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One Dimensional Convolution

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Abstract

The development of multi-core computers means that the characteristics of digital filters can be rapidly processed in software form. The filter has been implemented in software by an algorithm within the computer which effectively convolves the sampled sequence x(n) with the impulse response of the filter h(n) to produce the processed output sequence y(n). Convolution is the name given to the ordered combination of multiplication and summation.

Key word: convolution, digital filtering

Introduction

Convolution is the important and fundamental concept in signal processing and analysis. In order to understand the meaning of convolution, it should be started from the concept of signal decomposition. The input signal is decomposed into simple additive components, and the system response of the input signal results in by adding the output of these components passed through the system. In general, a signal can be decomposed as a weighted sum of basis signals. Furthermore, there is an important fact under convolution; the only thing needed to know about the system's characteristics is the impulse response of the system. By combining the properties of impulse response and decomposition, the equation of convolution can be constructed. In linear and time-invariant system, the response resulting from several inputs can be computed as the sum of the responses each input acting alone. A signal is decomposed into a set of impulses and the output signal can be computed by adding the scaled and shifted impulse responses. If we know a system's impulse response, then we can easily find out how the system reacts for any input signal.

Computerized Convolution

A linear system's characteristics are completely specified by the system's impulse response, as governed by the mathematics of convolution. This is the basis of many signal processing techniques. For example: Digital filters are created by designing an appropriate impulse response. Let's summarize this way of understanding how a system changes an input signal into an output signal. First, the input signal can be decomposed into a set of impulses, each of which can be viewed as a scaled and shifted delta function. Second, the output resulting from each impulse is a scaled and shifted version of the impulse response. Third, the overall output signals can be found by adding these scaled and shifted impulse responses. In other words, if system's impulse response is known then the output will be calculated for any possible input signal. This means we know everything about the system. There is nothing more that can be learned about a linear system's characteristics. The impulse response goes by a different name in some applications. If the system being considered is a filter, the impulse response is called the filter kernel, the convolution kernel, or simply, the kernel. Convolution is a formal mathematical operation, just as multiplication, addition, and integration. Addition takes two numbers and produces a third number, while convolution takes two signals and produces a third signal. Convolution is used in the mathematics of many fields, such as probability and statistics. In linear systems, convolution is used to describe the relationship between three signals of interest: the input signal, the impulse response, and the output signal.

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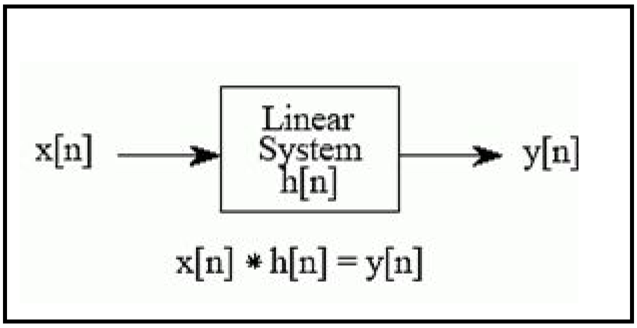


Figure 1. How convolution is used in DSP.
The output signal from a linear system is equal to the input signal convolved with the system's impulse response. Convolution is denoted by a star when writing equations.

Figure 1 show the notation when convolution is used with linear systems. An input signal, $x[n]$, enters a linear system with an impulse response, $h[n]$, resulting in an output signal, $y[n]$.

In equation form: $\boldsymbol{x[n] * h[n] = y[n]}$

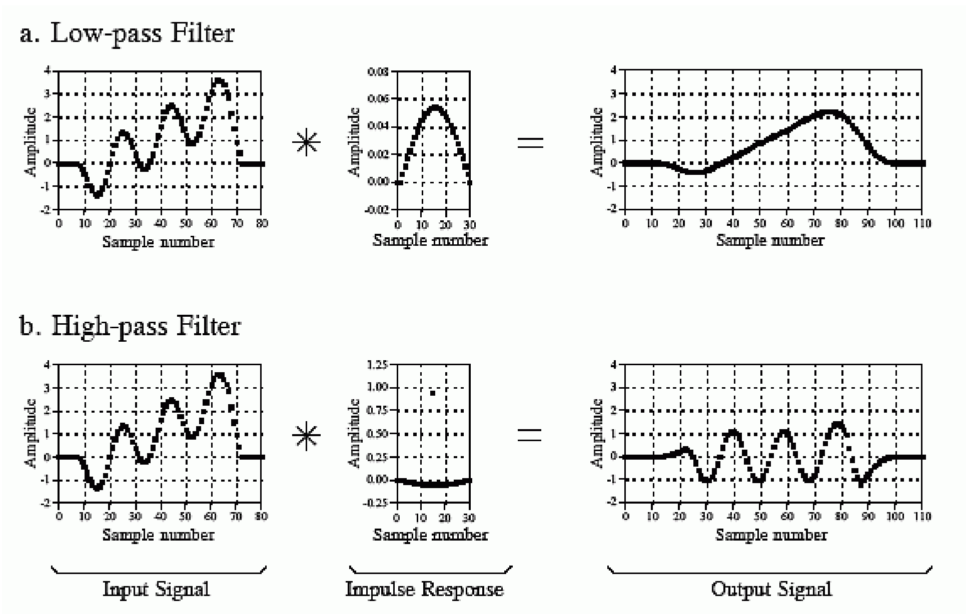


Figure 2. Examples of low-pass and high-pass filtering using convolution.
In this example, the input signal is a few cycles of a sine wave plus a slowly rising ramp. These two components are separated by using properly selected impulse responses.

