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Mathematical Model for Walking Pattern of Biped Robot

Aye Aye Thant [1]

ABSTRACT

In this paper, we give the mathematical modeling for walking pattern of biped robot. Biped robots have higher mobility than conventional wheeled robots, but they tend to tip over easily. To be able to walk stably in various environments, such as rough terrain, up and down slopes, or regions containing obstacles, we need to control its stability and walking.

In order to maintain its stability, we can prevent from biped robot's tipping over and falling down. To prevent this, we need to control Center of Gravity (CoM) and Gravity of Center of Mass (GCoM) of biped robot, in the case of static stability, and Zero Moment Point (ZMP) and Foot Rotational Indicator point (FRI) of it, in the case of dynamic stability. By using applied mechanic, Zero Moment Point and friction condition at the feet ensuring postural stability of the biped, as well as bounds on the joint angles and on the control torques, are treated as constraints.

The walking of biped robot can be determined by controlling foot and hip trajectories. To construct mathematical modeling for these walking trajectories, interpolation method which are based on Cubic polynomial and Cubic spline interpolation is applied.

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Mathematical Model for Walking Pattern of Biped Robot

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Keywords
Biped robot, stability, walking trajectory, Cubic polynomial, Cubic spline interpolation

1. INTRODUCTION

Biped robots have better mobility than conventional wheeled robots, but they tend to tip over easily. To be able to walk stably in various environments, such as on rough terrain, up and down slopes, or in regions containing obstacles, we need to maintain its stability and walking [10]. Among the several ways in which the static equilibrium of the robot foot may be disturbed- such as pure sliding, pure rotation about a boundary point [1]. It is necessary for the robot to adapt to the ground conditions with a foot motion, and maintain its stability with a torso motion. When the ground conditions and stability constraint are satisfied, it is desirable to select a walking pattern that requires small torque and velocity of the joint actuators [10].

Hip and foot trajectories are discussed on one walking cycle. Mathematical methods applied in this portion are first we formulate the constraint for positions of the breakpoints of hip and foot trajectories on various ground conditions. Then cubic polynomial and cubic spline interpolation method are applied to find the whole trajectory of one walking cycle.

Biped robot motion in 3D space, X axis points to the forward direction, Z axis points upward, and Y axis is cross product of the Z and X axis as shown in figure 1 [2]. The X-Z plane is the sagittal plane, X-Y plane is the transverse plane and Y-Z plane is frontal plane. In this research, trajectories are discussed only in the sagittal plane as shown in figure 2.

In this paper, we first briefly express about the general information for the walking cycle of biped robot. Secondly, the constraints of a complete foot trajectories and hip trajectory in various ground conditions are formulated. Then, the walking trajectories are generated by applying the cubic polynomial and cubic spline interpolation. Finally, conditions for contact stability are presented.

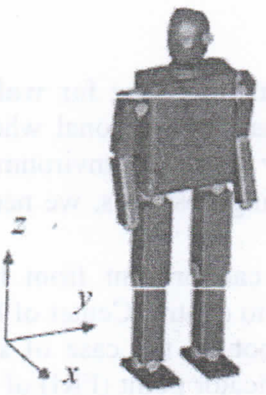

Fig.1. Biped Robot reference frames

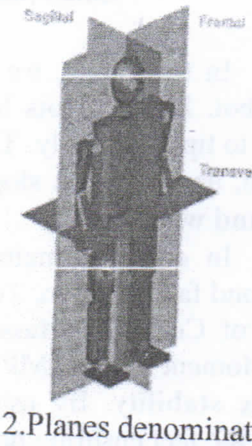

Fig. 2.Planes denomination

2. WALKING CYCLE

We considered an anthropomorphic biped robot with a trunk. Each leg consists of a thigh, a shin, and a foot, and has six degrees of freedom (DOF): three DOF in the hip joint, one in the knee joint, and two in the ankle joint as shown in Figure 3.

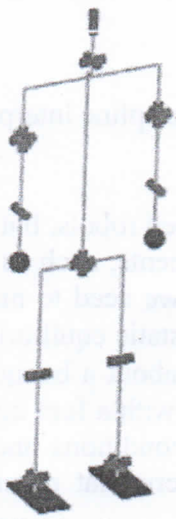

Fig.3. DOF of biped robot

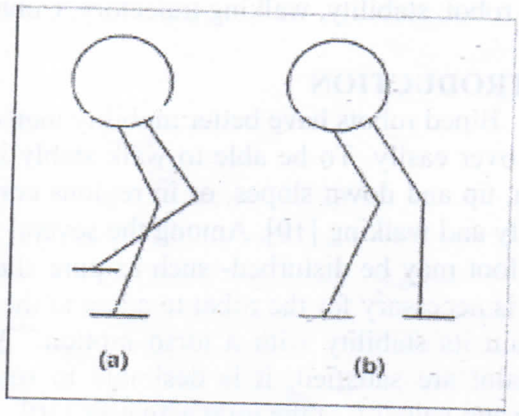

Fig.4. Walking phase,(a) single support and (b) double support

Biped walking is a periodic phenomenon. A complete walking cycle is composed of two phases. These two phases are double support phase and single support phase shown in Figure 4. During the **double support phase**, both feet are in contact with the ground. During the **single support phase**, while one foot is stationary with the ground, the other foot swings from the rear foot to the front (swing).One walking cycle may be classified into three phases. These are pre-swing phase, swing phase and post-swing phase as shown in figure 5.

