An EffectiveTechnique to Optimization

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Abstract

The purpose of this research is to find optimal solution of non-affine C^1 function by means of constructing respective Lagrange multiplier function together with Fritz John and Karush-Kuhn- Tucker conditions. These conditions are relating to problems P and Q. Problem P is the minimization of convex functional value with the constrained set of convex inequalities. Problem Q is the minimization of convex functional value with the constrained set of convex inequalities, linear inequalities and convex equalities. Lagrange multiplier function method is well-known but its manipulation is rather complicated. An effective way to handle Lagrange function is presented, in this paper. It is more convenient and more available than the Simplex method due to G.Dantzig. Most of the functions in this paper is non-affine C^1-functions. Moreover, some illustrative examples are also discussed where necessary.

Keywords

Convex Function, Concave Function, Linear Programming, Convex Programming, Karush-Kuhn-Tucker conditions, Lagrange Function and Fritz-John conditions.

1.Introduction

In the literature of optimization of functional value, there are two mains parts such as linear and non-linear optimization. Linear optimization has been developed as linear programming, since the middle years of 20^{th} century. Meanwhile, non-linear optimization has been developed as convex programming quadratic programming and fractional programming since nearly at the end of 20^{th} century. Many researchers concerning these phenomena have been trying in order to get new theories and new techniques which are available to social and environmental development of human societies. These research papers have been done by I.N.Gass, R.T Rockefeller, Mclindon and their followers. This research paper is one of the fruitful results of those researchers, especially for the research paper is organized as the technical development of programming methods. In the simplex method, pivot element is needed in any column of matrix in our consideration and the remaining elements must be vanished by row operations. The KKT method is not difficult as simplex method and KKT method gives us optimal solution by method of reducing impossible cases.

2. Non-affine and C^1 Function.

2.1 Non-affine Function

2.1.1 Definition [4]

Let $f: \mathbb{R}^n \longmapsto \mathbb{R}$.

$x \longmapsto f(x)$ is an affine function if there exists nonzero constant c:

$$f(x) = L(x) + c$$

where $L: \mathbb{R}^n \longmapsto \mathbb{R}$ is linear.

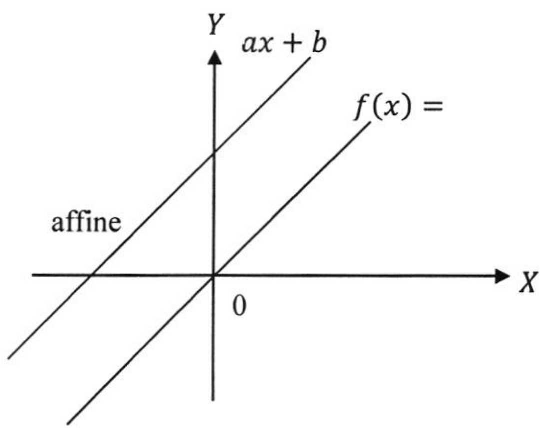
The function which is not an affine function is called a non-affine function.

Example

Every linear function is non-affine.

Every concave function is non-affine.

Every convex function is non-affine.



2.1.2 Definition [3]

Let $f: \mathbb{R}^n \longmapsto \mathbb{R}$

$x \longmapsto f(x)$

We say $f \in C^1$ if and f' are continuous.

Hence, $C^1(\mathbb{R}) = \{f | f: \mathbb{R} \to \mathbb{R}, f \text{ and } f' \text{are continuous}\}$ and

$C^\infty(\mathbb{R}) = \{f | f: \mathbb{R} \to \mathbb{R}, f, f', f'', \ldots \text{ are continuous}\}$.

Example

$|\bullet|: \mathbb{R} \longmapsto \mathbb{R}$

$$x \longmapsto |x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 1 & ,x > 0 \\ -1 & ,x < 0 \end{cases}$$

$f'(0)$ does not exist. So $f \notin C^1(\mathbb{R})$,

but $f \in C^1(R\backslash\{0\})$.

3. Fritz-John and Karush-Kuhn-Tucker Conditions [3]

In the first part, we study Fritz-John conditions and Karush-Kuhn-Tucker conditions with their proofs for problems P and Q.

3.1 Fritz-John Optimal Conditions for Problems P and Q [3]

3.1.1 Lagrange Functions [3]

Definition

Let $f_i: \mathbb{R}^n \mapsto \mathbb{R}$ and $h_j: \mathbb{R}^n \mapsto \mathbb{R}$ be non-affine and C^1- function. $(i = 0,1, \ldots, q)(j = 1, \ldots, r)$.The problem P is $minf_0(x): f_i(x) \leq 0$ and the problem Q is $minf_0(x): f_i(x) \leq 0$, $(i = 1, \ldots, q)$,...

Then **Lagrange function for problem P** is

$L : \mathbb{R}^{m+n+q+1} \mapsto \mathbb{R}$

$$(x, \lambda) \mapsto L(x, \lambda) = \lambda_0 f_0(x) + \lambda_1 f_1(x) + \cdots + \lambda_q f_q(x) + \sum_{k=1}^{m} v_k(a_k^T x - b)$$

Definition [3]

Let $f: \mathbb{R}^n \mapsto \mathbb{R}$

Then **Lagrange function for problem Q is**

$L:\mathbb{R}^{n+m+q+r+1} \mapsto \mathbb{R}$

$(x, \lambda, \mu, \mu^*) \mapsto L(x, \mu, \mu^*)$

$L(x, \lambda, \mu, \mu^*) = f_0(x) + \sum_{k=1}^{m} \mu_k(a_k^T x - b) + \sum_{i=1}^{q} \mu_i f_i(x) + \sum_{j=1}^{r} \mu_j^* h_j(x)$

Consider $\min_{x \in c} f(x)$

where $f: \mathbb{R}^n \to \mathbb{R}$

$x \mapsto f(x) \in C^1(R^n)$ and

Q: $\min_{x \in D} f(x)$

where , $C = \mathbb{R}^n \cap \{x | f_i(x) \leq 0, a_k^T x - b \leq 0\}$, $1 \leq i \leq q$, $1 \leq k \leq m$.

$$D = C \cap \{x \mid h_j(x) = 0 , 1 \leq j \leq r\}.$$

3.1.2 Definition [3][2]

We say that constraint set D as above admits Lagrange multipliers $\lambda_i^*$and μ_j^* at a point $x^* \in D$ if for every $f_0 \in C^1(\mathbb{R})$ for which x^* is a local minimizer of a problem, there exist vector $\lambda^* = (\lambda_1^*, \ldots, \lambda_m^*)$ and $\mu^* = (\mu_1^*, \ldots, \mu_r^*)$ that satisfy the following conditions.