Figure 2 shows convolution being used for low-pass and high-pass filtering. The example input signal is the sum of two components: three cycles of a sine wave (representing a high frequency), plus a slowly rising ramp (composed of low frequencies). In (a), the impulse response for the low-pass filter is a smooth arch, resulting in only the slowly changing ramp waveform being passed to the output. Similarly, the high-pass filter, (b), allows only the more rapidly changing.

Implementation

As used in Digital Signal Processing (DSP), convolution can be understood in two separate ways. The first looks at convolution from the viewpoint of the input signal. This involves analyzing how each sample in the input signal contributes to many points in the output signal. The second way looks at convolution from the viewpoint of the output signal. This examines how each sample in the output signal has received information from many points in the input signal. Keep in mind that these two perspectives are different ways of thinking about the same mathematical operation. The first viewpoint is important because it provides a conceptual understanding of how convolution pertains to DSP. The second viewpoint describes the mathematics of convolution. This typifies one of the most difficult tasks encountered in DSP: making the conceptual understanding fit with the jumble of mathematics used to communicate the ideas.

Table 2(a). Convolution is the name given to the ordered combination of multiplication for each time.

t = 0	t = T	t = 2T	t = 3T	t = 4T	Response due to
$x_0\ h_0$	$x_0\ h_1$	$x_0\ h_2$	$x_0\ h_3$	$x_0\ h_4$	x_0
	$x_1\ h_0$	$x_1\ h_1$	$x_1\ h_2$	$x_1\ h_3$	x_1
		$x_2\ h_0$	$x_2\ h_1$	$x_2\ h_2$	x_2
			$x_3\ h_0$	$x_3\ h_1$	x_3
				$x_4\ h_0$	x_4

Table 2(b). Convolution is the name given to the ordered combination of multiplication followed by summation.

Column	T	Sum
1	0	$y_0 = x_0\ h_0$
2	T	$y_1 = x_0\ h_1 + x_1\ h_0$
3	2T	$y_2 = x_0\ h_2 + x_1\ h_1 + x_2\ h_0$
4	3T	$y_3 = x_0\ h_3 + x_1\ h_2 + x_2\ h_1 + x_3\ h_0$
5	4T	$y_4 = x_0\ h_4 + x_1\ h_3 + x_2\ h_2 + x_3\ h_1 + x_4\ h_0$

Using the discrete form of convolution confirms the results shown in Table 2. To evaluate y(n), the output of the filter after n conversions a systematic approach is required. In Table 2(b) the rows represent the responses to the input sequence x(0), x(1), x(2), x(3), x(4), respectively. The columns show the terms present at times t=0,t=T, t=2T,etc. and the response y(n) is simply the sum of the terms in the nth column.

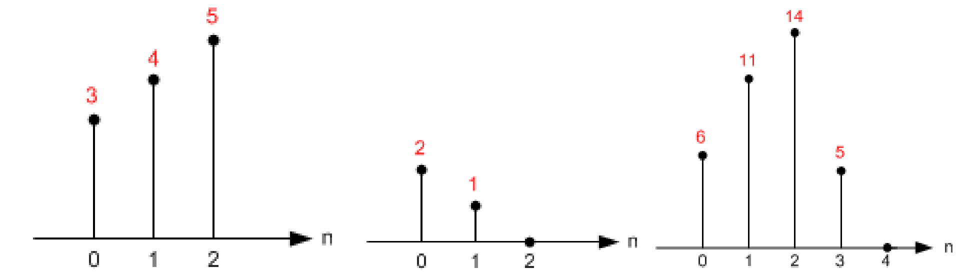
Graphical Approach

Convolution steps are: (1) Slide the center element of the kernel to the point to be computed, (2) Multiply the convolution weights in the kernel by the corresponding point values in the input, (3) Sum the individual products from the previous step. The result is the value for the output. This process is repeated for all of the points.

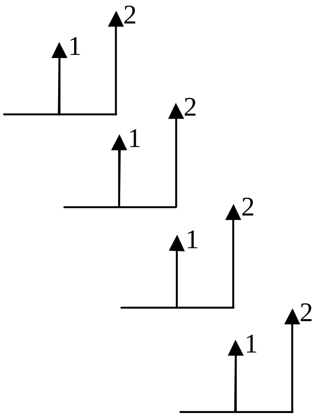
Let's start with an example of convolution of 1D signal, then find out how to implement into computer programming algorithms.

x[n] = { 3, 4, 5 }
h[n] = { 2, 1 }

x[n] has only non-zero values at n=0,1,2, and impulse response, h[n] is not zero at n=0,1. Others which are not listed are all zeros.



