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CONTENTS

COMPARISON OF TRIANGULAR AND RECTANGULAR ELEMENTS APPROXIMATION FOR FIELD PROBLEM

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***Abstract* -In this paper, two types of finite element method especially for Poisson's equations are thoroughly discussed with bounding conditions. It is the purpose herein to present brief derivations of the finite element approximation describing a discrete model.**

***Keywords* - Finite Difference Method, Gauss-Seidel Method, Partial Differential Equation, Variational Principle.**

I. INTRODUCTION

Let us consider the field problem $L\phi = f$ in R, with B $\phi = g$ on C, the bounding R, L and B are differential operators. The finite element method seeks on approximations, (x, y), to the exact solution, ϕ (x, y) in a piecewise manner, the approximations being sought in each of total E elements. This in the general element an approximation ϕ^e (x, y) is sought in such a manner that outside e, $\phi^e(x, y) = 0$, e = 1, 2, ---, E and it follow that the approximate solution may be written as $\phi(x,y) = \sum_e \phi^e(x,y)$ where the summation is taken over all the elements.

II. RECTANGULAR ELEMENT FOR POISSON'S EQUATION

A simplest rectangular element is one with just four nodes, one at each corner. Choose local coordinates (ξ, η) as shown is Figure 1. Since there are four nodes with one degree of freedom at each node, the displacement variation throughout the element is of the following bilinear form,

$$\phi^e(x, y) = c_0+c_1x+c_2y+c_3xy \tag{1}$$

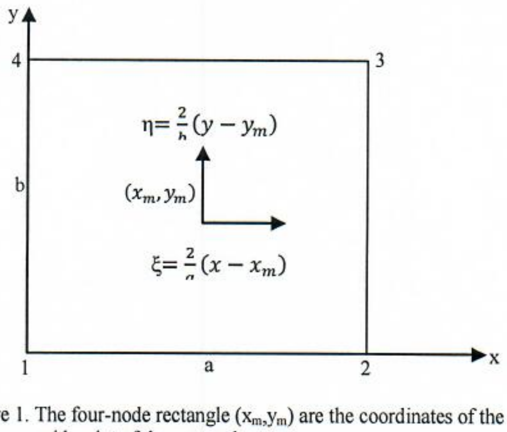

Figure 1. The four-node rectangle (x_m,y_m) are the coordinates of the mid-point of the rectangle

Using the Lagrangian interpolation polynomials, the shape functions are obtained as follows:

$$N_1 = \frac{\xi-1}{-1-1}\frac{\eta-1}{-1-1} = \frac{(1-\xi)(1-\eta)}{4}$$

Similarly $N_2 = \frac{(1-\xi)(1-\eta)}{4}$

$$N_3 = \frac{(1+\xi)(1+\eta)}{4}$$

$$N_4 = \frac{(1-\xi)(1+\eta)}{4}$$

Then $\phi^e(x, y) = N^e\delta^e = N_1\phi_1 + N_2\phi_2 + N_3\phi_3 + N_4\phi_4$

$\phi^e(x,y)$

$$= \frac{1}{4}[(1-(1-\xi))(1-\eta) \quad (1+\xi)(1-\eta) \quad (1+\xi)(1+\eta) \quad (1-\xi)(1-\eta)]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{bmatrix} \tag{2}$$

Now $\frac{\partial}{\partial x} = \frac{2}{a}\frac{\partial}{\partial \xi}$ and $\frac{\partial}{\partial y}(1-\eta) = \frac{2}{b}\frac{\partial}{\partial \eta}$.

Thus $\alpha = \frac{1}{2}\begin{bmatrix} \frac{-1(1-\eta)}{a} & \frac{(1-\eta))}{a} & \frac{(1+\eta)}{a} & \frac{-(1+\eta)}{a} \\ \frac{-(1-\xi)}{b} & \frac{-(1+\xi)}{b} & \frac{(1+\xi)}{b} & \frac{(1-\xi)}{b} \end{bmatrix}$,

$$k^e = \int_{-}^{1}\int_{-1}^{1} k\alpha^t\alpha\frac{a}{2}d\xi\frac{b}{2}d\eta,$$