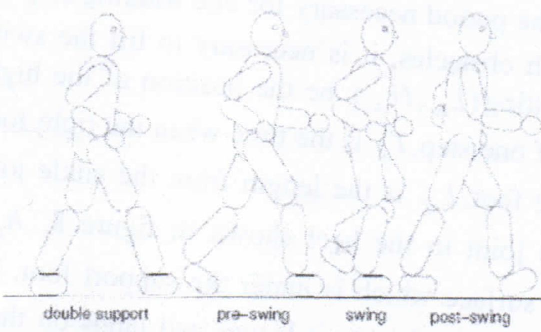

Fig. 5. Phases of dynamic biped gait

For a sagittal plane, each foot trajectory can be denoted by a vector $X_a = [x_a(t), z_a(t), \theta_a(t)]^T$, where $(x_a(t), z_a(t))$ is the coordinate of the ankle position, and $\theta_a(t)$ denotes the angle of the foot. The hip trajectory can be denoted by a vector $X_h = [x_h(t), z_h(t), \theta_h(t)]^T$, where $(x_h(t), z_h(t))$ denotes the coordinate of the hip position and $\theta_h(t)$ denotes the angle of the hip as shown in figure 6.

3. WALKING TRAJECTORIES

3.1. Foot Trajectories

Each foot trajectory can be denoted by a vector

$$X_a(t) = [x_a(t), z_a(t), \theta_a(t)]^T,$$

where $(x_a(t), z_a(t))$ is the coordinate of the ankle position, and $\theta_a(t)$ denotes the angle of the foot as shown in Figure 6.

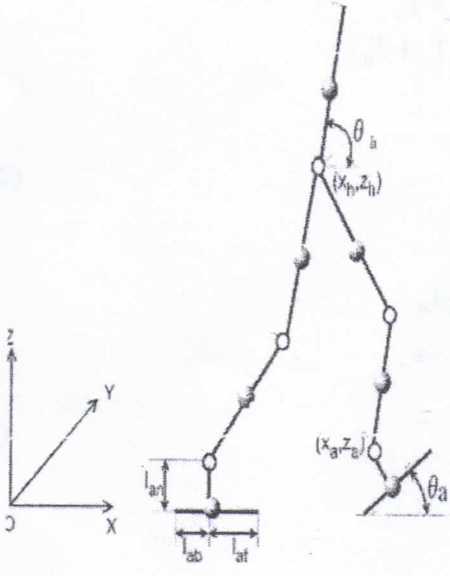

Fig. 6. Model of the biped robot

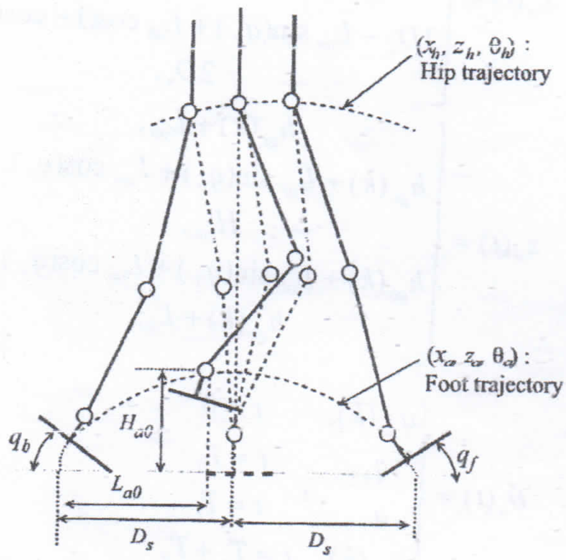

Fig. 7. Walking parameters

To simplify our analysis, we define the *one walking step*. It is defined as to begin with the heel of the right foot leaving the ground and with the heel of the right foot making first contact with the ground.

Assuming that the period necessary for one walking step is T_c. Over rough terrain or in environments with obstacles, it is necessary to lift the swing foot high enough to negotiate obstacles. Letting (L_{ao}, H_{ao}) be the position of the highest point of the swing foot , D_s is the length of one step, T_m is the time when the right foot is at its highest point, L_{an} is the height of the foot, L_{af} is the length from the ankle joint to the toe, L_{ab} is the length from the ankle joint to the heel shown in figure 8, $h_{gs}(k)$ and $h_{ge}(k)$ are the heights of the ground surface which is under the support foot. Letting q_b and q_f be the designated angles of the right foot as it leaves and lands on the ground respectively as shown in figure 7.