Input: x[n] Impulse Response: h[n] Output: y[n]



$y_0 = x_0\ h_0 = 2x3 = 6$

$y_1 = x_0\ h_1 + x_1\ h_0 = 4\times2+3\times1=8+3=11$

$y_2 = x_0\ h_2 + x_1\ h_1 + x_2\ h_0 = 5\times2+4\times1+3\times0=10+4=14$

$y_4 = x_0\ h_4 + x_1\ h_3 + x_2\ h_2 + x_3\ h_1 + x_4 = 0\times1+5\times1+4\times0+3\times0=5$

It can be figured out how this system behaves by only looking at the impulse response of the system. When the impulse signal is entered the system, the output of the system looks like amplifier and echoing. At the time is 0, the intensity was increased (amplified) by double and gradually decreased while the time is passed.

Analytical Approach

From the equation of convolution, the output signal y[n] will be

y[n]=∑ x[k] . h[n-k]

Let's compute manually each value of y[0], y[1], y[2], y[3], ...

$$y[0] = \sum_{k=-\alpha}^{\alpha} x[k].h[0-k] = x[0]\,h[0] = 3.2 = 6$$

$$y[1] = \sum_{k=-\alpha}^{\alpha} x[k].h[1-k] = x[0]\ .\ h[1-0] + x[1].h[1-1] + \cdots = \mathrm{x}[0].\mathrm{h}[1] + \mathrm{x}[1].\mathrm{h}[0]$$
$$= 3.1 + 4.2 = 11$$

$$y[2] = \sum_{k=-\alpha}^{\alpha} x[k].h[2-k] = x[0]\ .\ h[2-0] + x[1].h[2-1] + x[2].h[2-2] + \cdots$$
$$= \mathrm{x}[0].\mathrm{h}[2] + \mathrm{x}[1].\mathrm{h}[1] + x[2].h[0] = 3.0 + 4.1 + 5.2 = 14$$

$$y[3] = \sum_{k=-\alpha}^{\alpha} x[k].h[3-k] = x[0]\ .\ h[3-0] + x[1].h[3-1] + x[2].h[3-2] + x[3].h[3-3] + \cdots$$
$$= \mathrm{x}[0].\mathrm{h}[3] + \mathrm{x}[1].\mathrm{h}[2] + x[2].h[1] + \mathrm{x}[3].\mathrm{h}[0] = 0 + 0 + 5.1 + 0 = 5$$

$$y[4] = \sum_{k=-\alpha}^{\alpha} x[k].h[4-k] = x[0]\ .\ h[4-0] + x[1].h[4-1] + x[2].h[4-2] + \cdots = 0$$

Computational Approach

The convolution algorithm can be implemented by C++ programming. The code is listed.

```
for ( i = 0; i < sampleCount; i++ )
{
    y[i] = 0;                       // set to zero before sum
    for ( j = 0; j < kernelCount; j++ )
    {
     y[i] += x[i - j] * h[j];    // convolve: multiply and accumulate
    }
}
```

Result is:

```
INPUT
 3.00,  4.00,  5.00,  0.00,  0.00,

KERNEL
 2.00,  1.00,

OUTPUT
 6.00, 11.00, 14.00,  5.00,  0.00,
```

Results and Discussion

This study emphasized on the underlying principles of convolution, together with the acquisition of some computational skills. Although the example is based on a filter, it is assumed that the transfer function can be derived analytically: in certain instances, the system parameters can only be obtained by dynamic measurement. Clearly, this is equal to the differential of the step response and serves as useful reminder of the principle of linearity. Recognize the significance of the results, convolution is seen to be a powerful and incisive method of solving differential equations of the form. Convolution provides an alternative time-domain procedure which gets close to the physical reality behind the abstraction. The problem can be solved by many ways such as computational, graphical and analytical methods. But the finding is same result. Convolution and related operations are found in many applications of engineering and mathematics. In electrical engineering, the convolution of one function (the input signal) with a second function (the impulse response) gives the output of a linear time-invariant system (LTI). It is used in digital signal processing and image processing applications. In optics, many kinds of "blur" are described by convolutions. In linear acoustics, an echo is the convolution of the original sound with a function representing the various objects that are reflecting it. Convolution is used in mathematics of many fields such as probability and statistics.

Conclusion

Convolution in time domain corresponds to multiplication in frequency domain. This processing operation is equivalent to multiplying the frequency spectrum of the signal by the system frequency, the product corresponding to a filtering operation. Time domain convolution and frequency domain filtering can be perceived as complementary perspectives of the same processing operation. Essentially, the convolution construction is made up of a pair of nested loops, to provide the necessary multiply-summate and shift structure. Time domain convolution is a fundamental operation which provides a basis for more advanced signal-processing applications.

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