Replacement Policies of Items

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Abstract

This paper discusses two categories of replacement techniques to determine the best replacement strategies for the items that deteriorate with time and those do not deteriorate but fail suddenly. These models are discussed with respect to the parameters like maintenance cost, time and value of money.

Keywords: Replacement models, Deteriorate with time, Discount rate, Present value of the money, Fail suddenly.

Introduction

This paper is concerned with the replacement of items. In our daily life, all industrial and military equipment gets worn with time and usage and it functions with decreasing efficiency. For example, a machine requires higher operating cost, a transport vehicle such as a car or an airplane requires more and more maintenance cost, a railway timetable becomes more and more out of date with the passage of time. The ever increasing repair and maintenance cost necessitates the replacement of the equipment. However, there is no sharp, clearly defined time which indicates the need for this replacement. The replacement policy, in this case, consists of calculating the increased operating cost, maintenance cost, forced idle time cost together with cost of replacing the new equipment.

A separate but similar problem involves the replacement of items such as electric bulbs, radio tubes, television parts, etc. of the equipment which does not deteriorate with time but suddenly fails. The problem, in this case, is finding which items to replace and whether or not to replace them in a group and, if so, when. There is still another situation in which replacement becomes necessary. This is due to new discoveries and better design of the equipment. Thus in these situations, it is needed to formulate a replacement policy to determine the time or age at which the replacement of the given equipment is most economical.

Replacement of Items which Maintenance Costs Increase with Time

Quite often the repair and maintenance costs of items increase with time and a stage may come when these costs become so high that it is more economical to replace the item by a new one. Since both of these costs tend to increase with time, they are grouped while analyzing a problem.

If these costs decrease or remain constant with time, the best policy is never to replace the item. However, this condition is hardly met with in practice. If these costs fluctuate with time, the item should be replaced.

Generally, all costs that depend upon the choice or age of the equipment must be taken into account while analyzing the decision of the replacement. However, in special situations, certain costs may not be considered. For example, costs such as labour cost, electric cost, etc. that do not change with the age of the equipment and they may not be included in calculations.

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Optimal policy of items that deteriorate with time

Let us first consider a simple situation which consists of minimizing the average annual cost of equipment whose maintenance, repair costs increase with time and whose scrap value is constant. We ignore changes in the value of money during the period.

Let C = Capital cost of the item,

S = Scrap value of the item,

T_{ave} = Average annual total cost of the item,

n = Number of years the item is to be in use,

f(t) = Operating and maintenance cost of the item at time t.

In this situation, the total cost incurred during n years is

$$TC = C - S + \sum_{t=0}^{n} f(t). \tag{1}$$

Average annual cost incurred on the item is

$$ATC_n = \frac{1}{n}\left(C - S + \sum_{t=0}^{n} f(t)\right). \tag{2}$$

We want to find the value of n for which ATC_n is minimum.

Thus we have the inequalities:

$ATC_{n-1} > ATC_n < ATC_{n+1}$, which gives $ATC_{n-1} - ATC_n > 0$ and $ATC_{n+1} - ATC_n > 0$.

Rewriting Equation (2) for period n+1, we get

$$\begin{aligned} ATC_{n+1} &= \frac{1}{n+1}\left(C - S + \sum_{t=1}^{n+1} f(t)\right) \\ &= \frac{1}{n+1}\left(C - S + \sum_{t=1}^{n} f(t) + f(n+1)\right) \\ &= \frac{n}{n+1}\left(\frac{1}{n}\left\{C - S + \sum_{t=1}^{n} f(t)\right\}\right) + \frac{f(n+1)}{n+1} = \frac{n}{n+1}\cdot ATC_n + \frac{f(n+1)}{n+1}. \end{aligned}$$

$$\begin{aligned} ATC_{n+1} - ATC_n &= \frac{n}{n+1} ATC_n + \frac{f(n+1)}{n+1} - ATC_n \\ &= \frac{f(n+1)}{n+1} + ATC_n\left(\frac{n}{n+1} - 1\right) = \frac{f(n+1)}{n+1} - \frac{ATC_n}{n+1}. \end{aligned}$$

Since $ATC_{n+1} - ATC_n > 0$, we get

$$\frac{f(n+1)}{n+1} - \frac{ATC_n}{n+1} > 0 \text{ (or) } f(n+1) - ATC_n$$
$$> 0 \text{ (or) } f(n+1) > ATC_n.$$

Similarly, $ATC_{n-1} - ATC_n > 0$ yields $f(n) < ATC_{n-1}$.

