Existence of Atomic and Nuclear Hybrid State In $\Omega^- - \mathbf{Pb}$ System

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Abstract

The energy of hydrogen-like $\Omega^- - \mathrm{Pb}$ atom for various states was calculated by solving the Schrödinger equation. The $\Omega^- - \mathrm{Pb}$ atom is assumed as an ordinary hydrogen atom in which the electron is replaced by a negative omega hyperon. By assuming above approximation, the energy of hydrogen-like Pb^{81+} ionized atom was also calculated. To find out about their structure, the radius of hydrogen-like $\Omega^- - \mathrm{Pb}$ atom is determined. The energy of hydrogen-like $\Omega^- - \mathrm{Pb}$ atom is greater than Pb^{81+} ionized atom for various orbits and the inclusion of omega reduces the size of the atom in such a way that the smaller radius is obtained. In addition, the radius of $\Omega^- - \mathrm{Pb}$ atom is less than nuclear radius for lower orbits is observed. It is found that the existence of atomic state and nuclear state cannot be distinguished in lower orbits. In this way, the existence of atomic and nuclear hybrid state can be found in lower orbits of $\Omega^- - \mathrm{Pb}$ atom.

Key words: energy of hydrogen-like atom, radius of hydrogen-like atom.

Introduction

The Theory of Hydrogen Atom

The hydrogen atom is a simple mathematical problem in quantum mechanics, but any atom with more than one electron is so difficult that an exact solution is impossible. For the two-electron atom, helium, an elaborate approximate solution has been set up by Hylleraas, which gives result agreeing with experimental error. This agreement has convinced physicists that Schrödinger's equation for the many-body problem provides the correct starting point for a study of more complicated atoms. But the method used for helium is too complicated to apply to atoms with more than two electrons and the approximations must be made for many electron atoms. A good starting point is provided by assuming that each electron moves in a central or spherically symmetrical, force field produced by the nucleus and other electrons.

Hydrogen-Like Atom

The spectra of all atoms or ions with only one electron should be the same except for the factor Z^2 and the Rydberg number. Those of the ions $\mathrm{He}^+, \mathrm{Li}^{2+}, \mathrm{Be}^{3+}$ should be explained by means of the spectrum or any other ions which have only one electron. For Li^{2+}, Be^{3+} and still heavier

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highly ionized atoms, spectral lines have been observed which can be calculated by multiplying the frequencies of the lines of the H atom by Z^2 and insertion of the corresponding Rydberg constant. In 1916, the collected spectroscopic experience concerning the hydrogen–similarity of these spectra was generalized in displacement theorem of Somerfield and Kossel, which states: the spectrum of any atom is very similar to the spectrum of the singly charged positive ion which follows it in the periodic table.

Hydrogen–like heavy atoms are the heavy atoms from which all the electrons except one have been removed. Hydrogen–like heavy atoms can be prepared by accelerating the singly–ionized atoms to high energies and passing them through a thin foil; their electrons are “stripped off” on passing through the foil. For example, in order to strip all the electrons from a uranium atom and produce U^{92+} ions, they must be accelerated to energies greater than 10 GeV. By permitting the U^{92+} ions to recapture one electron each, one can then obtain hydrogen–like ion U^{91+}.The corresponding spectral lines are emitted as the captured electron makes transitions from orbits of high n to lower orbits. In the present work, we assumed above approximation, $\Omega^- - Pb$ atom to be like an ordinary hydrogen atom in which the electron is replaced by a negative omega. We calculated the energy of hydrogen–like Pb^{81+} ionized atom and $\Omega^- - Pb$ atom for various states by solving the Schrödinger equation. And then, we also calculated the radii of $\Omega^- - Pb$ atom to find out about their structure.

Strange Particle Ω –Hyperon

Omega Ω was discovered in the following reaction.

$$k + p \rightarrow \Omega + k^- + k^0$$

It has negative charge and its main decay modes are

$$\Omega^- \rightarrow \Xi^0 + \pi^-$$
$$\Omega^- \rightarrow \Xi^- + \pi^0$$
$$\Omega^- \rightarrow \Lambda^0 + k^-$$

Its half–life is 0.8×10^{-10}s and mass 1672.4 MeV/c^2. Its magnetic moment is –2.02 nuclear magneton. For Ω hyperon, hypercharge (Y) is –2 and strange number (S) is –3.

