FEDERATION OF MYANMAR ENGINEERING SOCIETIES

THE FIRST NATIONAL CONFERENCE ON ENGINEERING RESEARCH

IN

COMMEMORATION OF ANNUAL GENERAL MEETING OF FEDERATION OF MYANMAR ENGINEERING SOCIETIES

PROCEEDINGS

January 16, 2020

FEDERATION OF MYANMAR ENGINEERING SOCIETIES

NATIONAL CONFERENCE ON ENGINEERING RESEARCH

IN

COMMEMORATION OF ANNUAL GENERAL MEETING OF

FEDERATION OF MYANMAR ENGINEERING SOCIETIES

16th January 2020

Fed.MES

Preface for AGM Proceeding

The First National Conference on Engineering Research has been organized by Annual General Meeting Paper Committee of Federation of Myanmar Engineering Societies (Fed.MES). The Conference will be held in Fed.MES Building on 16th January 2020.

The Conference intends to being together Engineers, Researchers form Education and Industry to Share and Exchange their Knowledge, Experience, Information and Research on Engineering and Technology.

The First National Conference on Engineering Research will be held in accordance with the objectives of Fed.MES as:

1. To develop the Engineering Profession
2. To Raise the competitiveness of Myanmar Engineers
3. To lift the work of Human Resource Development and Capacity Building
4. To enhance the infrastructure and industrial development to the nation.

The aim and objective of the papers for various engineering field, will be discussed at this conference and those who are interested in Engineering are warmly welcome to participate in this conference. Participants are sincerely requested to put up the questions related to concern Engineering Fields. But it should be notice that it would be allow 5 min for each paper.

This National Conference would encourage Researchers and Engineers to present and discuss the recent advances in engineering fields. The paper committee of this National Conference has collected papers of various Engineering fields as: Civil, Engineering Education, Electrical Power, Engineering Geology, Information Technology, Mechatronics, Mechanical, Metallurgy, Mining, Petroleum Engineering, Renewable Energy, Environmental Engineering, and Earthquake Engineering.

The AGM Paper Committee of Fed.MES assures the participants to gain very informative and invaluable knowledge and information.

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16-1-2020

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Contents

Mathematical Approach to Stable Hip Trajectory for Ascending Stairs Biped Robot

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Abstract ---- The key contribution of this paper is to generate the stable hip trajectory for ascending stairs of the biped robot from the mathematical point of view. To approach this goal, we first construct the mathematical model for kinematics constraints of k^{th}step of ascending stairs and discuss the concept of hip trajectory relative to stability. From this, the stable hip trajectory for one cycle of ascending stairs step is derived by interpolation polynomial. Finally, the numerical result of the proposed trajectory is verified with Matlab figures. The present algorithm is also applicable for descending stairs of the biped robot.

Keywords: Ascending stairs, Biped Robot, Hip trajectory, Stable motion, Trajectory planning

I. INTRODUCTION

Biped robots have higher mobility than conventional wheeled robots, especially when moving on rough terrain, steep stairs, and in environments with obstacles [1]. Since a biped robot tends to tip over easily, it is necessary to take stability into account when determining a walking pattern. To walk stably in various environments, stability is the most important topic in bipedal robot walking. The zero moment point, (ZMP) is the point on the ground around which the sum of all the moments of the active force equals zero, has been widely used as a criterion to assure stability of bipedal walking in many studies [1].

The walk of biped robot can be determined by foot and hip trajectory. Zhang et al [2] have used the cubic polynomial for motion planning of biped robot for climbing stairs. However, in these methods do not ensure continuity of acceleration at the path points. So, generation of a smooth and continuous motion has not been guaranteed as cubic spline interpolation method. To alleviate this problem, foot and hip trajectory for level ground are generated by cubic spline interpolation in my previous research [6]. And then, generation of foot trajectory for ascending stairs by applying cubic spline interpolation has been derived [11]. Now, stable hip trajectory for ascending stairs of biped robot has been generated. To be able to get stable hip trajectory, either generates the smooth trajectory or points the two hip parameters for stability. From this concept, we formulate the problem of smooth hip motion with the largest stability margin using two parameters, and derived the hip trajectory. Also, both foot trajectories and the hip trajectory are smooth, and the ZMP trajectory is always near the center of the stability region, that is the robot has a large stability margin [1].

