Application of Hungarian Method for Travelling Top Seven Tourist Destinations in Myanmar

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Abstract **- In this paper, we introduce the Travelling Salesman Problem (TSP) and solve for the most efficient route of the problem by using the steps of the Hungarian method. Specifically, this paper discussed the properties of a TSP matrix, provided the steps for Hungarian method, and described a list of 7 cities and the distance between each pairs of cities that apply these concepts of a Travelling Salesman problem. We do not consider any constraint on the order in which the localities are visited, nor do we take into the account possible traffic at differing times. We used to travel for top seven problems to show how the Hungarian method is used and it is an efficient way to solve the Travelling Salesman Problem. At the end, Hungarian Algorithm method is used to find minimum distance for shortest possible route that visits each city and return of the origin city.**

Keywords-Travelling Salesman Problem, Hungarian Method, Matrix, Distance value, Minimize route

I. INTRODUCTION

The travelling salesman problem has been somewhat of an anomaly for mathematicians since the early 1900s [1]. The travelling salesman problem (TSP) considers multiples localities to be visited and solves for the shortest route among these stop. We will consider the constraints in which no locality is visited twice before all other localities are visited, and then the salesman returns to the city in which they started.

The idea of mathematically solving for the most efficient route between multiple stops appears to have first been developed in the 1930s [1] by Karl Menger, who stated that the problem would always be solvable with a finite number of trials. He believed that there were possible rules that would simplify the problem, allowing for less trails, but he did not know what they would entail. Some progress was made in 1954 [2] with linear programming method to solving a 49 city [2] TSP, which is presented in the “granddaddy” TSP papers, Solution of a large scale travelling-salesman problem. Over time, more and more algorithms were developed, each becoming more efficient than the last, but admittedly still not as efficient as desired [1]. In this paper, we discuss the efficiency of a linear approach to solving the Travelling Salesman problem by using matrices, especially applies to find in travelling for seven tourist destination in Myanmar by Hungarian algorithm method.

The Tourism master plan in Myanmar was created in 2013, targeting 7.5 million arrivals by 2020. In order to reach target arrivals, the country needs to upgrade the well-known of tourist spots. The most popular tourist destinations in Myanmar are the big cities such as Yangon and Mandalay. According to the religious sites, there are Mon State, Kyaiktiyo , Pindaya and Hpa-An. For nature trails, there are Inle Lake, Pyin Oo Lwin, Kalaw and Kengtung. As the ancient cities such as Bagan and Mrauk-U are also the most popular places in Myanmar as well as beaches in Ngapali, Maungmagan and Ngwe-Saung [3]. Among these destinations, in this paper, we only focus for effective travelling to top 7 places by using Hungarian method.

The ideas presented in this article are based most predominantly on the research in Classes of Matrices for the Travelling Salesman Problem by Richard H. Warren [1]. In section II, we review properties of matrices and, more specifically, how these properties correlate to the work we do with a TSP matrix. In Section III we explain and lay out the steps of the Hungarian Method [4] and provide a simple example solving a TSP with this method. The information and example used in section III are essential to find minimize distance between seven cities,[5] what is the shortest possible route that visits each city and returns to the origin city?[6] TSP working procedure is Section IV and to apply in Section V.

II. PROPERTIES OF A TSP MATRIX

The properties of a TSP matrix are similar to that of any matrix, but are essential to understand the material in Section III and step by step procedure in Section IV.

Recall from Section I that a TSP matrix A is $n \times n$, [7] meaning there are an equal number of rows and columns . Logically, since we have a certain amount of cities that are acknowledged as both “from”, our rows, and “to” our columns, this concept is natural.

The entries of a matrix are listed in rows and columns [7]. Each entry can be identified by the row and column [8] they are in. In the case of our TSP matrix A, any entry in A can be identified with the notation a_{ij} with i being the row of the entry and j being the column of the entry. For a TSP matrix, this means that entry a_{ij} is the distance from city i to city$j$. Thus, the template matrix for a TSP matrix A is the same as any simple matrix, but has a more distinct, applicable meaning.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

We learn that a TSP on n cities is characterized by an $n \times n$ matrix$A = [a_{ij}]$, [9] where a_{ij} is a real number and represents the distance from city i to city j. Since the distance from city i to city i is 0 (because there is no real distance from a location to itself) the main diagonal elements of A are always 0, and therefore are insignificant to the TSP [9]. For the TSP A, the length of a tour t is the sum of

$$a_{1t(1)} + a_{2t(2)} + \cdots + a_{nt(n)} \qquad (1)$$

We can find the optimal tour for the TSP A such that the sum of (1) is the minimum tour of t of all possible tours. As we are trying to solve the TSP for the shortest route among multiple stops, our goal is to solve for the optimal tour. As stated in the TSP A has the same set of optimal tours as its triangular bock form. Thus, the study of the TSP can actually be simplified to the study of triangular block form [9].

