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# A Study on Weakly Preopen and Weakly Preclosed Functions

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**Abstract**

In this paper we study two classes of functions called weakly preopen and weakly preclosed functions as generalization of weak openness and weak closedness respectively. We obtain their characterizations, their basic properties and their relationships with other types of functions between topological spaces.

**Key words:** regularopen set, regular closed set, strongly continuous function, weakly open function, weakly closed function, weakly preopen function, weakly preclosed function.

## 1. Introduction and Preliminaries

The aim of this paper is to define and study the class of weakly preopen functions and weakly preclosed functions as a new generalization of weakly open functions and weakly closed functions. We investigate some of the fundamental properties of this class of functions.

Throughout this paper, $(X,\tau)$ and $(Y,\sigma)$ (or simply, $X$ and $Y$ ) denote topological spaces on which no separation axioms are assumed unless explicitly stated. We recall the following definitions and properties that will be used in this paper.

**(viii) Interior and Closure Properties**

If $A$ is any subset of X, then the ***interior of a set A*** in $X$ is the union of all open sets contained in $A$ and is denoted by ***Int(A)***. The ***closure of a set A*** in $X$ is the intersection of all closed subsets of $X$ containing $A$, and is denoted by ***Cl(A).***Some properties of interior and closure of a set in general topology are described as the followings.

(1) $Int(\phi) = \phi$ and $Int(X) = X$ .

(2) $Int(A) \subseteq A$.

(3) $Int(Int(A)) = Int(A)$.

(4) If $A \subseteq B$, then $Int(A) \subseteq Int(B)$.

(5) $A$ is open $\Leftrightarrow A = Int(A)$.

(6) $Cl(\phi) = \phi$ and $Cl(X) = X$ .

(7) $A \subseteq Cl(A)$.

(8) $Cl(Cl(A)) = Cl(A)$.

(9) If $A \subseteq B$, then $Cl(A) \subseteq Cl(B)$.

(10) $A$ is closed $\Leftrightarrow A = Cl(A)$.

(11) $Cl(A)$ is the smallest closed super set of $A$.

(12) $Int(A)$ is the largest open subset of $A$.

(13) $X \setminus Int(A) = Cl(X \setminus A)$.

(14) $X \setminus Cl(A) = Int(X \setminus A)$.

**1.2 Some Generalizations of Open Set**

A subset $A$ of space $X$ is called

(ix) ***preopen***if $A \subseteq Int(Cl(A))$.

(ii) ***preclosed***if $Cl(Int(A)) \subseteq A$.

(x) ***regular open*** if $A = Int(Cl(A))$.

(iv) ***regular closed*** if $A = Cl(Int(A))$.

(v) The ***preinteriorof A*** is the union of all preopen sets contained in $A$ and is denoted by ***pInt(A)***. Since the union of preopen sets is preopen, then $pInt(A)$ is preopen.

(vi) The ***preclosureof A*** is the intersection of all preclosed sets containing $A$ and is denoted by

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***pCl*(*A*)**, which is the smallest preclosed set in *X* containing *A*.

**(xi) Preinterior and Preclosure Properties**

Let *A* and *B* be two subsets of a topological space $(X,\tau)$. Then the followings are hold:

(1) $pInt(\phi)=\phi$ and $pInt(X)=X$.

(2) $pInt(A)\subseteq A$.

(3) *A* is preopen $\Leftrightarrow A = pInt(A)$.

(4) $pCl(\phi)=\phi$ and $pCl(X)=X$.

(5) $A\subseteq pCl(A)$.

(6) *A* is preclosed $\Leftrightarrow A = pCl(A)$.

(7) $X \setminus pInt(X\setminus A) = pCl(A)$.

(8) $X \setminus pCl(X\setminus A) = pInt(A)$.

**1.4 Theorem**

(i) Every open set is preopen.

(ii) Every closed set is preclosed.

(xii) Every regular open set is open.