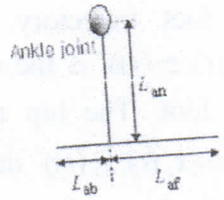

Fig.8. foot parameters

Assuming that entire sole surface of the right foot is in contact with the ground at $t = 0$ and $t = T_c + T_d$. The breakpoints of foot trajectory for one walking step on various ground conditions are described by following:

$$x_a(t) = \begin{cases} 0, & t = 0 \\ L_{an}\sin(q_b) + L_{af}\cos(1-\cos(q_b)), & t = T_d \\ L_{ao}, & t = T_m \\ 2D_s - L_{an}\sin(q_f) + L_{ab}\cos(1-\cos(q_f)), & t = T_c \\ 2D_s, & t = T_c + T_d \end{cases} \tag{1}$$

$$z_a(t) = \begin{cases} h_{gs}(k) + L_{an}, & t = 0 \\ h_{gs}(k) + L_{af}\sin(q_b) + L_{an}\cos(q_b), & t = T_d \\ H_{ao}, & t = T_m \\ h_{ge}(k) + L_{ab}\sin(q_f) + L_{an}\cos(q_f), & t = T_c \\ h_{ge}(k) + L_{an}, & t = T_c + T_d \end{cases} \tag{2}$$

$$\theta_a(t) = \begin{cases} q_{gs}(k), & t = 0 \\ q_b, & t = T_d \\ q_f, & t = T_c \\ q_{ge}(k), & t = T_c + T_d \end{cases} \tag{3}$$

where T_d is the interval of the double-support phase, $q_{gs}(k)$ and $q_{ge}(k)$ are the angles of the ground surface under the support foot. Above constraints of time interval for one walking cycle are classified into three walking phases as following.

I). Pre-Swing -Phase $(t \in [0, t_d))$: : The one walking cycle of biped robot starts with the right foot located flat on the ground at $t = 0$. The right foot rolls over the toes during $t \in (0, t_d)$.

II). Swing-Phase $(t \in [t_d, t_c))$: : The swing phase starts with the right foot just about to leave the ground at $t = t_d$ and then swings towards its new position.

III). Post-Swing-Phase $(t \in [t_c, t_c + t_d])$: : At $t = t_c$ the foot touches the ground and rolls around the heel during $t \in (t_c, t_c + t_d)$. The one walking cycle ends at $t = t_c + t_d$, when the right foot is flat on the ground again.

We can easily produce different foot trajectories, by varying the values of constraint parameters $q_{gs}(k), q_{ge}(k), h_{gs}(k), h_{ge}(k), q_b, q_f, H_{ao}$ and L_{ao} in equation (1), (2) and (3). For example,

I) if walking pattern of biped robot is on rough terrain, we can vary the values of $q_{gs}(k)$, $q_{ge}(k)$, $h_{gs}(k)$ and $h_{ge}(k)$ according to its ground conditions.

II) On level ground, $q_{gs}(k) = q_{ge}(k) = h_{gs}(k) = h_{ge}(k) = 0$.

III) Over obstacles, we can vary the values of L_{ao} and H_{ao} according to the obstacle.

IV) On climbing stairs, $x_{fe} = x_{fs} + 2L_s$, $z_{fe} = z_{fs} + 2S_h$,

where (x_{fs}, x_{fe}) and (z_{fs}, z_{fe}) are initial and final position of one walking cycle and L_s is step length and S_h is stair height.

3.2. Hip Trajectory

The hip trajectory can be denoted by a vector

$$X_h = [x_h(t), z_h(t), \theta_h(t)]^T,$$

where $(x_h(t), z_h(t))$ denotes the coordinate of the hip position and $\theta_h(t)$ denotes the angle of the hip as shown in figure 6 and 7.

A complete walking process is composed of three phases: a starting phase in which the walking speed varies from zero to a desired constant velocity, a steady phase with a desired constant velocity, an ending phase in which the walking speed varies from a desired constant velocity to zero.

Letting x_{sd} and x_{ed} denote distances along the x-axis from the hip to the ankle of the support foot at the start and end of the single-support phase, respectively as shown in figure 9. $x_h(t)$ can be described by the double support phase and the single support phase, during one-step cycle. We get the following equation

$$x_h(t) = \begin{cases} x_{ed}, & t = 0 \\ D_s - x_{sd}, & t = T_d \\ D_s + x_{ed}, & t = T_c \end{cases}$$

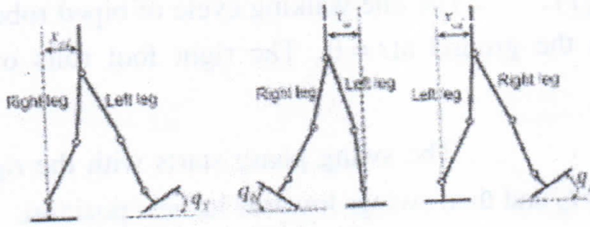

Fig.9. Walking cycle

Hip motion $x_h(t)$ hardly affects the position of the ZMP. By defining different values for x_{sd} and x_{ed} to vary within a fixed range, in particular

$$\begin{cases} 0.0 < x_{sd} < 0.5D_s \\ 0.0 < x_{ed} < 0.5D_s \end{cases} \tag{4}$$

Based on the trajectory of $x_h(t)$ and (4) and the ZMP, a smooth trajectory with the largest stability margin can be formulated as follows:

$$\max \quad d_{zmp}(x_{sd}, x_{ed}) \tag{5}$$
$$x_{sd} \in (0, 0.5D_s), x_{ed} \in (0, 0.5D_s)$$

where $d_{zmp}(x_{sd}, x_{ed})$ denotes the stability margin.