Energy and Radius for Omega Hyperonic Atom

The Schrödinger Equation for Omega Hyperonic Atom

The Schrödinger equation for the omega hyperonic atom is

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi \tag{1}$$

$$\left[\nabla^2 + \frac{2\mu}{\hbar^2}\left(E - V\right)\right]\psi = 0 \tag{2}$$

Where the potential energy V of the electron in the hydrogen atom is

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r} \quad (3)$$

$$\nabla^2 = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] \quad (4)$$

Substitution equation (4) into equation (2)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{2\mu}{\hbar^2}[E - V(r)]\psi = 0 \quad (5)$$

The wave function ψ is variable–seperable, i.e, ψ can be written as

$$\psi(r,\theta,\varphi) = P(\theta)Q(\varphi)R(r) \quad (6)$$

where $P(\theta)$ is a function of θ only, $Q(\varphi)$ of φ only and $R(r)$ of r only.

Substitution equation (6) into equation (5)

$$\frac{P(\theta)Q(\varphi)}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) + \frac{R(r)Q(\varphi)}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}P(\theta)\right) + \frac{P(\theta)R(r)}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Q(\varphi)$$
$$+\frac{2\mu}{\hbar^2}[E - V(r)]P(\theta)Q(\varphi)R(r) = 0 \quad (7)$$

Dividing by $P(\theta)Q(\varphi)R(r)$ on both sides,

$$\frac{1}{R(r)r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) + \frac{1}{P(\theta)r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}P(\theta)\right) + \frac{2\mu}{\hbar^2}[E - V(r)]$$
$$= -\frac{1}{Q(\varphi)r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Q(\varphi) \quad (8)$$

Multiplying equation (8) by $r^2\sin^2\theta$, we get

$$\frac{\sin^2\theta}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right) + \frac{\sin\theta}{P(\theta)}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) + \frac{2\mu r^2\sin^2\theta}{\hbar^2}[E - V(r)] = -\frac{1}{Q(\varphi)}\frac{d^2}{d\varphi^2}Q(\varphi) \quad (9)$$

According to equation (9), the R. H. S and L. H. S should equal to some arbitrary constant and we use m_ℓ^2.

Therefore R. H. S of equation (9) becomes

$$-\frac{1}{Q(\varphi)}\frac{d^2}{d\varphi^2}Q(\varphi) = m_\ell^2 \quad (10)$$

Similarly, L. H. S of equation (9) becomes

$$\frac{\sin^2\theta}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right) + \frac{\sin\theta}{P(\theta)}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) + \frac{2\mu r^2\sin^2\theta}{\hbar^2}[E - V(r)] = m_\ell^2 \quad (11)$$

Dividing equation (11) by $\sin^2\theta$, we get

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right) + \frac{2\mu r^2}{\hbar^2}[E - V(r)] = \frac{m_\ell^2}{\sin^2\theta} - \frac{1}{P(\theta)\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right) \quad (12)$$

Similarly, L. H. S and R. H. S should equal to some arbitrary constant.

Therefore, the R. H. S of equation (12) becomes

$$\frac{m_\ell^2}{\sin^2\theta}-\frac{1}{P(\theta)\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d}{d\theta}P(\theta)\right)=L^2 \tag{13}$$

and L. H. S becomes

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right)+\frac{2\mu r^2}{\hbar^2}[E-V(r)]=L^2 \tag{14}$$

Substitution the potential term

$$V(r)=-\frac{Ze^2}{4\pi\varepsilon_0 r} \tag{15}$$

We obtain the equation

$$\frac{1}{R(r)}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right)+\frac{2\mu r^2}{\hbar^2}\left[\frac{Ze^2}{4\pi\varepsilon_0 r}+E\right]=L^2 \tag{16}$$

By multiplying with $\frac{R(r)}{r^2}$, equation (16) becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}R(r)\right)+\left[\frac{2\mu}{\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0 r}+E\right)-\frac{\ell(\ell+1)}{r^2}\right]R(r)=0$$

Since the reduce mass $\mu=m$, multiplying with $\frac{\hbar^2}{2m}$, we get

$$\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\left(\frac{Ze^2}{r}+E\right)R-\frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}R=0 \tag{17}$$

where $4\pi\varepsilon_0=1$ in atomic unit.