For the special case k=1,

$$k^e = \frac{1}{6ab}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 2(r+1/r) & r-2/r & -r-1/r & 1/r-2r \\ r-2/r & 2(r+1/r) & 1/(r-2r) & -r-1/r \\ 1/r-2r & r-1/r & r-2/r & 2(r+1/r) \\ 1/r-2r & r-1/r & r-2/r & 2(r+1/r) \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \tag{3}$$

where $r = a/b$ is the aspect ratio of the element and

$$f^e = \int_{-1}^{1}\int_{-1}^{1} f(x,y)\, N^t\frac{a}{2}\frac{b}{2}d\xi d\eta \tag{4}$$

If the element is boundary element and a non-homogeneous mixed boundary condition holds there, than additions are needed to the stiffness and force matrices [1]. On each side the arc length s is such that

$$ds = -dx = -\frac{a}{2}d\xi,$$

$$\bar{k}^e = \int_{-1}^{1}\sigma(s)\begin{bmatrix}0\\0\\2(1+\xi)\\2(1-\xi)\end{bmatrix}[0 \quad 0 \quad 2(1+\xi) \quad 2(1-\xi)]\left(-\frac{a}{2}d\xi\right) \tag{5}$$

$$\bar{f}^e = \int_{-1}^{1} h\frac{1}{4}\{0 \quad 0 \quad 2(1+\xi) \quad 2(1-\xi)\}\left(-\frac{a}{2}\right)d\xi \tag{6}$$

There results will now be used to obtain one-element solution of the boundary-value problem.

III. TRIANGULAR ELEMENT FOR POISSON'S EQUATION

For irregular boundaries, rectangular elements are not appropriate since it is difficult to approximate the boundary geometry with such elements (see Figure 2.a). A more versatile element, as far as boundary geometry approximation is concerned, is the triangle, since any curve can be approximated arbitrarily closely by a polygonal arc, and the area enclosed by a polygon can be exactly covered by triangles (see Figure 2.b). This of course is true for rectangles, but a larger number will be required to achieve a given accuracy.

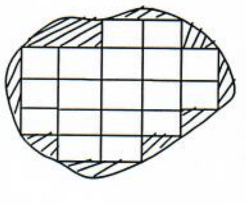

Figure.2-a. A typical geometry suitable for approximation with rectangular elements.

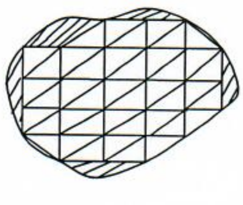

Figure.2-b. Approximation with triangular elements using the same number of nodes.

It is possible to set up the element matrices using the global coordinates(x,y); however, the algebra is simplified by using a set of triangle coordinates (L_1,L_2,L_3) as shown in Figure 3.

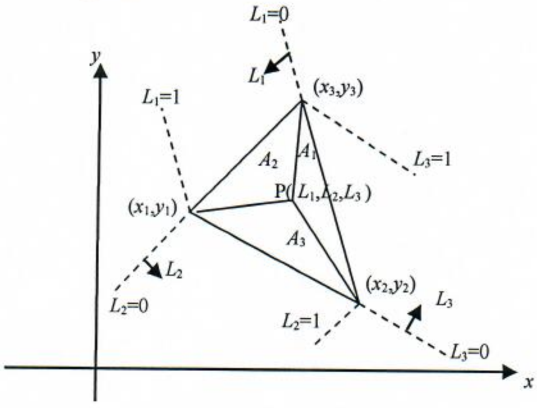

Figure3 Area coordinates for a triangular element

These coordinates are often area coordinates and

$L_1 = \frac{A_1}{A}$, $\quad L_2 = \frac{A_2}{A}$, $\quad L_3 = \frac{A_3}{A}$, (7)

where A is the area of the triangle and A_1, A_2, A_3 are the areas shown there. The position of p may thus be given by the coordinates (L_1, L_2, L_3). It follow that the three coordinates are not independent since they satisfy the equation

$L_1+L_2+L_3 = 1.$ (8)