These results provide the following replacement policy:

(i) If the running cost (operating and maintenance cost) for the next year, f(n+1) is more than the average annual cost of n^{th} year, ATC_n, then replace at the end of n years. That is,

$$f(n+1) > \frac{1}{n}\left(C - S + \sum_{t=0}^{n} f(t)\right).$$

(ii) If the running cost of the present year is less than the previous year's average annual cost, ATC_{n-1}, then do not replace.

That is, $f(n) < \frac{1}{n-1}\left(C - S + \sum_{t=0}^{n-1} f(t)\right)$. The above policy implies that n is optimal at the minimum average annual cost.

Example

Purchase price of a machine is 7,000 Kyats. Its running cost and resale price are given below;

Year	: 1	2	3	4	5	6	7	8
Running cost (Kyats)	: 1,100	1,300	1,500	1,900	2,400	2,900	3,500	4,100
Resale price (Kyats)	: 3,100	1,600	850	475	300	300	300	300

We want to determine that after how many years it will be economical to replace the machine by a new one.

Assuming that the loss of flexibility due to fewer vehicles is of no importance and that he will continue to have sufficient work for three of the old vehicles, we examine the replacement policy as follows:

The average annual cost for old machine is computed in Table 1.

Table 1:

(1) Years of service (n)	(2) Resale value (s) Kyats	(3) Purchase price – resale value (C-S) Kyats	(4) Annual maintenance cost f(t) Kyats	(5) Summation of maintenance cost $\sum_{t=0}^{n} f(t)$ Kyats	(6) Total cost (3)+(5) Kyats	(7) Average annual cost (6)/(1) Kyats
1	3,100	3,900	1,100	1,100	5,000	5,000
2	1,600	5,400	1,300	2,400	7,800	3,900
3	850	6,150	1,500	3,900	10,050	3,350
4	475	6,525	1,900	5,800	12,325	3,081
5	300	6,700	2,400	8,200	14,900	2,980
6	300	6,700	2,900	11,100	17,800	**2,967**
7	300	6,700	3,500	14,600	21,300	3,043
8	300	6,700	4,100	18,700	25,400	3,175

Observing from Table 1, it is found that the average annual cost is minimum 2,967 Kyats in the sixth year.

Thus, the old machine should be replaced at the end of sixth years by a new one.

Replacement of Items whose Maintenance Costs and Value of Money also Changes with Time

Discount rate and present value of the money

As the money value changes with time, we must calculate the present value or the present worth of the money to be spent a few years hence. If it is the interest rate (i may also be considered as the rate of inflation or the sum of the rates of interest and inflation) per year, a kyat invested at present will be equivalent to $(1 + i)$ a year hence, $(1 + i)^2$ two years hence, and $(1+i)^n$ in n years time. In other words, making a payment of one kyat after n years is equivalent to paying $(1+i)^{-n}$ now. The quantity $(1+i)^{-n}$ is called the **present worth** or **present value** of one kyat spent n years from now.

Present value of a kyat $=(1+i)^{-n}$, spent n years hence $= v^n$,

where $v=(1+i)^{-1} = \dfrac{1}{1+i}$ is called **discount rate** and is always less than unity.