The structure of the paper is as follows: Section 2 describes theoretical element for ascending stairs cycle of biped robot. . Section 3 represents kinematic constraints for hip trajectory. Section 4 includes stable hip trajectory planning of biped robot. Section 5 demonstrates of the resulting hip trajectory by applying Matlab.

II. THEORETICAL ELEMENT FOR ASCENDING STAIRS OF BIPED ROBOT

A biped robot is two-legged robot and is expected to eventually evolve into with a human-like body. Each leg of an anthropomorphic biped robot consists of a thigh, a shank, foot and has six degrees of freedom (DOF); three DOF in the hip joint, one in the knee joint and two in the ankle joint [1].

Biped walking is a periodic phenomenon. A complete climbing upstairs cycle is composed of two phases. These are single support phase and double support phase as shown in Figure 1.

Double support phase: During the double support phase, both feet are in contact with the stairs.

Single support phase: During the single support phase, while one foot is stationary with the stair, the other foot swings from the rear foot to the font.

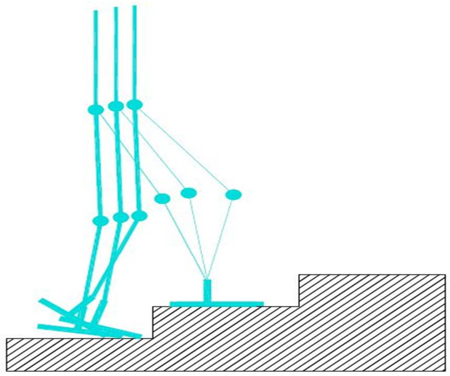

Figure 1(a)

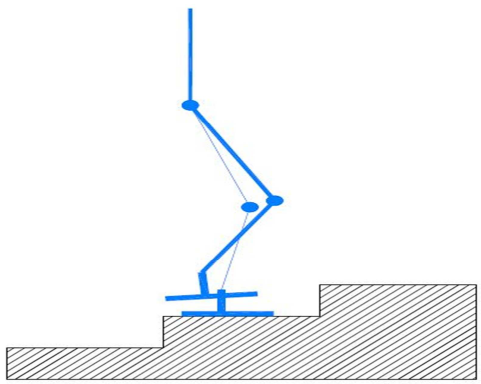

Figure 1(b)

Figure 1(a) double-support phase (b) single-support phase

To enable the robot to adapt to ascend stairs, first foot trajectories and then hip trajectory must be specified. If both foot and hip trajectories are known, all joint trajectories of the biped robot will be determined by kinematic constraints [1]. The walking pattern can therefore be denoted uniquely by both foot and hip trajectories. Biped robot motion in 3D space, x axis point to the forward direction, and z axis points upward, and y axis is cross product of z and x axis. The x-z plane is the sagittal plane (lateral plane), x-y plane is the transverse plane and y-z plane is frontal plane. In this paper, trajectories are discussed only in the sagittal plane.

III. KINEMATICS CONSTRAINTS FOR HIP TRAJECTORY

The kinematic constraints were also taken into consideration during the ascending stairs of the biped robot. In this section, kinematic constraints for k^{th} ascending step of hip trajectory are formulated.

1. Hip Trajectory

Each hip trajectory can be denoted by a vector

$$X_h = [x_h(t), z_h(t), \theta_h(t)]^T,$$

where $(x_h(t), z_h(t))$ is the coordinate of the hip position, and $\theta_h(t)$ denotes the ankle of the hip as shown in Figure 2.

