APPROXIMATE SOLUTIONS ON FREE SURFACE PROFILE OF HYDRAULIC JUMP BY MEANS OF SIMPLIFIED DEPTH AVERAGED FLOW MODEL

Kyoto University	Student Member	○Thin Thwe Thwe
Kyoto University	Fellow Member	Takashi Hosoda
Dept. of Civil Eng., IITRM	Non-Member	M. N. Langhi
Kawasaki Heavy Industries Ltd.	Member	Daiki Chifu

1. INTRODUCTION

A simple depth-averaged flow model is derived considering the deformation of stream-wise velocity to reproduce the characteristics of hydraulic jump. Initially, representing the stream-wise velocity distribution using a power series of dimensionless depth, the dependency of coefficients in the power series on the spatial coordinate is formulated using 2-D continuity and momentum equations. The water surface profile equation for a hydraulic jump is deduced using the depth-averaged continuity and momentum equation with iteration procedures. Then the calculated approximate solutions for water surface profiles are verified with the previous experimental data.

2. MODEL FORMULATION

The 2-D horizontal momentum equation is adopted in deriving the approximate solution for a hydraulic jump (see Fig.1).

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial h}{\partial x}+D_{my}\frac{\partial^2 u}{\partial y^2} \tag{1}$$

where $(x, y)=$ special coordinates, $u=$ horizontal velocity component, $v=$ vertical velocity component, $h=$ water depth, $g=$ gravitational acceleration, $D_{my}=$ eddy diffusivity ($\equiv \beta' Uh$), $U=$ depth averaged velocity.

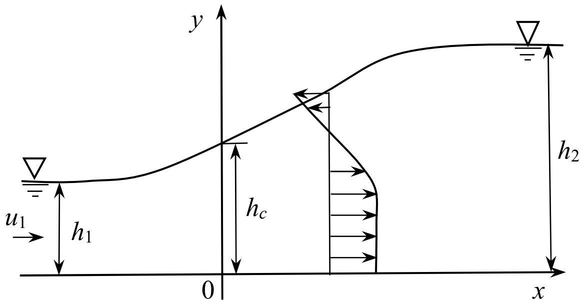

Fig.1 Coordinate system and explanation of symbols

The stream-wise velocity distribution is assumed to be represented by the power series Eq.(2).

$$u=u_0+u_1\eta+u_2\eta^2+.... \qquad \text{where} \quad \eta=y/h \tag{2}$$

The vertical velocity distribution is derived integrating the 2-D continuity equation as Eq.(3).

$$v=-\int_0^h\frac{\partial u}{\partial x}dy=-h\sum_{i=0}^{2}\left(\frac{\partial u_i}{\partial x}-i\frac{u_i}{h}\frac{\partial h}{\partial x}\right)\frac{\eta^{i+1}}{i+1} \tag{3}$$

Substituting Eq.(2) and Eq.(3) into Eq.(1) leads to the relation between u_2 and the other variables as Eq.(4).

$$u_2=\frac{h^2}{2D_{my}}\left(u_0\frac{\partial u_0}{\partial x}+g\frac{\partial h}{\partial x}\right) \tag{4}$$

Although Madsen et al.[1)] neglected the effect of bottom friction, the effect of bottom friction on the velocity distribution is taken into account through the relation between u_0, u_1 and the friction velocity u_* as $u_0=r_*u_*$ and $u_1=u_*^2/\beta' U$ where r_* is a constant.

The depth averaged momentum equation is obtained integrating Eq.(1) from the bottom to the water surface as Eq.(5).

$$\frac{d}{dx}\int_0^h u^2dy=-gh\frac{dh}{dx}+\frac{\tau_{xb}}{\rho} \tag{5}$$

where τ_{xb} is bottom shear stresses.

Although the bottom friction term on the right side of Eq.(5) is neglected for simplicity, the method of analysis in this study is applicable to the case with the term.