The relationship between the global coordinates (x, y) and the (local) triangular coordinates (L_1,L_2,L_3) is given by

$$x = L_1x_1 + L_2x_2 + L_3x_3 \quad (9)$$
$$y = L_1y_1 + L_2y_2 + L_3y_3 \quad (10)$$

and may be solved to obtain L_1, in terms of x and y as

$$L_1 = \frac{a_1+b_1x+c_1y}{2A} \quad (11)$$

where the area A is given by

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \quad (12)$$

The constants a_i, b_i and c_i are given in terms of nodal coordinates by

$$a_1 = x_2y_3 - x_3y_2$$
$$b_1 = y_2 - y_3$$
$$c_1 = x_3 - x_2 \quad (13)$$

the others being by cyclic permutation. From equation (11) the following relationships between derivatives may be obtained;

$$\frac{\partial L_i}{\partial x} = \frac{b_i}{2A} \text{ and } \frac{\partial L_i}{\partial y} = \frac{c_i}{2A} \quad (14)$$

Finally, a result concerning an integral involving the area coordinates is required; the proof is given in [1].

$$\iint_A L_1^m L_2^n L_3^p dxdy = \frac{2Am!n!p!}{(m+n+p+2)!} \quad (15)$$

Since the element has three nodes with one degree of freed at each node, the displacement variation throughout the element is linear, i.e. it is of the form.

$$\emptyset^e(x,y) = a_0 + a_1x + a_2y \quad (16)$$

Using nodal displacement and interpolating through the element in the usual manner gives

$$\emptyset^e = N^e\delta^e$$
$$\emptyset^e = [L_1 \quad L_2 \quad L_3]\{\emptyset_1 \quad \emptyset_2 \quad \emptyset_3\} \quad (17)$$

The shape functions are easily found since the value of L_i at node j is δ_{ij}, and each L_i varies linearly with x and y through the element. In [1], the element stiffness matrix is given by

$$k_{ij}^e = \iint_A k\left(\frac{\partial L_i}{\partial x}\frac{\partial L_j}{\partial x} + \frac{\partial L_i}{\partial y}\frac{\partial L_j}{\partial y}\right) dxdy$$

$$k_{ij}^e = \iint_A k\left(\frac{b_ib_j}{4A^2} + \frac{c_ic_j}{4A^2}\right) dxdy \quad (18)$$

In the special case $k = 1$,

$$k_{ij}^e = \frac{1}{4A}(b_ib_j + c_ic_j)$$

The element force vector is given by

$$f_i^e = \iint_A L_i f(x,y) dxdy \quad (19)$$

and from equations (9) and (10), $f(x, y)$ can be written in terms of L_1, L_2, L_3 and hence f_i^e may be obtained.

$$\bar{k}^e = \int_{c_2} \sigma(s) N^{e^t} N^e ds$$

$$\bar{f}^e = \int_{c_2} h(s) N^{e^t} ds$$

Suppose for example that side 3-1 is a boundary side (see Figure 4)

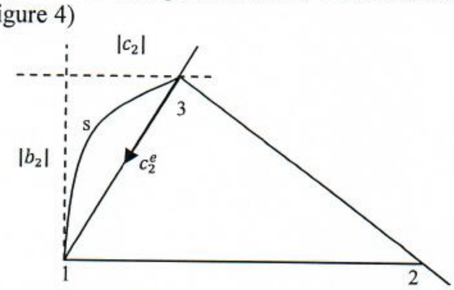

Figure 4. Boundary element with side 3-1 approximating the boundary

On side 3-1, $L_2 = 0$ and $s = (b_2^2 + c_2^2)^{1/2} L_1$

so that $ds = (b_2^2 + c_2^2)^{1/2} dL_1$

since $L_3 = 1 - L_1$

$$\bar{k}^e = \int_0^1 \sigma \begin{bmatrix} L^2 & 0 & L_1 - L^2 \\ 0 & 0 & 0 \\ L_1 - L_1^2 & 0 & (1-L_1)^2 \end{bmatrix} (b_2^2 + c_2^2)^{1/2} dL_1. \quad (20)$$

$$\bar{f}^e = \int_0^1 h\{L_1 \quad 0 \quad 1 - L_1\} (b_2^2 + c_2^2)^{1/2} dL_1 \quad (21)$$

Similar results are obtained by cyclic permutation when sides 1-2 and 2-3 are boundary sides.

These results will now be used to obtain a two-element solution using a single rectangular element.