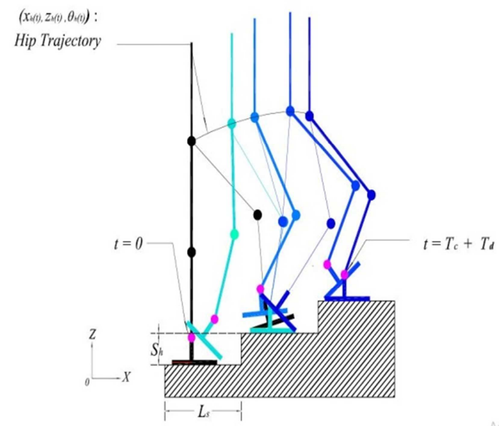

Figure 2 Hip trajectory of biped robot

The time intervals for one cycle of ascending stairs are denoted as following:

Place (entire sole contact) $(t = 0)$

Deploy (heel off) $(t = T_d)$

Deploy (heel contact) $(t = T_c)$

Place (entire sole contact) $(t = T_c + T_d)$ as shown in Figure 3.

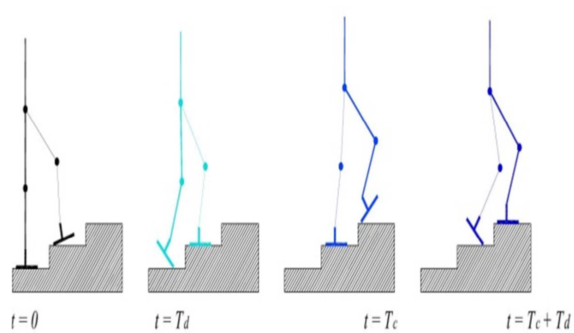

Figure 3 Time interval for one cycle of ascending stairs

The other hip parameters are listed as following:

x_{sd}	distances along x axis from the hip to the ankle of the support foot at the start of the single- support phase
x_{ed}	distances along x axis from the hip to the ankle of the support foot at the end of the single- support phase as shown in Figure 4.
H_{hmax}	the hip highest position at the middle of single support phase during one ascending cycle
H_{hmin}	the hip lowest position at the middle of double support phase during one ascending cycle
D_s	step` length
L_s	stair length
S_h	stair height
ZMP	zero moment point

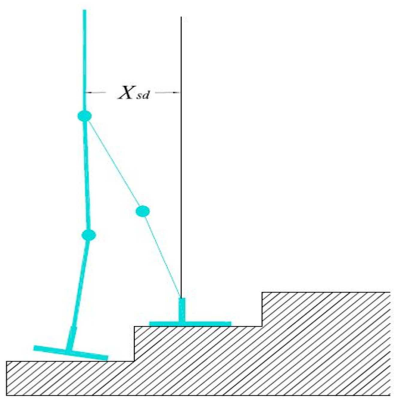

Figure 4(a)

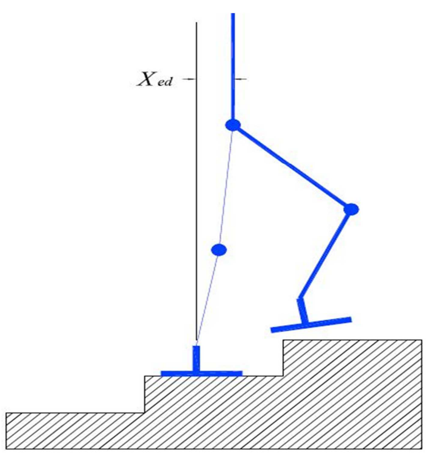

Figure 4(b)

Figure 4 Hip parameters (a) x_{sd} and (b) x_{ed}

For the hip trajectory for one cycle of ascending stairs, three phases are composed.: a starting phase in which the ascending speed varies from zero to a desired constant velocity, a steady phase with a desired constant velocity and an ending phase in which the speed varies from a desired constant velocity to zero.

