1.2 Fundamental interactions

In nature, there are four different types of fundamental interactions. These are gravitational, electromagnetic, weak and strong interactions. According to the quantum theory of field, all the interactions rely on the mechanism of exchange of quanta. All the forces are transmitted from one particle to the other by successive processes of emission, propagation and absorption of such mediators.

1. Gravitational interaction – It is the weakest of all the fundamental interactions and acts between all bodies having mass and is described by the long range inverse square type. This interaction is believed to be mediated through the quantum of interaction – graviton – which is yet to be discovered. This interaction provides a large attractive force between planets and produces the acceleration due to gravity in the vicinity of planets. It is of extreme importance for astral bodies in galaxies and on cosmological scale since large masses and distances are involved.

II. Collisions

2.1 Four-vectors

It is convenientat this point to introduce some simplifying notation. We define the position-time four-vector $x^{\mu}, \mu = 0,1,2,3$, as follows:

$$x^0 = ct \qquad x^1 = x \qquad x^2 = y \qquad x^3 = z$$

In term of x^{μ}, the Lorentz transformations take on a more symmetrical appearance:

$$\begin{aligned} x^{0} &= \gamma\left(x^0 - \beta x^1\right) \\ x^{1'} &= \gamma\left(x^1 - \beta x^0\right) \\ x^{2'} &= x^2 \\ x^{3'} &= x^3 \end{aligned} \qquad (2.1.1)$$

$$x^{\mu\nu} = \sum_{\nu=0}^{3} \Lambda^{\mu}_{\nu} x^{\nu} \; (\mu = 0,1,2,3) \qquad (2.1.2)$$

The coefficients Λ^{μ}_{p} may be regarded as the elements of a matrix Λ:

$$\Lambda = \begin{bmatrix} \gamma & -\gamma^{\beta} & 0 & 0 \\ -\gamma^{\beta} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2.1.3)$$

(i.e., $\Lambda^0_0 = \Lambda^1_1 = \gamma; \Lambda^1_0 = \Lambda^0_1 = -\gamma\beta; \Lambda^2_2 = \Lambda^3_3 = 1$; and all the rest are zero). To avoid writing lots of Σ's, we shall follow Einstein's "summation convention," which says that repeated Greek indices (one as subscript, one as superscript) are to be summed from 0 to 3. Thus equation (2.1.2) becomes, finally,

$$\chi^{\mu\nu} = \Lambda^{\mu}_{\nu}\chi^{\nu} \qquad (2.1.4)$$

$$I \equiv (\chi^0)^2 - (\chi^1)^2 - (\chi^2)^2 - (\chi^3)^2 = (\chi^{0'})^2 - (\chi^{1'})^2 - (\chi^{2'})^2 - (\chi^{3'})^2 \qquad (2.1.5)$$

Such a quantity, which has the same value in any internal system, is called an invariant. (In the same sense, the quantity $r^2 = x^2 + y^2 + z^2$ is invariant under rotations.) This invariant is in the form of a sum: $\sum_{\mu=0}^{3}\chi^{\mu}\chi^{\mu}$, but unfortunately there are those three irritating minus signs. To keep track of them, we introduce the metric, $g_{\mu\nu}$, whose components can be displayed as a matrixg:

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \qquad (2.1.6)$$

i.e., $g_{00} = 1; g_{11} = g_{22} = g_{33} = -1$; all the rest are zero. With the help of $g_{\mu\nu}$, the invariant can be written as a double sum: $I = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu}\chi^{\mu}\chi^{\nu} = g_{\mu\nu}\chi^{\mu}\chi^{\nu}$ (2.1.7)

Carrying things a step further, we define the covariant four-vector χ_{μ} (index down) as follows:

$$\chi_{\mu} \equiv g_{\mu\nu}\chi^{\nu} \qquad (2.1.8)$$

We define a four-vector, a^{μ}, as a four-component object that transforms in the same way does when we go from one inertial system to another, to wit:

$$a^{\mu'} = \Lambda^{\mu}_{\nu}a^{\nu} \qquad (2.1.9)$$

With the same coefficients Λ^{μ}_{ν}. To each such (contravariant) four-vector we associate a covariant four-vector a_{μ}, obtained by simply changing the signs of the spatial components, or, more formally

$$a_{\mu} = g_{\mu\nu}a^{\nu}$$

Of course, we can go back from covariant to contravariant by reversing the signs again:

$$a^{\mu} = g^{\mu\nu}a_{\nu} \qquad (2.1.10)$$

Where $g^{\mu\nu}$ are technically the elements in the matrix g^{-1} (however, since our metric is its own inverse, $g^{\mu\nu}$ is the same as $g_{\mu\nu}$). Given any two four-vectors, a^{μ} and b^{μ}, the quantity

$$a^{\mu}b_{\mu} = a_{\mu}b^{\mu} = a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3 \qquad (2.1.11)$$

is invariant (the same number in any internal system).

