Determination of the nuclear structure of Kaonic Helium System

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Abstract

The nuclear structure calculation of kaonic helium atom and kaonic helium nucleus by assuming the two-body system which is composed of kaon and helium core nucleus. We used the Coulomb potential for kaonic helium atom. It is found that the binding energy of the kaonic helium atom is 0.046 MeV. Structure of konic nuclear system with Akaishi's optical potential was also studied. The binding energy of kaonic nuclear state is found to be 37.046 MeV.

Key words: helium core nucleus, kaon-nucleus interaction

Introduction

An exotic atom is the analogue of a normal atom in which one or more of the negatively charged electrons found in an ordinary atom are replaced by other negatively charged particle, such as a muon or a pion.

A kaonic atom is formed when an incident negatively charged particle is stopped in a target and captured into a high atomic Bohr orbit the nucleus, replacing one of the outer electrons.

Kaonic atoms and kaon nuclei carried the knowledge of K^{-} nucleon interaction in nuclear medium. This knowledge is important to discover the kaon properties at finite density and to determine the constraints on kaon condensation in high density matter. Experimental studies of the kaonic nuclear states using in-flight $(\mathrm{K}^{-},\mathrm{N})$ reactions were proposed and performed by Kishimoto and his collaborators. Experiments employing stopped $(\mathrm{K}^{-},\mathrm{N})$ reactions were carried out by Iwasaki and their collaborators. In this experiment, it was some possible indications of the existence of kaonic nuclear states with significantly narrow widths were found. Another indications of the K^{-}pp bound state were reported by the FINUDA experiment. There are also theoretical studies of the structure and information of kaonic nuclear states related to these experimental activities. It should be noted that these theoretical studies predict the possible existence of ultra-high density states in kaonic nuclear system. A

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critical analysis of the latest stopped K^- experimental data was also reported by Oset and Toki. Thus, under these theoretical and experimental conditions, it is very significant to observe the structure of kaonic atom and kaonic nuclei.

Kaon–Nucleon Interaction

In order to solve the $(K^- - \alpha)$ two–body system, it is necessary to know the $(K^- - \alpha)$ interaction. We use the optical potential for $(K^- - \alpha)$ interaction of Akaishi *et al.,* which is derived form a phenomenology $(\overline{K} - N)$ interaction. The two–body $(\overline{K} - N)$ interaction is constructed so as to reproduce the free $(\overline{K} - N)$ interaction. The I=0, $(\overline{K} - N)$ interactions are described as follows

$$V\frac{1}{KN}(r) = V_D^I \exp\left\{-(r/0.66\,\mathrm{fm})^2\right\} \tag{1}$$

with $V_D^{I=0}$ = 436 MeV. The g matrix in a nuclear medium is defined by

$$g = v + v\frac{Q_N}{E_{st} - Q_N \hat{T} Q_N} g\,. \tag{2}$$

The binding effects of $\overline{K}$ and N are properly taken into account through the starting energy, E_{st} which is a quantity independent of k. The $Q_N \hat{T} Q_N$ prescription is employed for intermediate states in the g matrix equation. The optical potential for deeply bounded nuclear states, starting energy is taken to be –110 MeV and g matrix is

$$g_0^{I=0} = -1704 - i0\ \mathrm{MeV fm}^3\,. \tag{3}$$

The optical potential between $\overline{K}$ and the core nucleus is constructed by folding g matric of (2) with density $\rho(r)$,

$$\rho(r) = A\left(\frac{A}{A-1}\frac{\beta}{\pi}\right)^{\frac{3}{2}} \exp\left(-\frac{A}{A-1}\beta r^2\right) \tag{4}$$

where the parameter β is related to the rms nuclear radius (R_{core}) as

$$R_{core} = \sqrt{\frac{3(A-1)}{2A\beta}}\,. \tag{5}$$

or in terms of harmonic oscillator strength $\left(\frac{\hbar^2}{M_N}\right)\beta = \hbar\omega$. The bound–state energy,