**Proof**:

(xii) Let *A* be an open set. Then $Int(A)\subseteq Int(Cl(A))$, and we have $A\subseteq Int(Cl(A))$.
So *A* is preopen.

(xiii) Let *A* be a closed set.Then $Cl(Int(A))\subseteq Cl(A)$,and we have $Cl(Int(A))\subseteq A$.
So *A* is closed.

(xiv) Let *A* be a regular open set. Then $A=Int(Cl(A))$,so, $Int(A)=Int(Int(Cl(A)))$.
Thus $Int(A)=Int(Cl(A))=A$. Hence *A* is open.

**(xv) Some Generalizations of Continuous Functions**

A function $f:(X,\tau)\to(Y,\sigma)$ is called:

(xvi) ***precontinuous*** if for each open subsets V of Y, $f^{-1}(V)$ is preopen in X .

(ii) ***weakly open*** if $f(U)\subseteq Int(f(Cl(U)))$ for each open subset *U* of *X*.

(xvii) ***weakly closed*** if $Cl(f(Int(F)))\subseteq f(F)$ for each closed subset *F* of *X*.

(iv***) relatively weakly open*** if *f*(*U*) is open in *f*(*Cl*(*U*)) for every open subset *U* of *X*.

(v) ***strongly continuous*** if for every subset *A* of *X*, $f(Cl(A))\subseteq f(A)$.

(vi) ***open*** if for each open set *U* of *X*, *f*(*U*) is open in *Y*.

(xviii) ***closed*** if for each closed set *F* of X, *f*(*F*) is closed in *Y*.

(xix) ***preopen*** if for each open set *U* of *X*, *f* (*U*) is preopen in *Y*.

(xx) ***preclosed*** if for each closed set *F* of *X*, *f* (*F*) is preclosed in *Y*.

## 2.Weakly Preopen Functions

**2.1 Definition**. A function $f:(X,\tau)\to(Y,\tau)$ is said to be ***weakly preopen***
if $f(U)\subseteq pInt\,(f(Cl(U))$ for each open set *U* of *X*.

**2.2 Proposition** . A function $f:X\to Y$ is a preopen function if and only if
$f^{-1}(Cl(V)\subseteq Cl(f^{-1}(V))$ for each open subset *V* of *Y*.

**Proof**: Assume that *f* is a preopen. Let *V* be any open subset of *Y*.
Suppose $x\in f^{-1}(Cl(V))$,and *U* is an open subset of *X* containing *x*,
then $f(x)\in f(U)\cap(Cl(V))$ ,
$f(x)\in Int(Cl(f(U)))\cap(Cl(V))$

so that $V \cap Int(Cl(f(U))) \neq \phi$, since $Cl(V)$ is closed.

Thus $V \cap f(U) \neq \phi$ , so $U \cap f^{-1}(V) \neq \phi$.

Hence $x \in Cl(f^{-1}(V))$.

Conversely, suppose that $f^{-1}(Cl(V)) \subseteq Cl(f^{-1}(V))$ for each open subset $V$ of $Y$. If $f$ is not preopen then for some open subset $U$ of $X$, $f(U) \not\subset \text{Int}(Cl(f(U)))$ .Let $V = Y \setminus Cl(f(U))$. Then $f(U) \cap V = \phi$ but $f(U) \cap Cl(V) \neq \phi$.Thus $U \cap f^{-1}(Cl(V)) \neq \phi$, and, by hypothesis, $U \cap Cl(f^{-1}(V)) \neq \phi$. Hence $U \cap f^{-1}(V) = \phi$ is contradiction to the fact that $f(U) \cap V = \phi$.

**2.3 Proposition** . If $f : X \to Y$ is a preopen function, then $Cl(f(U)) \subseteq f(Cl(U))$ for each open set $U$ of $X$.

**Proof**: Let $U$ be an open set in $X$. Let $V = f(U)$. Then $U \subseteq f^{-1}(V)$. Since $f$ is preopen, then by proposition 2.2, $f^{-1}(Cl(V)) \subseteq Cl(f^{-1}(V))$. Thus $f^{-1}(Cl(V)) \subseteq Cl(U)$ and $Cl(V) \subseteq f(Cl(U))$. Therefore $Cl(f(U)) \subseteq f(Cl(U))$.