The unit width discharge q and the momentum flux M_0 are defined as Eq.(6) and (7).

$$\frac{q}{h}=u_0+\frac{1}{2}u_1+\frac{1}{3}u_2 \tag{6}$$

$$M_0=h\left(u_0^2+u_0u_1+\frac{1}{3}u_1^2+\frac{2}{3}u_0u_2+\frac{1}{2}u_1u_2+\frac{1}{5}u_2^2\right)+\frac{1}{2}gh^2 \tag{7}$$

Starting with $u_0=U$ as the first step, the iteration procedures with Eq.(6) are applied to obtain better approximation for u_0, u_1 and u_2. Eq.(8) is the momentum equation in non-dimensional form obtained after repeating the iteration procedures three times, where $h'=h/h_c$, $x'=x/h_c$, $h_c=$ critical depth..

$$1+\frac{1}{2}h'^3-h'Fr_1^{2/3}X_1+\frac{1}{12}\frac{1}{\alpha^2r_*^4}A^2+\frac{1}{45\alpha^2}\left(B+h'^3\frac{dh'}{dx'}\right)+\frac{1}{12\alpha^2r_*^2}A\left(B+h'^3\frac{dh'}{dx'}\right)=0 \tag{8}$$

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Dept. of Urban Management., Kyoto Univ. (C1-3-265, Kyodai Katsura, Nishikyo-ku, Kyoto 615-8540, Tel. 075-383-3266)

where $\beta' = \alpha = \frac{D_{my}}{q}$, $M_0 = \frac{q^2}{h_1} + \frac{1}{2} g h_1^2$, $X_1 = 1 + \frac{1}{2}\frac{1}{Fr_1^2}$, $X_2 = 1 - \frac{1}{2}\frac{1}{\alpha r_*^2}$, $F = 1 - h'^3$, $A = X_2^2 + \frac{1}{3\alpha} X_2 F \frac{dh'}{dx'} + \frac{1}{36\alpha^2} F^2 \left(\frac{dh'}{dx'}\right)^2$

$$B = -X_2^2 \frac{dh'}{dx'} - \frac{1}{6\alpha} X_2 \left(1 + 2h'^3 + F\right)\left(\frac{dh'}{dx'}\right)^2 - \frac{1}{36\alpha^2} F\left(1 + 2h'^3\right)\left(\frac{dh'}{dx'}\right)^3 + \frac{1}{6\alpha} X_2 F h' \frac{d^2 h'}{dx'^2} + \frac{1}{36\alpha^2} F^2 h' \frac{dh'}{dx'}\frac{d^2 h'}{dx'^2}$$

3. DERIVATION PROCESSES OF APROXIMATE SOLUTIONS FOR HYDRAULIC JUMP

Referring to Fig. 1, the profiles for the negative and positive regions are calculated separately to find out the approximate solutions considering the boundary conditions at the origin(x=0). The surface profiles are assumed to be represented in the following equations with three terms of exponential functions.

For the negative region $x \le 0$: $h' = h_1 + U_{1n} \exp(\gamma_1 x) + U_{2n} \exp(2\gamma_1 x) + U_{3n} \exp(3\gamma_1 x)$ (9a)

For the positive region $x > 0$: $h' = h_2 - U_{1p} \exp(\beta_1 x) - U_{2p} \exp(2\beta_1 x) - U_{3p} \exp(3\beta_1 x)$ (9b)

where h_1 and h_2 are the sequent depths before and after the jump. the coefficients $U_{1n}, U_{2n}, U_{3n}, U_{1p}, U_{2p}, U_{3p}$ and the constants β_1, γ_1 are calculated taking into account of the boundary conditions at the upstream and downstream boundaries.

For the upstream end $x = x_u$: $h' = h_1,\ d^2h'/dx'^2 = 0$ (9a)

For the downstream end $x = +\infty$: $h' = h_2,\ dh'/dx' = 0,\ d^2h'/dx'^2 = 0$ (9b)

All coefficients are calculated equating dh'/dx', d^2h'/dx'^2 and d^3h'/dx'^3 of the negative and positive regions at the origin.