$E_{\overline{K}}$ is obtained by solving the $\overline{K}$–core relative motion

$$\left[-\frac{\hbar^2}{2\mu_{\bar{K}A}} \frac{d^2}{dr^2} + V_{\bar{K}A}(r) \right] u_{\bar{K}}(r) = E_{\bar{K}}\, u_{\bar{K}}(r) \tag{6}$$

$$V_{\bar{K}A}(r) = \int g(\vec{r} - \vec{r}\,')\, \rho(\vec{r}\,')\, d\vec{r}\,' \tag{7}$$

repeatedly in such a way that E_{st}, g and $E_{\bar{K}}$ become to be self–consistent at a given β. The optical potential for $(\bar{K}-\alpha)$ interaction which is obtained in the above mentioned procedure is

$$V_{\bar{K}\alpha}(r) = \{(-285 - i\,22)\} \exp\left[-(r/1.18\,fm)^2 \right] \tag{8}$$

with $\hbar\omega$= 30 MeV. The potential is displaced in figure (1).

Two–Body Schroedinger Equation for $K^- - \alpha$ system

In order to calculate the energy eigen value of $(K^- - \alpha)$ we solved Schroedinger equation,

$$H\,\Psi = E\,\Psi\ ,$$

$$(T+V)\,\Psi = E\,\Psi \tag{9}$$

where T and V are kinetic energy and potential energy operator. H is Hamiltonian operator, E is energy eigen value and Ψ is eigen vector.

Since the interaction between kaon and the core nucleus ^4_2He is central force, we used the spherical coordinate for the wave function,

$$\Psi(\vec{r}) = R(r)\,\Theta(\theta)\,\Phi(\varphi)\,. \tag{10}$$

where R(r) is the radial part and $\Theta(\theta)\Phi(\varphi)$ is the angular part. By using the separable method, the following radial equation is obtained,

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu} \frac{\ell(\ell+1)}{r^2} \right] u(r) = E\, u(r)\,. \tag{11}$$

where, u(r) = r R(r), M_α= mass of α core nucleus,

M_{K^-}= mass of kaon, $\mu = M_\alpha M_{K^-} / M_\alpha + M_{K^-}$ = the reduced mass

We have to solve eq. (11), we expand the wave function u(r) with Gaussian basis,

$$u(r) = r^{\ell+1} \sum_{j=1}^{N_b} c_j\, e^{-(r/b_j)^2} \tag{12}$$

where, c_j's are expansion coefficients, b_j's are range parameter and N_b is the number of basis all of which are to be adjusted in the calculations. b_j's are chosen to be geometric progression as follow,

$b_2/b_1 = b_3/b_2 = b_4/b_3 = \ldots = c$,

$$c = (b_{N_b} / b_1)^{1/N_b - 1}$$

Multiplying both sides of the equation by $r^{\ell+1} e^{-(r/b_j)^2}$ from the left and integration through the equation,

$$\int r^{\ell+1} e^{-(r/b_i)^2} \left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{\hbar^2}{2\mu} \frac{\ell(\ell+1)}{r^2} + V(r) \right] \sum_j c_j r^{\ell+1} e^{-(r/b_j)^2} dr$$

$$= E \int r^{\ell+1} e^{-(r/b_i)^2} \sum_j c_j r^{\ell+1} e^{-(r/b_j)^2} dr$$

$$\sum_j \left[T_{ij}^{\ell} + F_{ij}^{\ell} + V_{ij}^{\ell} \right] c_j = E \sum_j N_{ij}^{\ell} c_j \qquad (13)$$

The Norm Matrix Element

$$N_{ij}^{\ell} = \int r^{2(\ell+1)} e^{-(r/b_i)^2} e^{-(r/b_j)^2} dr = \frac{(2\ell+1)!!}{2^{\ell+1}} \frac{\sqrt{\pi}}{\left(r/b_i^2 + r/b_j^2 \right)^{\ell+\frac{3}{2}}} \qquad (14)$$

The Kinetic Energy Matrix Element

$$T_{ij}^{\ell} = \int r^{\ell+1} e^{-(r/b_i)^2} \left\{ -\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2}) \right\} r^{\ell+1} e^{-(r/b_i)^2} dr$$

$$T_{ij}^{\ell} = -\frac{\hbar^2}{2\mu} N_{ij}^{\ell} \left[\frac{2(2\ell+3)}{b_j^4 \left(1/b_i^2 + 1/b_j^2 \right)} - \frac{(4\ell+6)}{b_j^4} + \frac{2\ell(\ell+1) \left(1/b_i^2 + 1/b_j^2 \right)}{(2\ell+1)} \right] \qquad (15)$$