After solving mathematical steps, we obtain the energy of omega hyperonic atom for the n^{th} orbit.

$$E_n=-\frac{Z^2 m_\Omega e^4}{2\hbar^2}\frac{1}{n^2} \tag{18}$$

Radius for Omega Hyperonic Atom

The Coulomb force between a stationary nucleus with charge +Ze and an orbiting electron with charge –e is

$$F=-\frac{kZe^2}{r^2} \tag{19}$$

From Newton's second law of motion, the centripetal force

$$F=-\frac{mv^2}{r} \tag{20}$$

From equation (19) and equation (20)

$$mv^2=\frac{kZe^2}{r} \tag{21}$$

But, from Bohr's postulate

$$mvr=n\hbar$$

$$v^2=\frac{n^2\hbar^2}{m^2r^2} \tag{22}$$

Substituting the equation (22) into equation (21) we get

$$r_n = \frac{n^2\hbar^2}{mkZe^2} \tag{23}$$

For omega hyperonic atom

$$r_n = \frac{n^2\hbar^2}{m_\Omega kZe^2}$$

Results and Discussions

The Energies of Hydrogen–Like $\mathbf{Pb^{81+}}$ Ionized Atom and $\Omega^- - \mathbf{Pb}$ Atom

Hydrogen–like heavy atoms are the heavy atoms from which all the electrons except one have been removed. We assumed $\Omega^- - \mathrm{Pb}$ atom to be like an ordinary hydrogen atom in which the electron is replaced by a negative omega hyperon. And then the energies of hydrogen–like Pb^{81+} ionized atom and $\Omega^- - \mathrm{Pb}$ atom for various states were calculated by solving the Schrödinger equation. From these results, the energy of $\Omega^- - \mathrm{Pb}$ atom is greater than Pb^{81+} ionized atom for each orbit. The calculated energies for these atoms are given in Table (1) and the energy levels of Pb^{81+} ionized atom are shown in Figure (1). Then the energy levels of $\Omega^- - \mathrm{Pb}$ atom are obtained as shown in Figure (2).

The Radii of Hydrogen–Like $\mathbf{Pb^{81+}}$ ionized atom and $\Omega^- - \mathbf{Pb}$ atom

The calculated radii of hydrogen–like Pb^{81+} ionized atom and $\Omega^- - \mathrm{Pb}$ atom are given in Table (2). From calculation, the radius of $\Omega^- - \mathrm{Pb}$ atom is very smaller than Pb^{81+} ionized atom for each orbit and the higher the orbits, the greater the size of these atoms. The inclusion of omega–minus hyperon reduces the size of the atom in such a way that the small radius is obtained. Moreover, the radius of $\Omega^- - \mathrm{Pb}$ atom is less than the nuclear radius for lower orbit is observed. Therefore, the existence of atomic state and nuclear state are in mixture. The mixture of the atomic state and nuclear state is called hybrid state. So we cannot distinguish the existence of the atomic state or the nuclear state of hydrogen–like $\Omega^- - \mathrm{Pb}$ atom for lower orbits.

Table (1) A comparison between energies of Hydrogen–like Pb^{81+} ionized atom and $\Omega^- - \mathrm{Pb}$ atom for various orbits

Orbital quantum number	Energy (MeV)	
	Hydrogen-like Pb^{81+} ionized atom	Hydrogen-like $\Omega^- - \mathrm{Pb}$ atom
1	- 0.0911625	- 298.5325
2	- 0.0227906	- 74.6331
3	- 0.0101292	- 33.1702
4	- 0.0056976	- 18.6582
5	- 0.0036465	- 11.9413
6	- 0.0025323	- 8.2925

Table (2) A comparison between radii of Hydrogen–like Pb^{81+} ionized atom and $\Omega^{-} - Pb$ atom for various orbits

