate the situation of their operations and profits.

Similarly meteorologist estimates the weather conditions and seismologist forecasts earthquake. However they can't predict exactly because the earth is very confuse. According to instrumental record, there have powerful earthquakes in Myanmar since 1900. They used earthquake engineering and today they apply seismometer. That is using by Probabilistic Seismic Hazard Assessment (PSHA). They also apply and refer Poisson model and normal distribution to get the earthquake's distance and magnitude for their estimation. So their predictions are nearly right and possible conditions.

VII. Conclusion

Normal distribution is very useful for statistical problems. If the trial of event is finite, your problem will approach normal distribution. To apply normal distribution, you must need to realize mean and variance. The top of the normal distribution curve is mean and both sides have equal distribution and your curve will be symmetric.

All the distributions in natural, most of these problems are normal probability distribution. If you don't know the situation of your testing area, you must apply this method.

Similarly you can test the IQ for people in any school, any community, new product distribution in business, and so on. Normal distribution will support your destination. You can use this method for others such as public health's situation, various forms of public activities, and the gap of public's properties. By using this method, it can support the nearly truth result for researchers. So normal distribution is a valuable equation for all people and that really changed the world.

References

[1] Erwin Kreyszig , Advanced Engineerind Mathematics, 9th Edition.

[2] 10 Equations that changed the world form Weikipidia

[3] Business Insider. Com

[4] Super Prof Magazine form Science and Engineering

[5] https://Simple . m. Wikipedia. Org

[6] Normal Distribution form Wikipedia.org

[7] Officially Earthquake Survey Data from Meteorological Department