The Centrifugal Potential Energy Matrix Element

$$F_{ij}^{\ell} = \frac{\hbar^2}{2\mu} \int r^{\ell+1} e^{-(r/b_i)^2} \frac{\ell(\ell+1)}{r^2} r^{\ell+1} e^{-(r/b_j)^2} dr$$

$$F_{ij}^{\ell} = \frac{\hbar^2}{2\mu} N_{ij}^{\ell} \frac{2\ell(\ell+1) \left(1/b_i^2 + 1/b_j^2 \right)}{(2\ell+1)} \qquad (16)$$

$$T_{ij}^{\ell} + F_{ij}^{\ell} = \frac{\hbar^2}{2\mu} N_{ij}^{\ell} \frac{(4\ell+6)}{b_i^2 + b_j^2} \qquad (17)$$

The Potential Energy Matrix Element

$$V_{ij}^{\ell} = \int r^{\ell+1} e^{-(r/b_i)^2} V(r) r^{\ell+1} e^{-(r/b_j)^2} dr \ , \quad V_{ij}^{\ell} = -e^2 \left[\frac{2\ell!!}{2^{\ell+\frac{3}{2}}} \frac{\sqrt{\pi}}{\left(1/b_i^2 + 1/b_j^2 \right)^{\ell+1}} \right] \qquad (18)$$

H_{ij}^{ℓ} is the Hamiltonian matrix element

$$H_{ij}^{\ell} = T_{ij}^{\ell} + F_{ij}^{\ell} + V_{ij}^{\ell},$$
$$\sum_j H_{ij}^{\ell} c_j = E \sum_j N_{ij}^{\ell} c_j \qquad (19)$$

The Schroedinger equation can be written as the following matrix equation.

$$\begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1N_b} \\ H_{21} & H_{22} & \cdots & H_{2N_b} \\ \vdots & \vdots & \cdots & \vdots \\ H_{N_b1} & H_{N_b2} & \cdots & H_{N_bN_b} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{N_b} \end{bmatrix} = E \begin{bmatrix} N_{11} & N_{12} & \cdots & N_{1N_b} \\ N_{21} & N_{22} & \cdots & N_{2N_b} \\ \vdots & \vdots & \cdots & \vdots \\ N_{N_b1} & N_{N_b2} & \cdots & N_{N_bN_b} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{N_b} \end{bmatrix}$$

$$[H]\,[c] = E\,[N]\,[c]$$
$$[N]^{-1}\,[H]\,[c] = E\,[c]$$
$$[A]\,[c] = E\,[c]$$

where [H] and [N] are square matrices and [c] is a column matrix. An eigenvalue equation with [A] = $[N]^{-1}$ [H], [N] and [H] are called the norm matrix and the Hamiltonian matrix. We used the power inverse iteration method to calculate eigenvalues E with corresponding eigenvectors c. The necessary computations are performed by writing a Fortran software.

Root–Mean–Square Distance of Kaonic–Helium Atom

In our calculation, we used the radial wave function which is expanded as

$$u(r) = r^{\ell+1} \sum_{j=1}^{N_b} c_j e^{-(r/b_j)^2}$$

To evaluate the normalization constant, the normalization condition is $\int u^* u \, dr = 1$

$$u = A \sum_j c_j r^{\ell+1} e^{-(r/b_j)^2} \text{ and } A = \frac{1}{\sqrt{\sum_i \sum_j c_i^* c_j N_{ij}^{\ell}}} \qquad (20)$$

where, u is the normalization wave function and A is the normalization constant. We calculated the Root–mean–square distance of kaonic–helium atom as follows

$$\left\langle r^2 \right\rangle = \left\langle u \middle| r^2 \middle| u \right\rangle = \int u^* r^2 u \, dr$$

$$\left\langle r^2 \right\rangle = \int A^* \sum_i c_i^* r^{\ell+1} e^{-(r/b_i)^2} A \sum_j c_j r^{\ell+1} e^{-(r/b_j)^2} \, dr$$

$$\left\langle r^2 \right\rangle = \left| A^2 \right| \sum_i \sum_j c_i^* c_j \frac{(2\ell+3)!!}{2^{\ell+3}} \frac{\sqrt{\pi}}{\left(r/b_i^2 + r/b_j^2 \right)^{\ell+\frac{5}{2}}} \qquad (21)$$

Root–mean–square distance = $\sqrt{\langle r^2 \rangle}$

Conclusion

The binding energy of kaonic helium atom and kaonic helium nucleus has been calculated by solving the two–body Schroedinger equation with Gaussian basis wave function. Where b_1 and b_N are Gaussian basic parameter and N is the number of basic. We used the parameters for this computations are N=40, b_1=0.1 fm and b_N =20 fm. The size of the matrix to be diagonalized is N×N and the total number of matrix elements is (1600).