Hip motion $z_h(t)$ to be constant, or to vary within a fixed range. Assuming that $H_{h\max}$ be the hip highest position at the middle of the single-support phase, and $H_{h\min}$ be the hip lowest position at the middle of the double- support phase during one walking step, $z_h(t)$ has the following constraints:

$$z_h(t) = \begin{cases} H_{h\min}, & t = 0.5T_d \\ H_{h\max}, & t = 0.5(T_c - T_d) \\ H_{h\min}, & t = T_c + 0.5T_d \end{cases}$$

From the view point of the stability, hip motion parameter $\theta_h(t)$ is constant when there is no waist joint; in particular $\theta_h(t) = 0.5\pi$ rad on level ground.

3.3. Cubic polynomials for a path with via points

In one walking trajectory, the path is described in terms of number of points greater than two. The points are to be satisfied more densely in those segments of the paths where obstacles have to be avoided or a high path curvature is expected. Therefore, the problem is to generate a trajectory when N points, termed *path points*, are specified and have to be walked at certain instants of time. For each joint variable there are N constraints, and then one might want to use (N-1) order polynomial. This choice, however, has the following disadvantages: 1) it is not possible to assign the initial and final velocities. 2) As the order of a polynomial increases, its oscillatory behavior increases, and this may lead to trajectories which are not natural for walking. 3) Numerical accuracy for computation of polynomial coefficients decreases as order increases. 4) Polynomial coefficients depend on all assign points; thus, if it is desired to change a point, all of them to be recomputed.

These drawbacks can be overcome if a suitable number of lower *order interpolating polynomials*, continuous at the path points, are considered in place of single high-order polynomials.

The interpolating polynomial of lowest order is the cubic polynomial, since it allows imposing continuity of velocities at the path points. With reference to the single joint variable, a function $q(t)$ is sought, formed by a sequence of N-1 cubic polynomials $\Pi_k(t)$ for $k = 1,...,N-1$, continuous with continuous first derivatives. The function $q(t)$ attains the values q_k for $t = t_k$ (k=1,...,N), and $q_1 = q_i$, $t_1 = 0$, $q_N = q_f$, $t_N = t_f$; the q_k is represent the path points describing the desired trajectory at $t = t_k$.

To do this, we need to specify the desired velocity at each via points. There are several ways in which the desired velocity at via points must be satisfied: 1) *the user specifies arbitrary values of velocities at the path points.* 2) *The system automatically chooses the velocities at the path points by applying a certain criterion.* 3) *The system automatically chooses the velocities at the path points to cause the acceleration shall be continuous*. Then three methods corresponding to the above three data are described.

Method 1: Interpolating Polynomials with Velocity Constraints at Path Points

This solution requires the user to be able to specify the desired velocity at each path points; the solution does not possess any novelty with respect to the above concepts.

The system of equations allowing the computation of the coefficients N-1 cubic polynomials interpolating the N path points is obtained by imposing the following equations on the generic polynomials $\Pi_k(t)$ interpolating q_k and q_{k+1}, for k=1,...,N-1:

$$\Pi_k(t_k) = q_k$$
$$\Pi_k(t_{k+1}) = q_{k+1}$$
$$\dot{\Pi}_k(t_k) = \dot{q}_k$$
$$\dot{\Pi}_k(t_{k+1}) = \dot{q}_{k+1}$$

The result is $N-1$ system of four equations in the four unknown coefficients of the generic polynomial; these can be solved one independently of the other. The initial and final velocities of the trajectory are typically set to zero ($\dot{q}_1 = \dot{q}_N = 0$) and continuity of velocity at the path points is ensured by setting

$$\dot{\Pi}_k(t_{k+1}) = \dot{\Pi}_{k+1}(t_{k+1})$$

for $k = 1,...,N-2$. In this method, the resulting discontinuity on the acceleration, since only continuity of velocity is guaranteed. Therefore a convenient system should include either method 2 or 3.

Method 2: Interpolating Polynomials with Computed Velocities at Path Points

In this case, the velocity at a path point has to be computed according to a certain criterion. Imagine via points connected with straight line segments. If the slope of these line changes sign at via points, choose zero velocity; if the slope of these line does not change sign, choose the average of the two slopes as the via velocity. In this way, from specification of the desired via points alone, the system can chooses the velocity at each points. By interpolating path points with linear segments, the relative velocities can be computed according to the following rules:

$$\begin{aligned} \dot{q}_1 &= 0 \\ \dot{q}_k &= \begin{cases} o & \operatorname{sgn}(v_k) \neq \operatorname{sgn}(v_{k+1}) \\ \frac{1}{2}(v_k + v_{k+1}) & \operatorname{sgn}(v_k) = \operatorname{sgn}(v_{k+1}) \end{cases} \\ \dot{q}_n &= 0, \end{aligned} \qquad (6)$$

where $v_k = \dfrac{(q_k - q_{k-1})}{(t_k - t_{k-1})}$ gives the slope of the segment in the time interval $[t_{k-1}, t_k]$. With the above settings the determination of the interpolating polynomials is reduced to the previous case. It is easy to recognize that the imposed sequence of path points leads to having zero velocity at the intermediate points.