$$a.b \equiv a_{\mu}b^{\mu}$$

$$a.b = a^0b^0 - \vec{a}.\vec{b} \qquad (2.1.12)$$

We also use the notation a^2 for the scalar product of a^{μ} with itself:

$$a^2 \equiv a.a = (a^0)^2 - a^2 \qquad (2.1.13)$$

2.2 Energyand Momentum

If we want to go from the lab system, S, to a moving system, S', both the numerator and the denominator must be transformed only the numerator transforms; $d\tau$, as we have seen, is invariant. In fact, proper velocity is part of a four-vector:

$$\eta^{\mu} = \frac{d\chi^{\mu}}{d\tau} \tag{2.2.1}$$

Whose zeroth component is

$$\eta^{0} = \frac{d\chi^{0}}{d\tau} = \frac{d(ct)}{\left(\frac{1}{\gamma}\right)dt} = \gamma c \tag{2.2.2}$$

Thus $\eta^{\mu} = \gamma(c, v_x, v_y, v_z)$

Incidentally, $\eta_{\mu}\eta^{\mu}$ should be invariant, and it is:

$$\eta_{\mu}\eta^{\mu} = \gamma^2\left(c^2 - v_x^2 - v_y^2 - v_z^2\right) = \gamma^2 c^2\left(1 - \frac{v^2}{c^2}\right) = c^2 \tag{2.2.3}$$

If we defined momentum as mv, then the law of conservation of momentum would be inconsistent with the principle of relativity. But if we define momentum as $m\eta$, then conservation of momentum is consistent with the principle of relativity. This doesn't guarantee that momentum is conserved; that's a matter for experiments to decide. But it does say that if we're hoping to extend momentum conservation to the relativistic domain, we had better not define momentum as mv, whereas $m\eta$ is perfectly acceptable.

The upshot is that in relativity, momentum is defined as mass times proper velocity:

$$p \equiv m\eta \tag{2.2.4}$$

Since proper velocity is part of a four-vector, the same goes for momentum:

$$p^{\mu} = m\eta^{\mu} \tag{2.2.5}$$

The spatial components of P^{μ} constitute the (relativistic) momentum three-vector:

$$\vec{p} = \gamma m v = \frac{mv}{\sqrt{1 - v^2/c^2}} \tag{2.2.6}$$

Meanwhile, the "time" component is

$$p^{0} = \gamma mc \tag{2.2.7}$$

For reasons that will appear in a moment, we define the "relativistic energy", E, as

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{2.2.8}$$

The zeroth component of p^{μ}, then, is E/c. Thus energy and momentum together make up a four-vector-the energy- momentum four-vector:

$$p^{\mu} = \left(\frac{E}{c}, p_x, p_y, p_z\right) \tag{2.2.9}$$

Incidentally, from equations (2.2.9) and we obtain

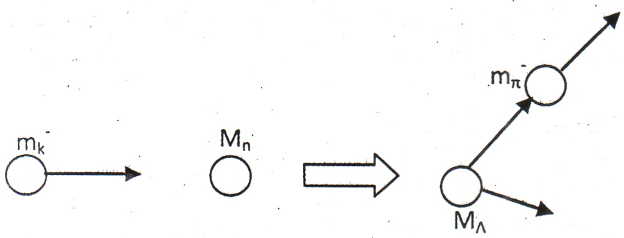
$$p_\mu p^\mu = \frac{E^2}{c^2} - p^2 = m^2c^2$$

which, again, is manifestly invariant.

III. Calculation of Momentum Transfer

3.1 Calculation of Threshold Energy of $K^- + n \rightarrow \pi^- + \Lambda$ Reaction

We calculate general formulation of threshold energy of $K^- + n \rightarrow \Lambda + \pi^-$ reaction as shown in figure.



Figure(3.1) Schematic diagram of $K^- + n \rightarrow \Lambda + \pi^-$ reaction

Let p^μ_{TOT} be the total energy-momentum four-vector in the laboratory; it is conserved. We calculate the total energy-momentum four-vector before collision in the laboratory frame.

$$p^\mu_{TOT} = \left(\frac{E_{tot}}{c}, \vec{p}_{tot}\right) \qquad (3.1.1)$$

E_{tot} = total energy of before collision

$\vec{p}_{tot}$ = total momentum of before collection

Where E = total energy of before collision

m_n = mass of N particle

$$\vec{p}_{tot} = \vec{p}_{K^-} + \vec{p}_n$$

$\vec{p}_n = 0$, target particle is at rest.

$$\therefore \vec{p}_{tot} = \vec{p}_{K^-} \qquad (3.1.2)$$

By substituting equation (3.1.2) into equation (3.1.1), we get

$$P^\mu_{tot} = \left(\frac{E + m_n c^2}{c}, \vec{P}_{K^-}\right) \qquad (3.1.3)$$

Let be the total energy-momentum four-vector in the centre of mass frame. We calculate the total energy-momentum four-vector after collision in centre of mass frame.