III. HUNGARIAN METHOD

The Hungarian Method [4] was developed by Kuhn and is a method of simplifying the rows and columns of a matrix A to reach optimal assignment. This essentially means that we can simplify the entries in our matrix so that we can more easily figure out what our shortest route will be for our TSP. Although the optimally assigned matrix B not exactly equal to our original TSP matrix A, this inequality is insignificant. What matters is that the zero entries in B correlate to the positions of the entries in A, which provides us with the shortest route, and therefore solves our TSP in a much more simple process.

We learn the steps of the Hungarian method and how to transform our matrix using these steps[10].

Now we will work through to solve seven tourist destinations in Myanmar [3] to apply the Hungarian method to a TSP.

IV. WORKING PROCEDURE FOR SOLVING TSP

Based on Hungarian method, [4] here we introduce new method for travelling Salesman problem in which is described as follows.

The Hungarian algorithm method [11] for non-square arrays:

If an array is not square, start by adding a dummy row or column to make the array square. Fill this with the largest number in the existing array. Then apply the standard Hungarian algorithm method:

Step1: Reduce the array by both row and column subtractions.

Step 2: Cover the zero elements with the minimum number of lines. If the number of lines is the same as the size of the array, go to step 4.

Step 3: Augment the elements. To do this identifies the minimum uncovered elements. Subtract this element from all uncovered elements, and add this elements covered by two lines .Then return to step 2.

Step 4: Identify the maximal matching which uses only zero elements and apply this matching to the original array to find the minimum distance [12].

V. APPLY HUNGARIAN METHOD FOR SEVEN CITIES

From/ To	1	2	3	4	5	6	7
1	_	201	402	591	869	629	626
2	201	_	581	609	853	644	610
3	402	581	_	715	400	695	547
4	591	609	715	_	671	262	353
5	869	853	400	671	_	650	503
6	629	644	695	262	650	_	180
7	626	610	547	353	503	180	_

This table is distance (km) value table from one city to one city.

1. Yangon

2. Kyaiktiyo

3. Ngapali

4. Inle-Lake
5. Mrauk-U
6. Mandalay
7. Bagan

Using the information described in the table, this table values represent distance (km) for the respective routes from Google Map.

Travelling Salesman problem, stated as follows: A sale man is required to visit a number of cities during a trip. Given the distances between the cities, in what order should he travel so as to visit every cities precidely once and return home, with the minimum mileage traveled? [13]

From Section I, the main diagonal of the TSP matrix is insignificant to the problem. This gives the TSP matrix for this problem.

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
\_ & 201 & 402 & 591 & 869 & 629 & 626 \\
201 & \_ & 581 & 609 & 853 & 644 & 610 \\
402 & 581 & \_ & 715 & 400 & 695 & 547 \\
591 & 609 & 715 & \_ & 671 & 262 & 353 \\
869 & 853 & 400 & 671 & \_ & 650 & 503 \\
629 & 644 & 695 & 262 & 650 & \_ & 180 \\
626 & 610 & 547 & 353 & 503 & 180 & \_
\end{bmatrix}
\end{array}
$$

We will use the Hungarian method [4] from Section III to simplify this TSP matrix A.

Step 1: Subtract the smallest entry in each row from all of the entries in its row.

(Row minimization)

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
\_ & 0 & 201 & 390 & 668 & 428 & 425 \\
0 & \_ & 380 & 408 & 652 & 443 & 409 \\
2 & 181 & \_ & 315 & 0 & 295 & 147 \\
329 & 347 & 453 & \_ & 409 & 0 & 91 \\
469 & 453 & 0 & 271 & \_ & 250 & 103 \\
449 & 464 & 515 & 82 & 470 & \_ & 0 \\
446 & 430 & 367 & 173 & 323 & 0 & \_
\end{bmatrix}
\end{array}
$$

	1	2	3	4	5	6	7
1	_	0	210	390	668	428	425
2	0	_	380	408	652	443	409
3	2	181	_	315	0	295	147
4	329	347	453	_	409	0	91
5	469	453	0	271	_	250	103
6	449	464	515	82	470	_	0
7	446	430	367	173	323	0	_

Step 2: Subtract the smallest entry in each column from all of the entries in its column. (Column minimization)

	1	2	3	4	5	6	7
1	_	0	201	308	668	428	425
2	0	_	380	326	652	443	409
3	2	181	_	233	0	295	147
4	329	347	453	_	409	0	91
5	469	453	0	189	_	250	103
6	449	464	515	0	470	_	0
7	446	430	367	91	323	0	_