**2.4 Proposition**. If the function $f : X \to Y$ is preopen and if for each open set $U$ of $X$, then $f$ is weakly open.

**Proof**: Since $f$ is preopen, then for each open subset U of X such that $f(U) \subseteq Int(Cl(f(U)))$. From Proposition 2.3, we have $f(U) \subseteq Int(f(Cl(U)))$. Thus $f$ is weakly open.

**2.5 Proposition**.

(i) Every weakly open function is weakly preopen.

(ii) Every preopen function is weakly preopen.

**Proof(i):** Let $f : (X, \tau) \to (Y, \sigma)$ be a weakly open function. Since $f$ is weakly open, then $f(U) \subseteq Int(f(Cl(U)))$ for each open $U$ of $X$.

Moreover, $Int(f(Cl(U)))$ is open and then we have $Int(f(Cl(U)))$ which is preopen.

Thus $Int(f(Cl(U))) \subseteq pInt(Int(f(Cl(U))))$.

Furthermore, $pInt(Int(f(Cl(U)))) \subseteq pInt(f(Cl(U)))$.

Hence, $f(U) \subseteq pInt(f(Cl(U)))$ and we have $f$ which is weakly preopen.

**Proof(ii)**: Let $f : (X, \tau) \to (Y, \sigma)$ be a preopen function. Since $f$ is peropen, then $f(U) \subseteq Int(Cl(f(U)))$, for each open $U$ of $X$.

By proposition 2.3, $f(U) \subseteq Int(f(Cl(U)))$, so we have, $f(U) \subseteq pInt(f(Cl(U)))$.

Hence $f$ is weakly preopen.

**2.6 Example**. A weakly preopen function need not be weakly open.

Let $X = \{a, b\}$, $\tau = \{\phi, \{a\}, \{b\}, X\}$ , $Y = \{x, y\}$ and $\sigma = \{\phi, y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be given by $f(a) = x$ and $f(b) = y$.

We have $\phi$ , $X$ , $\{a\}$ , $\{b\}$ are preopen sets in $X$ and $\phi$ , $Y$ , $\{x\}$ , $\{y\}$ are preopen sets in $Y$.

$pInt(f(Cl(\phi))) = pInt(f(\phi)) = pInt(\phi) = \phi$,

$pInt(f(Cl(X))) = pInt(f(X)) = pInt(Y) = Y$,

$pInt(f(Cl(\{a\}))) = pInt(f(\{a\})) = pInt(\{x\}) = \{x\}$,

$pInt(f(Cl(\{b\}))) = pInt(f(\{b\})) = pInt(\{y\}) = \{y\}$.

Hence $\phi = f(\phi) \subseteq pInt(f(Cl(\phi))) = \phi$,

$Y = f(X) \subseteq pInt(f(Cl(X))) = Y$,

$\{x\} = f(\{a\}) \subseteq pInt(f(Cl\{a\})) = \{x\}$,

$\{y\} = f(\{b\}) \subseteq pInt(f(Cl(\{b\}))) = \{y\}$

and so f is weakly preopen. But, $f$ is not weakly open, since

$\{y\} = f(\{b\}) \not\subset Int(f(Cl(\{b\}))) = \phi$.

**2.7 Definition**. A topological space $(X,\tau)$ is said to be a ***regular space*** if for any closed subset $A$ of $X$ and any point $x \in X \setminus A$, there exist open sets $U$ and $V$ such that $x \in U, A \subseteq V$ and $U \cap V = \phi$.

**2.8 Theorem**. Let $X$ be a regular space. Then $f:(X,\tau) \to (Y,\sigma)$ is weakly preopen if and only if $f$ is preopen.