Method 3: Interpolating Polynomials with Continuous accelerations at Path Points (Splines)

Both the above two solutions do not ensure continuity of acceleration at the path points. This system chooses velocities in such a way that acceleration is continuous at via points. To do this, a new approach is needed. In this kind of spline, we replace the two velocity constraints at the connection of two cubics with two constraints that velocity be continuous and acceleration be continuous. The following equations have them to be satisfied:

$$\begin{aligned} \Pi_{k-1}(t_k) &= q_k \\ \Pi_{k-1}(t_k) &= \Pi_k(t_k) \\ \dot{\Pi}_{k-1}(t_k) &= \dot{\Pi}_k(t_k) \\ \ddot{\Pi}_{k-1}(t_k) &= \ddot{\Pi}_k(t_k) \end{aligned}$$

The resulting system for N path points, including the initial and final path points, cannot be solved. Hence the introduction of the end point constraints implies the determination of N-1 cubic polynomials. According to the end point constraint, we can classified into various type of spline such as natural spline, periodic spline, clamp spline and so on.

Then we construct the mathematical model for trajectory planning on one walking step on **level ground** by using the above three methods. Firstly, the values of parameter for walking trajectory are proposed in Table 1. Then, we briefly described the foot trajectory for the following proposed program. The following derivative must be satisfied:

$$\begin{array}{lll} \dot{x}_a(0)=0 & \dot{z}_a(0)=0 & \dot{\theta}_a(0)=0 \\ \dot{x}_a(T_c+T_d)=0 & \dot{z}_a(T_c+T_d)=0 & \dot{\theta}_a(T_c+T_d)=0 \end{array} \quad (7)$$

Table.1. Proposed parameter for walking algorithm

parameter	value
T_d	0.15s
T_m	0.5s
T_c	0.9s
L_{an}	7cm
L_{ab}	8cm
L_{af}	8cm
L_{ao}	24cm
H_{ao}	12cm
q_b	.5rad
q_f	.5rad

By using method 1, we let the velocity constraint for via points as the following equation (8).

$$\dot{x}_a(t)=\begin{cases} v_{xd} & t=T_d \\ v_{xm} & t=T_m \\ v_{xc} & t=T_c \\ v_{xcd} & t=T_c+T_d \end{cases} \quad \dot{z}_a(t)=\begin{cases} v_{zd} & t=T_d \\ v_{zm} & t=T_m \\ v_{zc} & t=T_c \\ v_{zcd} & t=T_c+T_d \end{cases} \quad \dot{\theta}_a(t)=\begin{cases} v_{\theta d} & t=T_d \\ v_{\theta m} & t=T_m \\ v_{\theta c} & t=T_c \\ v_{\theta cd} & t=T_c+T_d \end{cases} \quad (8)$$

Finally, we get the solution for foot trajectory.

$$x_a(t)=\begin{cases} (-8.69712+.15v_{xd})\dfrac{t^3}{(.15)^3}+(13.045-.15v_{xd})\dfrac{t^2}{(.15)^2}, & t\in(0,.15) \\ [(-39.30288+(v_{xd}+v_{xm})(35)]\dfrac{(t-.15)^3}{(.35)^3}+[58.95432-(2v_{xd}+v_{xm})(35)]\dfrac{(t-.15)^2}{(.35)^2} & \\ +v_{xd}(t-.15)+4.34856 & t\in(.15,.5) \\ [-43.30288+(v_{xm}+v_{xc})(.4)]\dfrac{(t-.5)^3}{(.4)^3}+[64.95432-(2v_{xm}+v_{xc})(.4)]\dfrac{(t-.5)^2}{(.4)^2} & \\ +v_{xm}(t-.5)+24, & t\in(.5,.9) \\ (-8.69712+.15v_{xc})\dfrac{(t-.9)^3}{(.15)^3}+(13.04568-.3v_{xc})\dfrac{(t-.9)^2}{(.15)^2}+v_{xc}(t-.9)+45.65, & t\in(.9,1.05) \end{cases}$$

$$
z_a(t) = \begin{cases}
(-5.956 + .15v_{zd})\dfrac{t^3}{(.15)^3} + (8.934 + .15v_{zd})\dfrac{t^2}{(.15)^2} + 7, & t \in (0,.15) \\
[-4.044 + (v_{zm} + v_{zd})(.35)]\dfrac{(t-.15)^3}{(.35)^3} + [6.066 + (2v_{zd} + v_{zm})(.35)]\dfrac{(t-.15)^2}{(.35)^2} & t \in (.15,.5) \\
+ v_{zd}(t - .15) + 9.978, & \\
[4.044 + (v_{zm} + v_{zc})(.4)]\dfrac{(t-.5)^3}{(.4)^3} + [-6.066 + (2v_{zm} + v_{zc})(.4)]\dfrac{(t-.5)^2}{(.4)^2} & t \in (.5,.9) \\
+ v_{zm}(t - .5) + 12, & \\
(5.956 + .15v_{zc})\dfrac{(t-.9)^3}{(.15)^3} + (8.934 - 3v_{zc})\dfrac{(t-.9)^2}{(.15)^2} - v_{zc}(t - .9) + 9.978, & t \in (.9,1.05)
\end{cases}
$$