Determination of optimal policy of replacement model

In order to find the optimal policy of replacement, we assume that the machine is replaced after n years. Let C be the purchase price of the machine and $R_1, R_2,\ldots, R_n$ be the running cost at $1^{st}, 2^{nd}, \ldots, n^{th}$ year respectively. Assuming that scrap value of the machine is Zero and that all payments (cash outflows) are made at the beginning of each year, the present worth of expenditure in n years is $P_n = C + R_1 + vR_2 + v^2R_3 + \ldots + v^{n-1}R_n$. Thus, P_n is the amount of money required now to pay all future costs of acquiring and operating the machine. Now, P_n increase as n increases which means that the present worth, if the machine is replaced after n+1 years is greater than if it is replaced after n years. Thus for any additional amount spent, we get an extra year's service. We are, therefore, interested in finding some function of the replacement interval which allows for this. In order to do so, let us assume that the manufacture invests the amount P_n by borrowing money at the interest rate i and repays it off in fixed annual payments, each of value x, throughout the life of the machine. Thus after n years, he will have paid off the total cost P_n of the machine.

The present worth of fixed annual payments, each of value x, for n years is

$$x+ vx+ v^2x + \ldots + v^{n-1}x = \frac{1-v^n}{1-v}x,$$

since this is equal to the sum P_n borrowed, $P_n = \dfrac{1-v^n}{1-v}x$, (or)

$$x = \frac{1-v}{1-v^n}P_n. \tag{3}$$

Thus the best period to replace the machine is the period n which minimizes

$x = \frac{1-v}{1-v^n} P_n$. However, since $(1-v)$ is a positive constant, the period at which to replace the machine is the period n which minimizes the function: $F_n = \frac{P_n}{1-v^n}$.

Since n can have only discrete values, method of finite differences can be used to calculate its optimal value. By this method, n will be optimal. That is, F_n will be minimum if

$$\Delta F_{n-1} < 0 < \Delta F_n. \tag{4}$$

Now $\Delta F_n = F_{n+1} - F_n = \frac{P_{n+1}}{1-v^{n+1}} - \frac{P_n}{1-v^n}$. (5)

$$\Delta F_n = \left[\frac{\left(1-v^n\right)P_{n+1} - \left(1-v^{n+1}\right)P_n}{\left(1-v^{n+1}\right)\left(1-v^n\right)}\right] = \frac{1}{\left(1-v^{n+1}\right)\left(1-v^n\right)} (P_{n+1} - P_n) + (v^{n+1}P_n - v^nP_{n+1}). \tag{6}$$

Further, $P_{n+1} = (C + R_1 + vR_2 + \ldots + v^{n-1}R_n) + v^nR_{n+1} = P_n + v^nR_{n+1}$.

From (6), we get

$$\Delta F_n = \frac{1}{\left(1-v^{n+1}\right)\left(1-v^n\right)}\left[\left(v^nR_{n+1}\right) + v^{n+1}P_n - v^n\left\{P_n + v^nR_{n+1}\right\}\right]$$

$$= \frac{v^n\left(1-v\right)}{\left(1-v^{n+1}\right)\left(1-v^n\right)}\left[\frac{1-v^n}{1-v}R_{n+1} - P_n\right] \tag{7}$$

$$= K\left[\frac{1-v^n}{1-v}R_{n+1} - P_n\right],$$

where $K = \frac{v^n\left(1-v\right)}{\left(1-v^{n+1}\right)\left(1-v^n\right)}$ is a positive constant . F_n has always the same sign as the quantity in brackets. From (4), n will be optimal if

$$\frac{1-v^{n-1}}{1-v}R_n - P_{n-1} < 0 < \frac{1-v^n}{1-v}R_{n+1} - P_n. \tag{8}$$

From (8), we have

$$\frac{1-v^n}{1-v}R_{n+1} - P_n > 0 \text{ (or) } R_{n+1} > P_n \Big/ \frac{1-v^n}{1-v} \text{ (or)}$$

$$R_{n+1} > \frac{C + R_1 + vR_2 + v^2R_3 + \ldots + v^{n-1}R_n}{1 + v + v^2 + \ldots + v^{n-1}} \tag{9}$$

(or) $$R_{n+1} > \frac{C + \sum_{r=1}^{n} R_r v^{r-1}}{\sum_{r=1}^{n} v^{r-1}} \tag{10}$$

(or) next periods cost > weighted average of previous cost.

The expression on the right-hand side of (9) is the weighted average of all costs up to and including period $n-1$. The weights 1, v, v^2, …, v^{n-1} are the discount factors applied to the costs in each period.