**Proof**: The sufficiency is clear. We will prove that the necessity.

Let $W$ be a nonempty open subset of $X$. For each $x$ in $W$, let $U_x$ be an open set such that $x \in U_x \subseteq Cl(U_x) \subseteq W$. Hence we obtain that $W = \cup\{U_x \mid x \in W\} = \cup\{Cl(U_x) \mid x \in W\}$ and $f(W) = \cup\{f(U_x) \mid x \in W\} \subseteq \cup\{pInt(f(Cl(U_x))) \mid x \in W\} \subseteq pInt(f(\cup\{Cl(U_x) \mid x \in W\}))$
$= pInt(f(W))$.

Thus, $f$ is preopen.

**2.9 Theorem**. For a bijective function $f:(X,\tau) \to (Y,\sigma)$, the following conditions are equivalent:

(xxi) $f$ is weakly preopen.

(ii) For each $x \in X$ and each open subset $U$ of $X$ containing $x$, there exists a preopen set $V$ containing $f(x)$ such that $V \subseteq f(Cl(V))$.

(xxii) For each closed subset $F$ of $X$, $f(Int(F)) \subseteq pInt(f(F))$.

(iv) For each open subset $U$ of $X$, $f(Int(Cl(U))) \subseteq pInt(f(Cl(U)))$.

(v) For every preopen subset $U$ of $X$, $f(U) \subseteq pInt(f(Cl(U)))$.

**Proof**:

(i)$\Rightarrow$(ii) Let $x \in X$ and $U$ be an open set in $X$ with $x \in U$. Since $f$ is weakly preopen, then $f(x) \in f(U) \subseteq pInt(f(Cl(U)))$. Let $V = pInt(f(Cl(U)))$. Hence $V \subseteq f(Cl(U))$, with $V$ containing $x$.

(ii)$\Rightarrow$(i) Let $U$ be an open set in $X$ and let $y \in f(U)$. Since $V \subseteq f(Cl(U))$, for some preopen $V$ in $Y$ containing $y$. Hence, we have $y \in V \subseteq pInt(f(Cl(U)))$. This shows that $f(U) \subseteq pInt(f(Cl(U)))$, i.e., $f$ is a weakly preopen function.

(i)$\Rightarrow$(iii) Let $f$ be a weakly preopen set and $U$ be an open set in $X$. So, we have $f(U) \subseteq pInt(f(Cl(U)))$. Let $F$ be a closed subset of $X$ and $U = Int(F)$.

Then we have $f(Int(F)) \subseteq pInt(f(Cl(Int(F))))$. Since $F$ is closed then $F$ is also preclosed, so we obtain $f(Int(F)) \subseteq pInt(f(F))$.

(iii)$\Rightarrow$(iv) Let $F$ be a closed set of $X$ and $f(Int(F)) \subseteq pInt(f(F))$. Let $U$ be an open subset of $X$. Then $Cl(U)$ is closed. So, we choose $F = Cl(U)$. Thus $f(Int(Cl(U))) \subseteq pInt(f(Cl(U)))$.

(iv)$\Rightarrow$(v) Let $U$ be a preopen subset of $X$. So, $U \subseteq Int(Cl(U))$ and so we have $f(U) \subseteq f(Int(Cl(U)))$. Since $f(Int(Cl(U))) \subseteq pInt(f(Cl(U)))$, then $f(U) \subseteq pInt(f(Cl(U)))$.

(v)$\Rightarrow$(i) Let $U$ be an open subset of $X$. Then $U$ is preopen and from (v), $f(U) \subseteq pInt(f(Cl(U)))$.

**2.10 Theorem**. Let $f:(X,\tau) \to (Y,\sigma)$ be a bijection function. Then the following statements are equivalent.

(i) $f$ is weakly preopen.

(ii) $pCl(f(Int(F))) \subseteq f(F)$, for each $F$ closed in $X$.

(iii) $pCl(f(U)) \subseteq f(Cl(U))$, for each $U$ open in $X$.