$$
\theta_a(t) = \begin{cases}
(-1 + .15v_{\theta l})\dfrac{t^3}{(.15)^3} + (1.5 - .15v_{\theta l})\dfrac{(t)^2}{(.15)^2}, & t \in (0,.15) \\
[-1.004 + .15v_{\theta c}]\dfrac{(t-.9)^3}{(.15)^3} + [1.506 - .3v_{\theta c}]\dfrac{(t-.9)^2}{(.15)^2} + v_{\theta c}(t - .9) + 2.64 & t \in (.9,1.05)
\end{cases}
$$

By using method 2, we get the solution for foot trajectory by cubic polynomial.

$$
x_a(t) = \begin{cases}
-678.2569016\,t^3 + 294.4193475\,t^2, & t \in (0,.15) \\
-119.6154396\,(t-.15)^3 + 80.840666866\,(t-.15)^2 & t \in (.15,.5) \\
+ 42.54346338\,(t-.15) + 4.335318275, & \\
-72.61400594\,(t-.5)^3 + 26.51672663\,(t-.5)^2 & t \in (.5,.9) \\
+ 55.17325463\,(t-.5) + 24, & \\
-723.2113895\,(t-.9)^3 + 24.28359511\,(t-.9)^2 & t \in (.9,1.05) \\
+ 41.53191307\,(t-.9) + 45.6646873, &
\end{cases}
$$

$$
z_a(t) = \begin{cases}
-1195.401336\,t^3 + 311.6899213\,t^2 + 7, & t \in (0,.15) \\
10.3144554\,(t-.15)^3 - 23.72230412\,(t-.15)^2 & t \in (.15,.5) \\
+ 12.81615665\,(t-.15) + 9.9782242, & \\
-14.66432713\,(t-.5)^3 - 6.7703676\,(t-.5)^2 + 12, & t \in (.5,.9) \\
1211.310429\,(t-.9)^3 \quad 231.0276088\,(t-.9)^2 & t \in (.9,1.05) \\
-12.45517134\,(t-.9) + 9.9782242, &
\end{cases}
$$

$$
\theta_a(t) = \begin{cases}
-222.22296296\,t^3 + 55.55566667\,t^2, & t \in (0,.15) \\
222.222963\,(t-.9)^3 \quad 44.44466667\,(t-.9)^2 & t \in (.9,1.05) \\
-1.66665\,(t-.9) + .5, &
\end{cases}
$$

By **using method 3**, we get the foot trajectory by cubic spline interpolation.

$$x_a(t)=\begin{cases} 186.4047223\,t^3+24{,}70801558\,t, & t\in(0,.15) \\ -85.42263817\,(t-.15)^3+83.88212505\,(t-.15)^2 \\ +37.29033434\,(t-.15)+4.3335318275\,, & t\in(.15,.5) \\ -50.80400006\,(t-.5)^3-5.811645025\,(t-.5)^2 \\ +64.61500235\,(t-.5)+24, & t\in(.5,.9) \\ 148.3921002\,(t-.9)^3-66.7764451\,(t-.9)^2 \\ +35.57976631\,(t-.9)+45.666468173\,, & t\in(.9,1.05) \end{cases}$$

$$z_a(t)=\begin{cases} -92.88998202\,t^3+21.94657288\,t+7, & t\in(0,.15) \\ 38.60765798\,(t-.15)^3-41.80049191\,(t-.15)^2 \\ +15.67649909\,(t-.15)+9.978482242\,, & t\in(.15,.5) \\ -32.20801903\,(t-.5)^3-1.262451029\,(t-.5)^2 \\ +.6044690609\,(t-.5)+12, & t\subset(.5,.9) \\ 88.69349747\,(t-.9)^3-39.91207386\,(t-.9)^2 \\ -15.86534089\,(t-.9)+9.978482242\,, & t\in(.9,1.05) \end{cases}$$

$$\theta_a(t)=\begin{cases} -8.71372548\,t^3+3.529058824\,t, & t\in(0,.15) \\ 8.71372549\,(t-.9)^3-3.921176471\,(t-.9)^2 & t\in(.9,1.05) \\ -2.9408824\,t+.5, \end{cases}$$

By using matlab program, the two result trajectory of method 2 and 3 is demonstrated as shown in figure 10, 11 and 12.