In our calculation, we used the Coulomb potential for kaonic helium atom and optical potential for kaonic helium nucleus. The binding energy of kaonic helium atom is 0.046 MeV and root–mean–square distance is 27 fm. For kaonic helium nucleus, the binding energy is 37.046 MeV, level width is 8.71 MeV and root–mean–square distance is 1.32 fm. The density distribution of relative motion between kaonic helium atom is shown in figure (2).

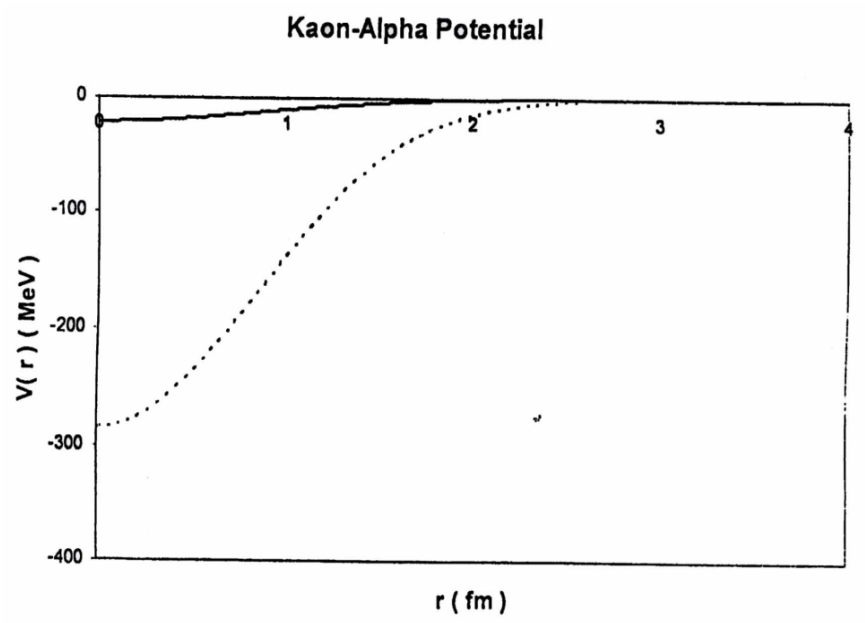

Figure 1. $K^- - \alpha$ optical potential where solid line and dotted line represent the imaginary part and the real part

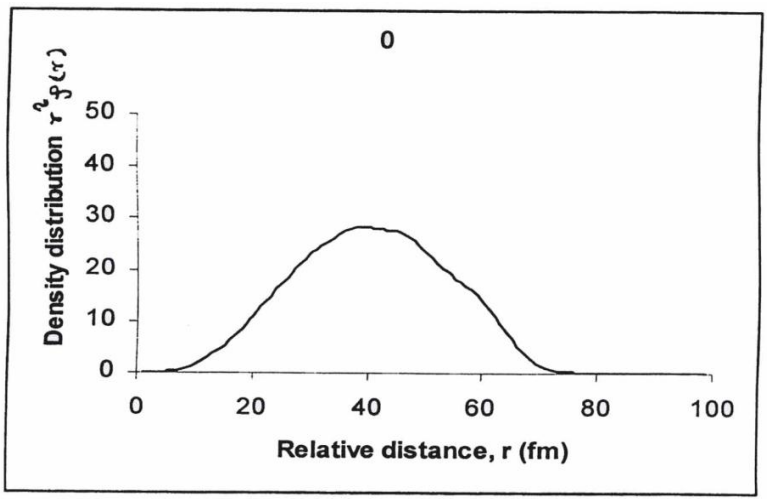

Figure 2. The density distribution of relative motion between kaon–alpha in kaonic helium atom

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