**Proof**:

(i)$\Rightarrow$(ii) Let $F$ be a closed set in $X$. Then, we have $f(X \setminus F) = Y \setminus f(F)$. Since $f(X \setminus F) \subseteq pInt(f(Cl(X \setminus F)))$, then $Y \setminus f(F) \subseteq Y \setminus pCl(f(Int(F)))$. Hence $pCl(f(Int(F))) \subseteq f(F)$.

(ii)$\Rightarrow$(iii) Let $U$ be an open set in $X$. Since $Cl(U)$ is a closed set and $U \subseteq Int(Cl(U))$, then $pCl(f(U)) \subseteq pCl(f(Int(Cl(U)))) \subseteq f(Cl(U))$.

(iii)$\Rightarrow$(ii) Let $F$ be a closed set in $X$. Since $Int(F)$ is open and $Cl(Int(F)) \subseteq F$, then $pCl(f(Int(F))) \subseteq f(Cl(Int(F))) \subseteq f(F)$.

(ii)$\Rightarrow$(i) Let $U$ be an open set in $X$ and $F = X \setminus U$. Since $pCl(f(Int(X \setminus U))) \subseteq f(X \setminus U)$, then $Y \setminus pInt(f(Cl(U))) \subseteq Y \setminus f(U)$. Thus, we have $f(U) \subseteq pInt(f(Cl(U)))$.

**2.11 Theorem**. If $f:(X,\tau) \to (Y,\sigma)$ is weakly preopen and strongly continuous, then $f$ is preopen.

**Proof**: Let $U$ be an open subset of $X$. Since $f$ is weakly preopen, then $f(U) \subseteq pInt(f(Cl(U)))$. Moreover, $f$ is strongly continuous, so we have $f(U) \subseteq pInt(f(U))$ and therefore, $f(U)$ is preopen. Hence $f$ is preopen.

**2.12 Example**. A preopen function need not be strongly continuous.

Let $X = \{a,b,c\}$ and $\tau$ be the indiscrete topology for $X$. Then the identity function of $(X,\tau)$ onto $(X,\tau)$ is a preopen function which is not strongly continuous.

**2.13 Theorem**. A function $f:(X,\tau) \to (Y,\sigma)$ is preopen if $f$ is weakly preopen and relatively weakly open.

**Proof**: Suppose that $f$ is weakly preopen and relatively weakly open. Let $U$ be an open subset of $X$ and let $y \in f(U)$. Since $f$ is relatively weakly open, then there is an open subset $V$ of $Y$ for which $f(U) = f(Cl(U)) \cap V$. Because $f$ is weakly preopen, it follows that $f(U) \subseteq pInt(f(Cl(U)))$.

Then $y \in pInt(f(U)) = pInt(f(Cl(U)) \cap V) \subseteq f(Cl(U)) \cap V = f(U)$ and therefore $f(U)$ is preopen.

## 3.Weakly Preclosed Functions

(xxiii) **Definition**. A function $f:(X,\tau) \to (Y,\sigma)$ is said to be ***weakly preclosed*** if $pCl(f(Int(F))) \subset f(F)$ for each closed set $F$ in $X$.

**3.2 Proposition**.

(i) Every closed function is preclosed.

(ii) Every weakly closed function is weakly preclosed.

(xxiv) Every preclosed function is weakly preclosed.

**Proof:** (i) Let $f:(X,\tau) \to (Y,\sigma)$ be a closed function and $F$ be a closed set in $X$. Then $f(F)$ is also a closed set in $Y$. By Theorem(1.4(ii)), $f(F)$ is preclosed in $Y$. Therefore, we have $f$ is a preclosed function.

Proof (ii) and (iii) it can be proved similar to proposition (2.5)(i) and (ii).

**3.3 Example**.

(i) Every preclosed function is not closed.

(ii) Every weakly preclosed function is not weakly closed.

**Proof:** (i) Let $X = \{x, y, z\}$ and $\tau = \{\phi, \{x\}, \{x, y\}, X\}$. Then a function $f:(X,\tau)\to(X,\tau)$ which is defined by $f(x) = x$, $f(y) = z$ and $f(z) = y$ . This function is preclosed. But it is not closed, since $f(\{z\}) = \{y\}$ is not a closed set in *Y*.