4. CALCULATED RESULTS

The calculated surface profiles with two and three exponential terms are compared with the previous experimental data used by Madsen et al. (1983) in Fig.2. The surface profiles with different parameters are displayed in Fig. 3. It is pointed out that the values of $\alpha = 0.09$ and $r_* = 10$ gives the reasonable profilein accordance with the experimental results.

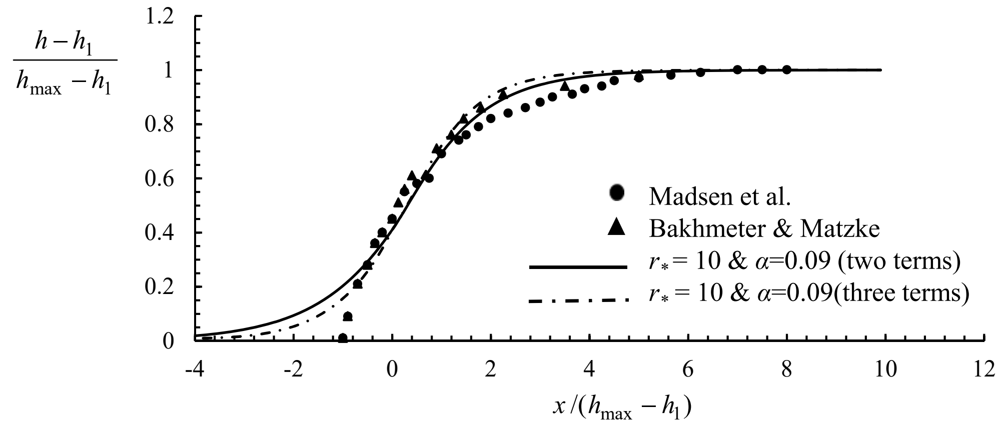

Fig.2 Comparison of approximate solutions and experiments

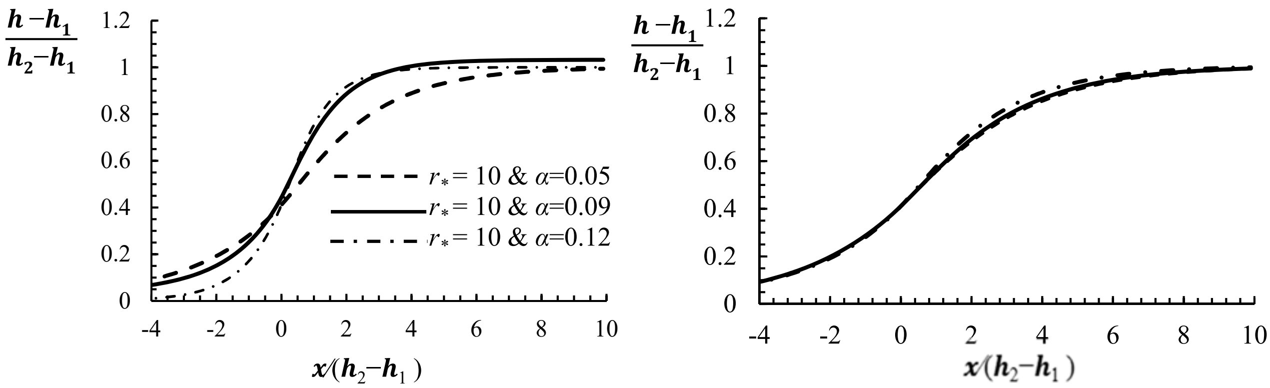

Fig.3 Comparisons of surface profiles with different parameters

5. CONCLUSIONS

It can be pointed out that the approximate solutions of water surface profiles show shows good agreement with the experimental results, although the velocity distributions of the solutions should be compared to the experimental ones.

REFERENCES

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