The time intervals for k^{th} ascending step are as follows:

i. Starting position $(t = kT_c)$

ii. Steady position $(t = kT_c + T_d)$

iii. Ending position $(t = (k+1)T_c)$

The hip trajectory on k^{th} walking step is as follows:

$$x_h(t) = \begin{cases} kD_s + x_{ed}, & t = kT_c \\ (k+1)L_s - x_{sd}, & t = kT_c + T_d \\ (k+1)L_s + x_{ed} & t = (k+1)T_c \end{cases} \quad (1)$$

$$z_h(t) = \begin{cases} H_{hmin}, & t = kT_c + 0.5T_d \\ s_h + H_{hmax}, & t = kT_c + 0.5(T_c - T_d) \\ 2s_h + H_{hmin}, & t = (k+1)T_c + 0.5T_d \end{cases} \quad (2)$$

To be able to get stability motion, the hip motion parameter $\theta_h(t)$ is constant.

1.1 The Concept of Ascending Trajectory Relative to Stability

Hip motion $x_h(t)$ hardly affects the position of the zero moment point. x_{sd} and x_{ed} to vary within a fixed range is specified [1], in particular

$$\begin{cases} 0.0 < x_{sd} < 0.5D_s \\ 0.0 < x_{ed} < 0.5D_s \end{cases} \quad (3)$$

To obtain a smooth periodic $x_h(t)$ of the steady phase, the following constraints must be specified.

$$\begin{cases} \dot{x}_h(kT_c) = \dot{x}_h(kT_c + T_c) \\ \ddot{x}_h(kT_c) = \ddot{x}_h(kT_c + T_c) \end{cases} \quad (4)$$

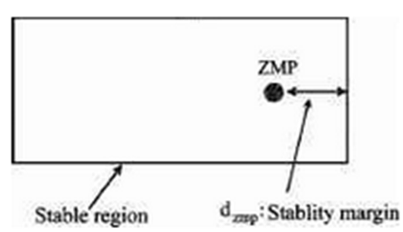

Figure 5 Zero Moment Point

Based on the whole smooth trajectory $x_h(t)$ and (4) and condition for stability (ZMP) as shown in Figure 5, a smooth trajectory with the largest stability margin can be formulated as follows:

$$\text{Max} \quad d_{zmp}(x_{sd}, x_{ed}) \tag{5}$$
$$x_{sd} \epsilon (0.0.5D_s), x_{ed} \epsilon (0.0.5D_s)$$

where $d_{zmp}(x_{sd}, x_{ed})$ denotes the stability margin.

If the ZMP is within the convex hull of all contact points_(stable region) between the feet and the ground, the biped robot is possible to walk.

IV. TRAJECTORY PLANNING OF BIPED ROBOT

In this section, the trajectory planning of ascending stairs of biped robot is introduced. Trajectory planning is crucial for stable motion of biped robot [5]. A trajectory is the path followed by the manipulator, plus the time profile along the path. A not trivial problem in bipedal robot ascending stairs is instability produced by the violent transition between the different dynamic walk phases. The mathematical interpolation is one of the simplest methods used for providing the suitable curves with respect to the given break points that the system must undergo. Considering a single higher order polynomial for the whole trajectory has many disadvantages. To avoid them, lower order interpolating polynomials are considered in place of single higher polynomial. These are cubic polynomial and cubic spline interpolation.

The cubic spline interpolation method (see Appendix) is applied to find the trajectory planning of biped walking. The result for applying this interpolation method is that the resulting trajectory be smooth and continuous. So, the ascending of biped robot can get high path curvature. Cubic spline interpolation guarantees velocity and acceleration continuous in control trajectories.

In biped ascending phase, there are always start condition and end condition. So, the starting and ending accelerations of walking are always zero. This start and end points condition is correspond to natural spline interpolation. So, cubic natural spline is applied to generate trajectory planning of biped robot.

1. Kinematics Constraints for one cycle of ascending stairs

In this study the complete one cycle of ascending stairs stable hip trajectory will be generated. The definition of one ascending step is now given as follows:

One cycle of Ascending Stairs: The one cycle of ascending stairs of the biped robot is defined as to begin with the heel of the right foot leaving the stair and with the heel of the right foot making first contact with the next up stair.