(xxv) Let $f:(X,\tau)\to(Y,\sigma)$ be the function from Example (2.6). It can be shown that f is weakly preclosed, but it is not weakly closed. For, $Y = Cl(f(Int(\{a\}))) \not\subset f(\{a\}) = \{x\}$.

**3.4 Theorem** . For a function $f:(X,\tau)\to(Y,\sigma)$, the following conditions are equivalent.

(i) $pCl(f(Int(Cl(U)))) \subseteq f(Cl(U))$ for each set U in X,

(ii) *f* is weakly preclosed,

(iii) $pCl(f(U)) \subseteq f(Cl(U))$ for every open set *U* of *X*,

(iv) $pCl(f(U)) \subseteq f(Cl(U))$ for each preopen set *U* of *X*,

(v) $pCl(f(Int(F))) \subseteq f(F)$ for each closed subset *F* in *X*,

(vi) $pCl(f(Int(F))) \subseteq f(F)$ for each preclosed subset *F* in *X*,

(vii) $pCl(f(U)) \subseteq f(Cl(U))$ for each regular open subset *U* of *X*.

(xxvi) For each subset *F* in *Y* and each open set *U* in *X* with $f^{-1}(F) \subseteq U$, there exists a preopen
set *A* in *Y* with $F \subseteq A$ and $f^{-1}(F) \subseteq Cl(U)$,

(xxvii) For each point $y \in Y$ and each open set *U* in *X* with $f^{-1}(y) \in U$, there exists a preopen
set *A* in *Y* containing *y* and $f^{-1}(A) \subseteq Cl(U)$.

**Proof**:

(i)$\Rightarrow$(ii) Let *U* be any set in *X* and $pCl(f(Int(Cl(U)))) \subseteq f(Cl(U))$. Since $Cl(U)$ is closed and choose $F = Cl(U)$. Then we obtain, $pCl(f(Int(F))) \subseteq f(F)$ and $f$ is weakly preclosed.

(ii) $\Rightarrow$(iii) Let *f* be a weakly preclosed function and *F* be a closed set of *X*. Then F is preclosed. Since we have $pCl(f(Int(F))) \subseteq f(F)$, and we choose $U = Int(F)$ ,then *U* is open and we obtain, $pCl(f(U)) \subseteq f(F) \subseteq f(Cl(Int(F))) \subseteq f(Cl(U))$.

(iii)$\Rightarrow$(iv) Let *U* be an open subset of *X* and $pCl(f(U)) \subseteq f(Cl(U))$. Since, *U* is also preopen, then we obtain $pCl(f(U)) \subseteq f(Cl(U))$.

(iv)$\Rightarrow$(v) Let *U* be any preopen of *X* and $pCl(f(U)) \subseteq f(Cl(U))$. Let *F* be a closed set in *X*. Then $Int(F)$ is open and it is also preopen. Choose $U = Int(F)$ .
So, we have $pCl(f(Int(F))) \subseteq f(Cl(Int(F)))$ .Since *F* is preclosed, then we obtain $pCl(f(Int(F))) \subseteq f(F)$.

(v)$\Rightarrow$(vi) Let *F* be a closed set in *X* and $pCl(f(Int(F))) \subseteq f(F)$. Since *F* is also preclosed, then we obtain $pCl(f(Int(F))) \subseteq f(F)$. .

(vi)$\Rightarrow$(vii) Let *F* be any preclosed set in *X* and $pCl(f(Int(F))) \subseteq f(F)$ . Suppose that
$U = Int(F)$ and $F = Cl(U)$. Then $U = Int(F) = Int(Cl(U))$ and we have *U* is regular open. Since, we obtain $pCl(f(Int(F))) \subseteq f(F)$ , then $pCl(f(U)) \subseteq f(Cl(U))$.