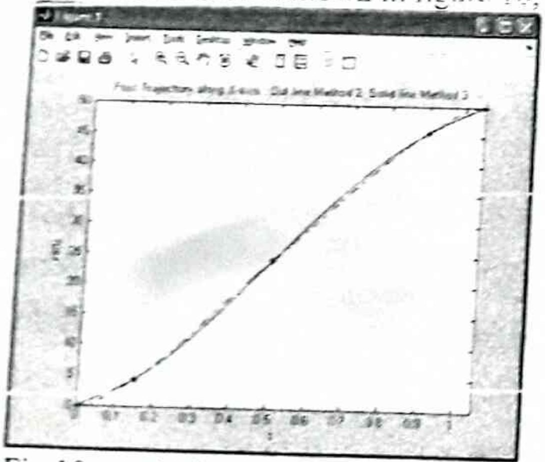

Fig.10.graph for $x_a(t)$, dot line method 2, solid line method 3

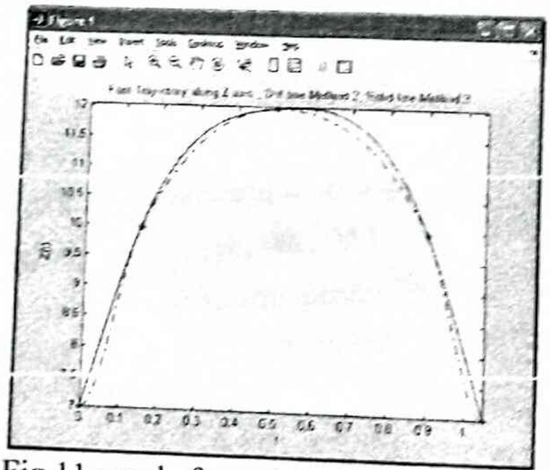

Fig.11.graph for $z_a(t)$, dot line method 2, solid line method 3

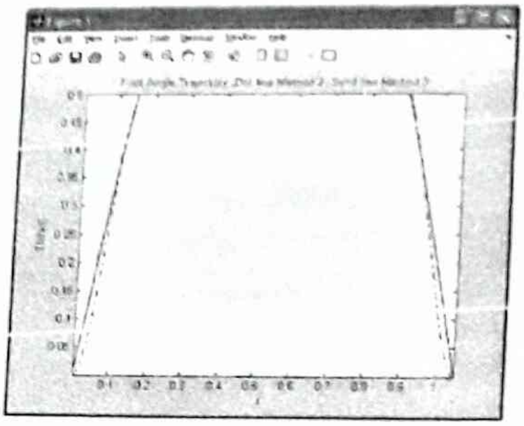

Fig.12.graph for $\theta_a(t)$, dot line method 2, solid line method 3

4. STABILITY

Since a biped robot inherently suffers from instability and always risks falling down, ensuring stability is the most important goal from the perspective of locomotion. There are two types of stability motion such as static stability and dynamic stability.

Static stable locomotion is characterized by the center of mass (CoM) of the machine being within the stable region, which is the convex hull consisting of its supporting feet. The projection of CoM on the ground is called the ground projection of CoM (GCoM). Dynamic stable locomotion is can be implemented by maintaining the zero moment point (ZMP) inside the stable region. If ZMP outside the support the polygon, it is called Foot rotation indicator (FRI).

4.1 Conditions for Contact Stability

An important feature of all forms of walking is, that physical contacts between a foot and the ground are unilateral [5]. Stability of the contact situation during the three walking phases defined in Sec. 3.1 thus requires the occurring contact forces to conform with the contact stability conditions summarized next. The sum of all forces on a contact surface is thereby represented by the resultant contact forces $F=[F_x, F_y, F_z]^T$ and moments $M=[M_x, M_y, M_z]^T$ acting in the points r_L (Cartesian coordinates of left leg), r_{Rf} (Cartesian coordinates of right leg on the pre swing phase), r_{Rb} (Cartesian coordinates of right leg on the post swing phase), depending on the current walking phase, (see figure 13).

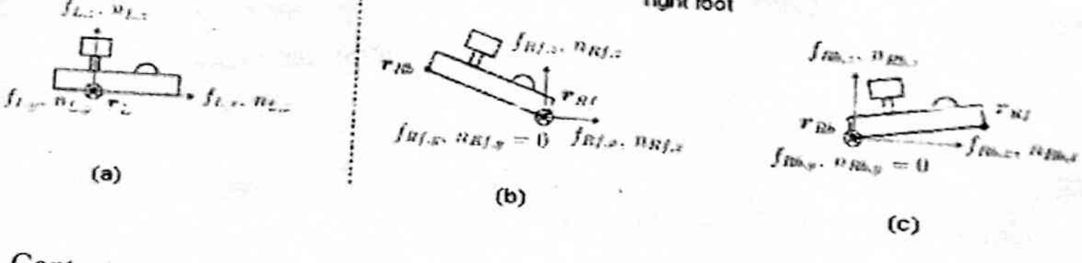

Fig.13. Contact constraints and forces during walking phases (a) supporting left foot (b) pre-swing phase (right foot) (c) post-swing phase (right foot).

Unilaterality Conditions on the resultant normal contact forces ensure, that a desired contact situation does not change by a foot lifting off the ground:

$$F_{l,z} \geq 0, \quad \forall t \in [0, t_c + t_d]$$

$$F_{Rf,z} \geq 0, \quad \forall t \in [0, t_d)$$

$$F_{Rb,z} \geq 0, \quad \forall t \in [t_c, t_c + t_d].$$

ZMP Conditions are used in this work to prevent a foot from beginning to rotate around its edges. The ZMP is defined as the point on the contact surface, where the resultant moments n_x; n_y of all contact forces are zero. The contact situation is stable, if the ZMP remains inside the contact area.