IV. COMPARISON OF RECTANGULAR ELEMENTS SOLUTION AND TRIANGULAR ELEMENTS SOLUTION

IV.I FOR POISSION'S EQUATION

Let us consider the problem $-\nabla^2\phi = 2(x+y) - 4$ in the square whose vertices are at (0,0), (1,0) , (1,1) , (0,1) . The boundary condition are $\phi(0,y) = y^2$, $\phi(1,y) = 1-y$, $\phi(x,0) = x^2$, $\phi(x,1) = 1-x$.

IV.1A. RECTANGULAR ELEMENTS SOLUTION

Suppose that the square is divided into four square elements as shown in Figure 5.

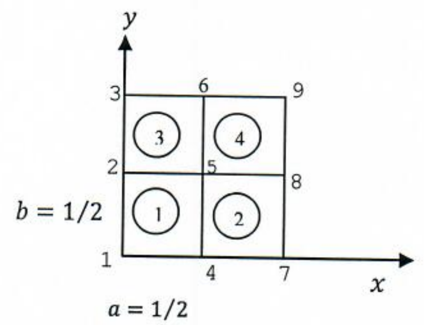

Figure 5. Four element discretization

Using equations (3) and (4), the element stiffness matrices and force vectors are

$$k^1 = \frac{2}{3}\begin{array}{c} \begin{matrix} 1 & 4 & 5 & 2 \end{matrix} \\ \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 4 \\ 5 \\ 2 \end{matrix}$$

$$k^2 = \frac{2}{3}\begin{array}{c} \begin{matrix} 4 & 7 & 8 & 5 \end{matrix} \\ \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \end{array} \begin{matrix} 4 \\ 7 \\ 8 \\ 5 \end{matrix}$$

$$k^3 = \frac{2}{3}\begin{array}{c} \begin{matrix} 2 & 5 & 6 & 3 \end{matrix} \\ \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \end{array} \begin{matrix} 2 \\ 5, \\ 6 \\ 3 \end{matrix}$$

$$k^4 = \frac{2}{3}\begin{array}{c} \begin{matrix} 5 & 8 & 9 & 6 \end{matrix} \\ \begin{bmatrix} 4 & -1 & -2 & -1 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ -1 & -2 & -1 & 4 \end{bmatrix} \end{array} \begin{matrix} 5 \\ 8 \\ 9 \\ 6 \end{matrix}$$

and the over all stiffness matrix $K = k^1 + k^2 + k^3 + k^4$.

$$K = \frac{2}{3}\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix} \\ \begin{bmatrix} 4 & -1 & 0 & -1 & -2 & 0 & 0 & 0 & 0 \\ -1 & 8 & -1 & -2 & -2 & -2 & 0 & 0 & 0 \\ 0 & -1 & 4 & -2 & -1 & 0 & 0 & 0 & 0 \\ -1 & -2 & 0 & 8 & -2 & 0 & -1 & -2 & 0 \\ -2 & -2 & -2 & -2 & 16 & -2 & -2 & -2 & -2 \\ 0 & -2 & -1 & 0 & -2 & 8 & 0 & -2 & -1 \\ 0 & 0 & 0 & -1 & -2 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & -2 & -2 & -2 & -1 & -8 & -1 \\ 0 & 0 & 0 & 0 & -2 & -1 & 0 & -1 & 4 \end{bmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix}$$

$$f^e = \tfrac{1}{8}(x_m y_m - 2)\{1, \ 1, \ 1, \ 1\} + \tfrac{y_m}{96}\{-1, \ 1, \ 1, \ -1\} + \tfrac{x_m}{96}\{-1, \ -1, \ 1, \ 1\} + \tfrac{1}{1152}\{1, \ -1, \ 1, \ -1\}$$

Now node 5 is the only where a Dirichlet boundary condition does not act; thus only the contributions to the equation corresponding to node 5 need by assembled. Row five of K is 2/3 [-2 -2 -2 -2 16 -2 -2 -2 -2]