(vii)$\Rightarrow$(viii) Let *F* be a subset in *Y* and *U* be open in *X* with $f^{-1}(F) \subseteq U$. Then
$f^{-1}(F) \cap Cl(X \setminus Cl(U)) = \phi$ and consequently, $F \cap f(Cl(X \setminus Cl(U))) = \phi$.Since $X \setminus Cl(U)$ is regular open, $F \cap pCl(f(X \setminus Cl(U))) = \phi$ by(vii). Let $A = Y \setminus pCl(f(X \setminus Cl(U)))$. Then *A* is preopen with $F \subseteq A$ and $f^{-1}(F) \subseteq X \setminus f^{-1}(pCl(f(X \setminus Cl(U))))$

$\subseteq X \setminus f^{-1}(f(Cl(X \setminus Cl(U))))$ by(vii)

$\subseteq X \setminus (Cl(X \setminus Cl(U))) \subseteq X \setminus (X \setminus Cl(U)) = Cl(U)$.

(viii)$\Rightarrow$(ix) Let $y$ be any point in $Y$ and each open set $U$ in $X$ with $f^{-1}(y) \in U$.
Then there exists a preopen set $A$ in $Y$ and we obtain
$A = Y \setminus pCl(f(X \setminus Cl(U)))$,by(viii)
$f^{-1}(A) = X \setminus f^{-1}(pCl(f(X \setminus Cl(U)))) \subseteq X \setminus f^{-1}(f(X \setminus Cl(U))) \subseteq Cl(U)$.
(ix)$\Rightarrow$(i) Let $U$ be any set in $X$ and $F$ be a closed set in $X$. Suppose that $F = Cl(U)$ and $y \in Y \setminus f(Cl(U))$. Since $f^{-1}(y) \subseteq X \setminus Cl(U)$, then there exists a preopen set $A$ in $Y$ with $y \in A$ and $f^{-1}(A) \subseteq Cl(X \setminus Cl(U)) = X \setminus Int(Cl(U))$.
Therefore, $A \cap f(Int(Cl(U))) = \phi$, and $y \in Y \setminus pCl(f(Int(Cl(U)))$. So, we obtain
$pCl(f(Int(Cl(U)))) \subseteq f(Cl(U))$.


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## References


Baker,C.W., *Decomposition of openness*, Int.J.Math & Math. Sci. 17(1994), 413-415.

Caldas. M; Navalagi.G.B., *On weak forms of preopen and preclosed functions*, Archivum Mathematicum, Vol.40 (2004), No.2, 119-128.

El-Deeb, S.N, Hasanein, I.A., Mashhour,A.S and Noiri, T., *On p-regular spaces*, Bull. Math. Soc.sci. Math.R.S.Roum . 27(1983), 311-315.

Levine, N., *Strong continuity in topological spaces*, Amer. Math. Monthly 67 (1960), 269.

Mashhour,A.S., Abd El-Monsef, M.E. and El-Deeb, S.N.,*Precontinuous and weak pre-continuous mappings*, Proc. Math.Phys.Soc.Egypt 53 (1982), 47-53.

Rose, D.A. *Weak openness and almost openness*, Int.J.Math & Math. Sci. 7(1984), 35-40.

Rose, D.A. and Jankovic, D.S, *Weakly closed functions and Hausdorff spaces*, Math. Nachr. 130(1987), 105-110.


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# Functions and Their Graphical Representation

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## Abstract

In this paper we shall define one of the most fundamental concepts in mathematics, the notation of a function. We shall discuss the notation used to describe functions and investigate some of their graphs.


**Key words:** Function, graph, relation, domain, range.

## Introduction

In mathematics, the concept of a function is very important and useful. It appears in almost every branch of the subject. We shall use the word *function* to denote a certain specific type of correspondence or association between the elements of two sets. And then, we shall show how to represent a functions geometrically by graphs. Such graphs provide a useful way of visualizing the behavior of a function. We shall also develop some basic techniques for using graphs of simple functions to constant graphs of more complicated functions.


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