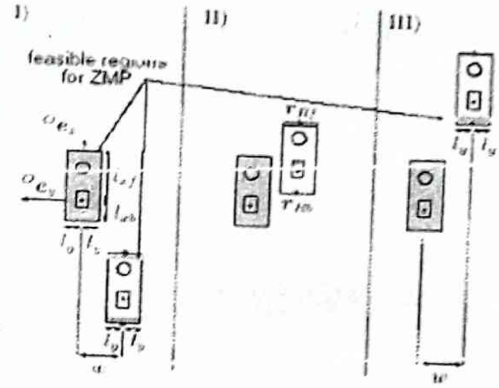

Fig. 14.Three walking phases I) pre-swing phase, II) swing phase, III) post-swing phase and feasible regions for ZMP, top view

With the resultant contact forces sketched in Figure14 the ZMP of the left foot denoted by ($x_{L,zmp}$ and $y_{L,zmp}$), which should remain flat on the ground during all three phases, can be expressed in the left foot frame L as

$$x_{L,zmp} = -\frac{M_{L,y}}{F_{Lz}} \quad , \quad y_{L,zmp} = \frac{M_{L,x}}{F_{Lz}} ;$$

The following constraints result from the area of valid ZMP positions $\forall t \in [0, t_c + t_d]$: illustrated in Fig. 14.

$$-l_{xb} \leq \frac{-M_{Ly}}{F_{Lz}} \leq l_{xf} \quad , -l_y \leq \frac{M_{Lx}}{F_{Lz}} \leq l_y$$

where l_y is length of foot along y axis.

As the right foot rotates around its front (back) edge during *pre-swing I* (*post-swing III*) there is no resultant moment M_y and the contact surface degenerates to a line. The ZMP of right foot (denoted by $x_{R,zmp}, y_{R,zmp}$) moves along this line while its coordinates in frame R are given by

$$y_{R,zmp} = \frac{M_{Rf,x}}{F_{Rf,z}}, \quad x_{R,zmp} = l_{xf}, \quad \forall t \in [0, t_d)$$

$$y_{R,zmp} = \frac{M_{Rb,x}}{F_{Rb,z}}, \qquad x_{R,zmp} = l_{sb}, \qquad \forall t \in [t_c, t_c + t_d]$$

where l_{sf} is length of angle joint to the toe along x-axis and l_{sb} is length of angle joint to the heel along x-axis.

Considering the areas designated in Fig. 14 now leads to the constraints

$$-l_y \le \frac{-M_{Rf,x}}{F_{Rf,z}} \le l_y, \qquad \forall t \in [0, t_d)$$

$$-l_y \le \frac{-M_{Rb,x}}{F_{Rb,z}} \le l_y, \qquad \forall t \in [t_c, t_c + t_d]$$

which ensure that the front (back) edge of the right foot remains flat on the ground.

Friction Conditions ensure that a supporting foot neither begins to slip on the ground nor starts to rotate around the normal axis e_z of the contact surface. The resultant tangential forces F_x, F_y and the resultant moments M_z cannot be treated independently, because their effects combine. Thus the friction condition

$$\sqrt{F_x^2 + F_y^2} + \left|\frac{M_z}{k}\right| \le \mu F_z \qquad (9)$$

is applied, which has to be satisfied by the resultant contact forces F_L of the left foot during all three phases, by F_{Rf} at the right foot during *pre-swing I*, and by F_{Rb} during *Post-Swing III*. The first term in (9) defines the usual friction cone, while the second is an additional tangential force induced by the moment M_z. The constant $0 < \mu < 1$ denotes the friction coefficient of the rubbing surfaces and k is the frictional radius. The assumed frictional radius for the left foot is $k_L = 0.5\sqrt{(2l_y)^2 + (l_{sb} + l_{sf})^2}$ and for the right foot $k_{Rf} = k_{Rb} = l_y$ during both pre-swing phase and post-swing phase.

4.2 Bounds on the joint angles and on the control torques

Physical admissibility of the walking phases also demands compliance with restrictions given by the performance limits of joint angle, joint velocity and torques. This is regarded by the inequality constraints

$$\theta_{min} \le \theta(t) \le \theta_{max}$$

$$\dot{\theta}_{min} \le \dot{\theta}(t) \le \dot{\theta}_{max}, \forall t = [o, t_c + t_d]$$

$$\tau_{min} \le \tau(t) \le \tau_{max}$$

where θ is the joint angle, $\dot{\theta}$ is the joint velocity and τ is the joint torques.

CONCLUSION

The result represented in this paper demonstrate that dynamically stable, physically feasible and naturally looking walking phase can be generated by mathematical modeling using cubic polynomial, cubic spline interpolation and applied mechanics. Future work will be considered the method for trajectory planning and concept of stability criteria points (CoM, GCoM, ZMP and FRI).

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