Also $F_5 = f_3^1 + f_4^2 + f_2^3 + f_1^4$

where subscripts refer to the local node numbering given in Figure 1.and

$$f_3^1 = \frac{1}{8}\left(\frac{1}{16} - 2\right) + \frac{1}{192} + \frac{1}{1152}$$
$$f_4^2 = \frac{1}{8}\left(\frac{3}{16} - 2\right) + \frac{1}{192} - \frac{1}{1152}$$
$$f_3^2 = \frac{1}{8}\left(\frac{3}{16} - 2\right) + \frac{1}{192} - \frac{1}{1152}$$
$$f_1^4 = \frac{1}{8}\left(\frac{9}{16} - 2\right) - \frac{1}{192} + \frac{1}{1152}$$

thus $F_5 = -\frac{7}{8}$

since

$$\phi_1 = 0, \phi_2 = \frac{1}{4}, \phi_3 = 1, \phi_4 = \frac{1}{4}$$
$$\phi_6 = \frac{1}{2}, \phi_7 = 1, \phi_8 = \frac{1}{2}, \phi_9 = 0$$

it follows that the equation for ϕ_5 which gives $\phi_5 = 0.355$

IV.IB. TRIANGULAR ELEMENTS SOLUTION

The problem has symmetry about the line $y = x$, $\frac{\partial\phi}{\partial n} = 0$

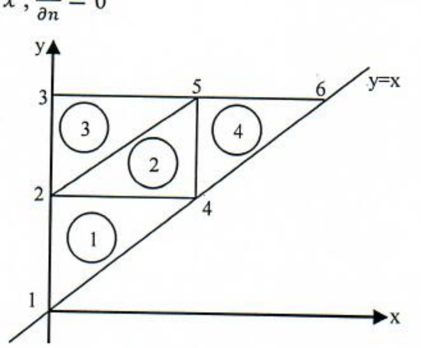

Figure.6. Four-triangular elements

The element stiffness matrices may be obtained directly from equation(18) and area of each triangle 1/8. Then

$$k^1 = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} 1 & 4 & 2 \end{array}$$
$$k^1 = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}\begin{array}{c} 1 \\ 4 \\ 2 \end{array}$$

$$\begin{array}{ccc} 5 & 2 & 4 \end{array}$$
$$k^2 = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}\begin{array}{c} 5 \\ 2 \\ 4 \end{array}$$

$$\begin{array}{ccc} 2 & 5 & 1 \end{array}$$
$$k^3 = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}\begin{array}{c} 2 \\ 5 \\ 1 \end{array}$$

$$\begin{array}{ccc} 4 & 6 & 5 \end{array}$$
$$k^4 = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}\begin{array}{c} 4 \\ 6 \\ 5 \end{array}$$

The element force vector is given by equation (19) as

$$f_i = \iint_A L_i\,[2(x+y) - 4]dxdy, \quad i = 1,2,3$$

where 1, 2, 3 is the local nodal numbering defined in Figure 3. Then using equations (9) and (10)

$$f_i = \int_A L_i[2L_1(x_1+y_1) + 2L_2(x_2+y_2) + 2L_3(x_3+y_3) - 4]\,dxdy$$

$$f_i = \frac{(x_i+y_i)}{24} + \frac{(x_j+y_j)}{48} + \frac{(x_k+y_k)}{48} - \frac{1}{6}$$

The only node without an essential boundary condition is node 4, thus only the equation associated with node 4 is assembled. This equation is

$$\sum_{j=1}^{6} K_{1j}\ \phi_j = F_4$$

where K and F are the over-all stiffness and force matrices respectively. Row 4 of K is [0 -1 0 2 -1 0]: and F4 = $f_2^1 + f_3^2 + f_1^4$, where the subscribe refer to the local nodal numbers of Figure 3.

$$f_2^1 = (\tfrac{1}{2}+\tfrac{1}{2}) / 48 + (0 + \tfrac{1}{2}) / 48 + (0+0) / 48 - \tfrac{1}{6} = -\tfrac{11}{96},$$

$$f_3^2 = (\tfrac{1}{2}+\tfrac{1}{2}) / 24 + (\tfrac{1}{2} + 1) / 48 + (0+\tfrac{1}{2}) / 48 - \tfrac{1}{6} = -\tfrac{8}{96},$$

$$f_1^4 = (\tfrac{1}{2}+\tfrac{1}{2}) / 24 + (1+ 1) / 48 + (\tfrac{1}{2}+1) / 48 - \tfrac{1}{6} = -\tfrac{5}{96}.$$