The hip trajectory for one cycle of ascending stairs of the biped robot can be easily followed from (1) and (2).

$$x_h(t) = \begin{cases} x_{ed,} & t = 0 \\ L_s - x_{sd}, & t = T_d \\ L_s + x_{ed} & t = T_c \end{cases} \tag{6}$$

$$z_h(t) = \begin{cases} H_{hmin,} & t = 0.5T_d \\ s_h + H_{hmax}, & t = 0.5(T_c - T_d) \\ 2s_h + H_{hmin,} & t = T_c + 0.5T_d \end{cases} \tag{7}$$

The parameter $\theta_h(t)$ can be determined as a constant. And then, parameters of hip trajectory are defined as follows:

Table 1 Parameters for the one cycle of ascending stairs

parameter	value
T_d	0.18 s
T_m	0.4 s
T_c	0.9 s
x_{sd}	19.39 cm
x_{ed}	10.31 cm
H_{hmin}	86.75 cm
H_{hmax}	87.8 cm
D_s	50 cm
L_s	39 cm
S_h	13 cm

In order to achieve stable motion, the two hip parameters are satisfied (3).

$$\begin{cases} 0.0 < x_{sd} < 25 \\ 0.0 < x_{ed} < 25 \end{cases} \tag{8}$$

By substituting the values of hip parameters to the one ascending step constraints (6) and (7), we get

$$x_h(t) = \begin{cases} 10.31, & ,t = 0 \\ 19.61, & ,t = 0.18, \\ 49.31, & ,t = 0.9 \end{cases} \tag{9}$$

$$z_h(t) = \begin{cases} 86.75, & t = 0.09 \\ 100.8, & t = 0.36 \\ 112.75 & t = 0.99 \end{cases} \tag{10}$$

The ascending of biped robot's velocity and acceleration are always zero at the initial and final positions of one cycle of ascending stairs. From this concept, natural spline condition

$m_1 = m_3 = 0$ will be used according to its initial and final position.

By using (13), the values of time interval can be determined and these values are as follows:

$$h_1 = 0.18 - 0 = 0.18$$

$$h_2 = 0.9 - 0.18 = 0.72$$

From this, d_1 and d_2 can be calculated by using (14),

$$d_1 = \frac{19.61 - 10.31}{0.18} = 51.667$$

$$d_2 = \frac{49.31 - 19.61}{0.72} = 41.25$$

For natural spline condition, the value of u_2 is determined by using (14),

$$u_2 = 6(41.25 - 51.667) = -62.52$$

Now the value of acceleration m_2 is obtained as follows from (14):

$$2(h_1 + h_2)m_2 + h_2 m_3 = u_2$$

$$2(0.18 + 0.72)m_2 = -62.52$$

$$m_2 = -34.733$$

For the pre-swing hip trajectory $t\epsilon(0.0.18)$, cubic spline interpolation can be expressed as

$$x_h(t) = a_{1,0}t^3 + a_{1,1}t^2 + a_{1,2}t + a_{1,3} \quad (15)$$

Use (12) for the values of coefficients,

$$a_{1\prime 0} = \frac{m_2 - m_1}{6h_1} = \frac{-34.733 - 0}{6(0.18)} = -32.16$$

$$a_{1,1} = \frac{m_1}{2} = 0$$

$$a_{1,2} = d_1 - \frac{h_1(2m_1 + m_2)}{6}$$

$$= 51.667 - \frac{(0.18)(-34.733)}{6} = 52.709$$

$$a_{1,3} = f_1 = 10.31$$

Substitute the values of coefficients in (15),

$$x_h(t) = -32.160t^3 + 52.709t + 10.31.$$

For the hip trajectory $t\epsilon(0.18.0.9)$, cubic spline interpolation can be expressed as

$$x_h(t) = a_{2,0}(t - 0.18)^3 + a_{2,1}(t - 0.18)^2 + a_{2,2}(t - 0.18) + a_{2,3}. \quad (16)$$

Similarly for the pre-swing phase, the values of the coefficients $a_{2,0}, a_{2,1}, a_{2,2}$ and $a_{2,3}$ can be determined.