Step 3: Draw vertical and horizontal lines through matrix A to connect all of the 0's in our matrix with as few lines as possible (Calculating panelties of all 0's)

	1	2	3	4	5	6	7
1	_	0^{382}	201	308	668	428	425
2	0^{328}	_	380	326	652	443	409
3	2	181	_	233	0^{325}	295	147
4	329	347	453	_	409	0^{91}	91
5	469	453	0^{304}	189	_	250	103
6	449	464	515	0^{91}	470	_	0^{91}
7	446	430	367	91	323	0^{91}	_

Max panelty is 382

1-2 (From First Row to Second column)

Reduce matrix

	1	3	4	5	6	7
2	_	380	326	652	443	409
3	2	_	233	0	295	147
4	329	453	_	409	0	91
5	469	0	189	_	250	103
6	449	515	0	470	_	0
7	446	367	91	323	0	_

All rows and column have at least on zero

	1	3	4	5	6	7
2	_	54	0^{83}	326	117	83
3	0^{327}	_	233	0^{323}	295	147
4	327	453	_	409	0^{91}	91
5	467	0^{157}	189	_	250	103
6	447	515	0^{0}	470	_	0^{91}
7	444	367	91	323	0^{91}	_

Calculate the panelties of all 0's

Max panelty is 327

3-1 (From third row to first column)

Reduced matrix

	3	4	5	6	7
2	54	0	326	117	83
4	453	_	409	0	91
5	0	189	_	250	103
6	515	0	470	_	0
7	367	91	323	0	_

All rows and column have at least on zero

	3	4	5	6	7
2	54	0^{3}	3	117	83
4	453	_	86	0^{86}	91
5	0^{157}	189	_	250	103
6	515	0^{0}	147	_	0^{83}
7	367	91	0^{3}	0^{0}	_

Calculating panelties of all 0's

Max panelty is 157

5-3(From fifth row to third column)

Reduced matrix

	4	5	6	7
2	0^{3}	3	117	83
4	_	86	0^{86}	91
6	0^{0}	147	_	0^{83}
7	91	0^{3}	0^{0}	_

Calculating panelties of all 0's

Max panelty is 86

4-6 (From fourth row to sixth column)

Reduced matrix

	4	5	7
2	0^{83}	3	83
6	_	147	0^{230}
7	91	0^{94}	_

Calculating panelties of all 0's

Max panelty is 230

6-7 (From sixth row to seventh column)

Reduced Matrix

	4	5
2	0	3
7	91	0

Calculating 2-4 (second row to fourth column)

7-5 (seventh row to fifth column)

So, 1-2, 3-1, 5-3, 4-6, 6-7, 2-4, 7-5

Now we will make a travel, each node just once and finish up the node from where we got started. From this result we find that one city started from one city, then we go to final end city and then back to start city without traveling to other cities , ie, 1→7→1, but it does not complete the whole travel, and so we have to go through the following method by Hungarian method [12]. This result is

1-2-4-6-7-5-3-1

Yangon→ Kyaiktiyo → Inle lake → Mandalay → Bagan →Mrauk-U →Nagapali →Yangon

Total minimum distance route = 201+609 +262 +180 +503 + 400 +402 = 2557 km [14]

This minimum distance route result is a Traveling Salesman planned to visit seven cities. He wants to begin from start city and visit other cities once and then return to start city [4].

VI. CONCLUSION

Hungarian Algorithm Method is applied for using Dynamic programming Hold-Karp technique to find the minimum distance, visit each city and return to the origin city and minimum distance to the Top seven destinations in Myanmar. We have also considered that there is a possible, more efficient algorithm or method to solving the Travelling Salesman Problem. Much like a mathematicians before us, we recognize that there may be a better, undiscovered algorithm to solve a given TSP. Research could be done to compare the efficiency of this and other method to solving the given TSP to find the most efficient algorithm .The Hungarian Method can be applied to many travelling salesman problem in real life problem to find better solutions. However, these approaches are used to the Hungarian method to straight forward and apply linear algebra concepts to a real life problem.

ACKNOWLEDGMENT

The author would like to acknowledge her thank to Dr. Lin Lin Naing, Professor and head, Faculty of Computing, University of Computer Studies, Hinthada for his encouragement and permission to do research paper, to write papers and to submit in conferences or journal. The author would like to express her gratitude to Dr. Thet Mon Win, Associate Professor, Faculty of Computing, University of Computer Studies, Hinthada. The author thanks to other authors and all teachers in the world as she can

use the reference of calculation method. Finally, the author thanks to her beloved mother, brother, sister and her friends because her research cannot be done successfully unless they support it.

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