Therefore, $F = -\frac{1}{4}$

The essential boundary condition gives

$$\phi_1 = 0, \phi_2 = \frac{1}{4}, \phi_3 = 1, \phi_5 = \frac{1}{2}, \phi_6 = 0$$

so that

$$-\frac{1}{4} + 2\phi_4 - \frac{1}{2} = -\frac{1}{4}$$

which gives

$$\phi_4 = -\frac{1}{4}$$

The solution at $(\frac{1}{4}, \frac{1}{4})$, $(\frac{1}{4}, \frac{1}{4})$ and $(\frac{1}{4}, \frac{1}{4})$ is found by linear interpolation between nodes 1 and 4, 2 and 5 , 4 and 6 respectively. The results are compared with the corresponding results using rectangular elements as shown in Table 1.

IV.II. FINITE DIFFERENT SOLUTION

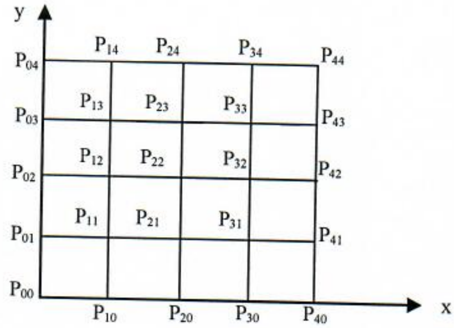

Figure 7. Finite Different Solution Mesh

$$U(x+h,y)+U(x-h,y)+U(x,y+h)+U(x,y-h)-4(x,y)=h^2\ f(x,y)$$

For P_{11}, $u_{21}+u_{01}+u_{12}+u_{10}-4u_{11} = 0.0625\{2(1/4+1/4)-4\}$

For P_{12}, $u_{22}+u_{02}+u_{13}+u_{10}-4u_{11} = 0.0625\{2(1/4+1/2)-4\}$

For P_{13}, $u_{23}+u_{03}+u_{14}+u_{12}-4u_{13} = 0.0625\{2(1/4+3/4)-4\}$

For P_{21}, $u_{31}+u_{11}+u_{22}+u_{20}-4u_{21} = 0.0625\{2(1/2+1/4)-4\}$

For P_{22}, $u_{32}+u_{12}+u_{23}+u_{21}-4u_{22} = 0.0625\{2(1/2+1/4)-4\}$

For P_{23}, $u_{33}+u_{13}+u_{24}+u_{22}-4u_{23} = 0.0625\{2(1/2+3/4)-4\}$

For P_{31}, $u_{41}+u_{21}+u_{32}+u_{30}-4u = 0.0625\{2(3/4+1/4)-4\}$

For P_{32}, $u_{42}+u_{22}+u_{33}+u_{31}-4u_{32} = 0.0625\{2(3/4+1/2)-4\}$

For P_{33}, $u_{43}+u_{23}+u_{34}+u_{32}-4u_{33} = 0.0625\{231/4+3/4)-4\}$.

By using the Gauss-Seidel Method,

u_{11}= 0.203125, u_{12}= 0.2890625, u_{13}= 0.484375
u_{22}= 0.28125, u_{23}= 0.335925, u_{33}= 0.265625.

TABLE1. COMPARISON OF SOLUTIONS

(x,y)	(1/4 , 1/4)	(1/4 , 1/4)	(1/4 , 1/4)	(1/4 , 1/4)
four rectangular elements	0.241	0.356	0.527	0.339
four triangular elements	0.125	0.25	0.375	0.125
Finite difference solutions	0.203125	0.28125	0.483475	0.265625
Exact Solutions	0.094	0.25	0.25	0.281

V. CONCLUSION

Triangular elements approximation is better than rectangular elements approximation for field problems. Finite element approximation for field problems improvements are made by refining the finite element mesh. Higher-order element may be introduced to get a better polynomial approximation but integrands involved would be necessary to use numerical integration.

For compressible and incompressible stokesian fluids, finite elements equation describing the motion of the fluids may be developed without resorting to variational principles by considering energy balances over an element.

It is reasonable to expect that the proposed finite elements model is a very good approximation of the continuum. This method is now very widely used, and forms the basis of most calculations in stress analysis and for many fluid flows.

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