By substituting the values of $a_{2,0}, a_{2,1}, a_{2,2}$ and $a_{2,3}$ to (16),

$$x_h(t) = 8.04(t - 0.18)^3 - 17.37(t - 0.18)^2 + 49.59(t - 0.18) + 19.61.$$

Finally the complete one cycle of standing stairs hip trajectory $x_h(t)$ is obtained as follows:

$$x_h(t) = \begin{cases} -32.16t^3 + 52.71t + 10.31 & , t \in (0,0.18) \\ 8.04(t - 0.18)^3 - 17.37(t - 0.18)^2 + 49.59(t - 0.18) + 19.61 & t \in (0.18,0.9) \end{cases}.$$

Similarly, the hip trajectory $z_h(t)$ can be derived as follows:

$$z_h(t) = \begin{cases} -68.043(t - 0.09)^3 + 57(t - 0.09) + 86.75 & , t \in (0.09,0.36) \\ 29.16(t - 0.36)^3 - 55.115(t - 0.36)^2 + 42.116(t - 0.36) + 100.8 & , t \in (0.36,0.99) \end{cases}.$$

V. DEMONSTRATION OF HIP TRAJECTORY BY APPLYING MATLAB

In this section, the numerical results of hip trajectory along x-axis and z axis are demonstrated as shown in Figure 6 and 7.

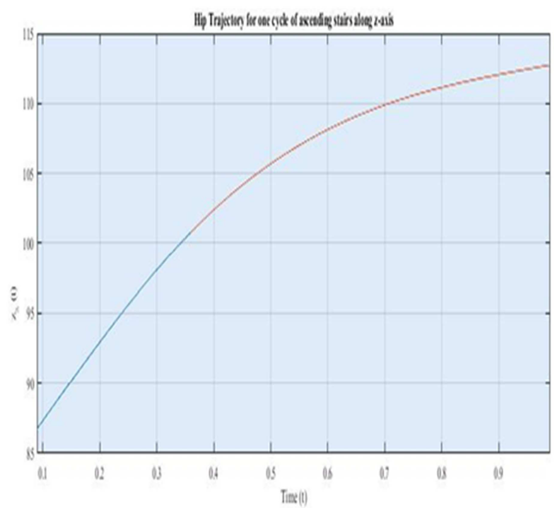

Figure 6 one-cycle of ascending stairs along x-axis

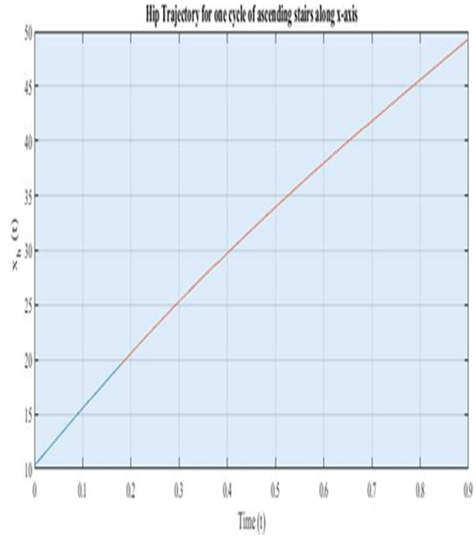

Figure 7 one cycle of ascending stairs along z-axis

VI. CONCLUSION

In this paper, mathematical modeling to stable hip trajectory for ascending stairs of a biped robot has been generated. As theoretical background of ascending stairs hip trajectory has been discussed. And then, kinematics constraints for k^{th} ascending stairs have been described. Finally, the goal of this research, the one cycle of ascending stairs hip trajectory has been generated by cubic spline interpolation. The resulting trajectories for applying this interpolation method are stable, smooth and continuous has been verified by Matlab figures.