Orbital quantum number	Radius (fm)	
	Hydrogen–like Pb^{81+} ionized atom	Hydrogen–like $\Omega^{-} - Pb$ atom
1	647. 6339	0.197767
2	2590.5356	0.791068
3	5828.7051	1.779030
4	10362.1424	3.164272
5	16190.8475	4.944175
6	23314.8204	7.119612

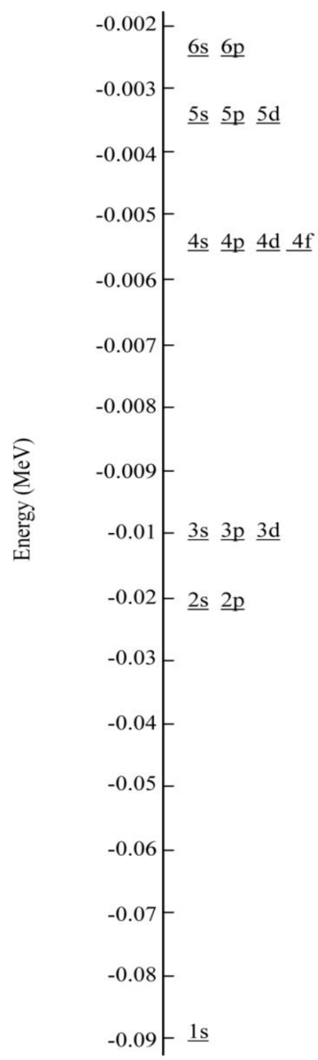

Figure (1) Energy levels of the Pb^{81+} ionized atom

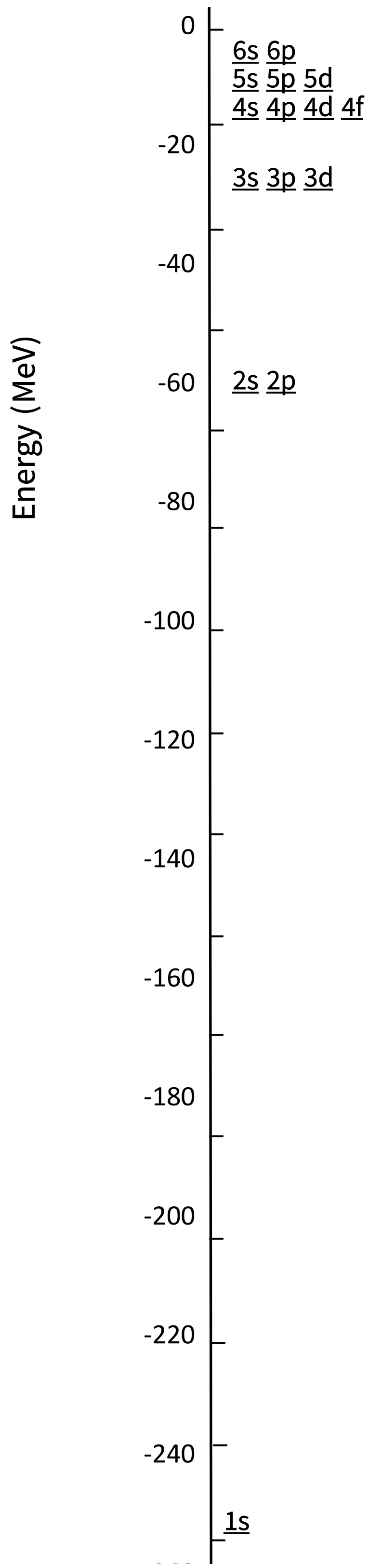

Figure (2) Energy levels of the $\Omega^{-} - \text{Pb}$ atom

Conclusion

We calculated the energies of hydrogen–like Pb^{81+} ionized atom and $\Omega^- - Pb$ atom for various states by solving the Schrödinger equation. And then, we also calculated the radii of hydrogen–like Pb^{81+} ionized atom and $\Omega^- - Pb$ atom to know about their structure. In our calculation, it was assumed that $\Omega^- - Pb$ atom is like an ordinary hydrogen atom in which the electron is replaced by a negative omega hyperon. From the calculations, it was concluded that the radius of $\Omega^- - Pb$ atom is less than nuclear radius and we cannot distinguish the existence of atomic state and nuclear state (hybrid state).

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