APPENDIX

Cubic Spline Interpolation

Suppose that $\{(t_j, f_j)\}_{j=1}^{n}$ are n points, where $t_1 < t_2 < \cdots < t_n$. The function $S(t)$ is called cubic spline if there exist $n-1$ cubic polynomials $S_j(t)$ with coefficients $a_{j,0}, a_{j,1}, a_{j,2}$ and $a_{j,3}$ that satisfies the following properties [4]:

I. $S(t) = S_j(t) = a_{j,0}(t-t_j)^3 + a_{j,1}(t-t_j)^2 + a_{j,2}(t-t_j) + a_{j,3}$ for $t \in [t_j, t_{j+1}]$ and $j = 1,2,\cdots,n-1$

II. $S(t_j) = f_j$ for $j = 1,2,\cdots,n.$

III. $S_j(t_{j+1}) = S_{j+1}(t_{j+1})$ for $j = 1,2,\cdots,n-2.$

IV. $S_j'(t_{j+1}) = S_{j+1}'(t_{j+1})$ for $j = 1,2,\cdots,n-2.$

V. $S_j''(t_{j+1}) = S_{j+1}''(t_{j+1})$ for $j = 1,2,\cdots,n-2.$

Property I states that $S(t)$ consists of piecewise cubies. Property II states that piecewise cubies interpolate the given sets of data points. Property III and IV require that the piecewise cubies represent a smooth continuous function. Property IV states that the second derivatives of the resulting function are also continuous.

A piecewise function is constructed as follows [4]:

$$S(t) = \begin{cases} S_1(t) & if \quad t_1 \le t \le t_2 \\ S_2(t) & if \quad t_2 \le t \le t_3 \\ & \vdots \\ S_{n-1}(t) & if \quad t_{n-1} \le t \le t_n \end{cases}$$

where $S_j(t)$ is a cubic polynomial defined by

$$S_j(t) = a_{j,0}(t-t_j)^3 + a_{j,1}(t-t_j)^2 + a_{j,2}(t-t_j) + a_{j,3}, j = 1,2,\cdots,n-1. \quad (11)$$

By using the properties, I, II, III and IV, we can get the coefficients of cubic spline interpolation as follow:

$$a_{j,0} = \frac{m_{j+1}-m_j}{6h_j}, \quad a_{j,1} = \frac{m_j}{2}, \quad a_{j,2} = a_{j,3} - \frac{h_j(2m_j+m_{j+1})}{6}, \quad a_{j,3} = f_j \quad (12)$$

where $m_j = S_j''(t_j)$ and $h_j = t_{j+1} - t_j$. (13)

Construction of natural spline interpolation

There exists a unique cubic spline with the free boundary conditions $m_1 = m_n = 0$ Mathews& Fink [3]. The linear system equations for finding $m_2, m_3, \dots, m_{n-1}$ connected with natural spline condition are

$$2(h_1 + h_2)m_2 + h_2 m_3 = u_2$$

$$h_{j-1}m_{j-1} + 2(h_{j-1} + h_j)m_j + h_j m_{j+1} = u_j$$

for $j = 3,4,\dots,N-2$

$$h_{n-2}m_{n-2} + 2(h_{n-2} + h_{n-1})m_{n-1} = u_{n-1}$$

where $u_j = 6(d_j - d_{j-1}), j = 2,3,\dots,n-1$ and $d_j = \frac{f_{j+1}-f_j}{h_j}$. (14)

ACKNOWLEDGEMENT

I am also grateful to my respectable teachers, Dr.Khaing Khaing Aye and Dr, Ni Ni Win, Heads of Department of Engineering Mathematics at YTU, Faculty of computing at MIIT who provided a support and encouragement. I also wish to express gratitude to my adorable benefactor parents for their encouragement and support and my husband for his AutoCAD drafting for this paper.

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