The other part of (8) can, similarly, be expressed as

$$R_n < \frac{C + R_1 + vR_2 + v^2R_3 + \ldots + v^{n-2}R_{n-1}}{1 + v + v^2 + \ldots + v^{n-2}} \tag{11}$$

(or)
$$R_n < \frac{C + \sum_{r=1}^{n} R_r v^{r-2}}{\sum_{r=1}^{n} v^{r-2}}\,. \tag{12}$$

From expressions (10) and (12), we conclude that

(a) The machine should be replaced if the next period's cost is greater than the weighted average of previous one.

(b) The machine should not be replaced if the present period's cost is less than the weighted average of previous one.

Example

A company is offered two minicomputers: MINICOMP and CHIPCOMP. MINICOMP has cost price of $2,500 its running and maintenance cost is $400 for each of the first 5 years and increases by $100 every subsequent year. CHIPCOMP, having the same capacity as MINICOMP, cost $1,250, has running and maintenance cost of $600 for 6 years, increasing by $100 per year. We want to determine which computer should be purchased if the money is worth 10% per year and the optimal replacement periods for each computer.

Table 2: MINICOMP

(1) Years of service (r)	(2) Running cost (R_r) $	(3) Discount Factor (v^{r-1}) $	(4) Discounted running cost $\left(R_r v^{r-1}\right)$ $	(5) Cumulative total discounted cost $C+\sum_{r=1}^{n} R_r v$ $	(6) Dividing factor $\sum_{r=1}^{n} v^{r-1}$ $	(7) Weighted average annual cost $\frac{(5)}{(6)}$ $
1	400	1.0000	400.00	2900.00	1.0000	2,900.00
2	400	0.9091	363.64	3263.64	1.9091	1,709.45
3	400	0.8264	330.56	3594.20	2.7355	1,313.84
4	400	0.7513	300.52	3894.72	3.4868	1,116.93
5	400	0.6830	273.20	4167.92	4.1698	999.50
6	500	0.6209	310.45	4478.37	4.7907	934.80
7	600	0.5645	338.70	4817.07	5.3552	899.40
8	700	0.5132	359.24	5176.31	5.8684	881.92
9	800	0.4665	373.20	5549.51	6.3349	**875.86 Replace**

As money is worth 10% per year, the discount rate for both computers is $v = \frac{1}{1+r} = \frac{1}{1+0.10}$ = 0.9091. The calculations for two minicomputers are entered in Table 2 and Table 3 respectively.

From Table 2, we conclude that for MINICOMP, \$800 < 875.86 < \$900, where \$800 is the running cost during 9th year and \$900 is that in 10th year. Hence MINICOMP should be replaced after 9th year.

Table 3 CHIPCOMP

(1) Years of service (r)	(2) Running cost (R_r) \$	(3) Discount Factor $\left(v^{r-1}\right)$ \$	(4) Discounted running cost $\left(R_r v^{r-1}\right)$ \$	(5) Cumulative total discounted cost $C+\sum_{r=1}^{n} R_r v$ \$	(6) Dividing factor $\sum_{r=1}^{n} v^{r-1}$ \$	(7) Weighted average annual cost $\frac{(5)}{(6)}$ \$
1	600	1.0000	600.00	1850.00	1.0000	1,850.00
2	600	0.9091	545.46	2395.46	1.9091	1,254.75
3	600	0.8264	495.84	2891.30	2.7355	1,056.95
4	600	0.7513	450.78	3342.08	3.4868	958.49
5	600	0.6830	409.80	3751.88	4.1698	899.77
6	600	0.6209	372.54	4124.42	4.7907	860.92
7	700	0.5645	395.15	4519.57	5.3552	843.96
8	800	0.5132	410.56	4930.13	5.8684	**840.11 Replace**
						844.52
9	900	0.4665	419.85	5349.98	6.3349	854.28
10	1000	0.4241	424.10	5774.08	6.7590	

Similarly, from Table 3 for CHIPCOMP we find that \$800 < 840.11 < \$900, where \$800 is the running cost in 8th year and \$900 is that in 9th year. Hence CHIPCOMP should be replaced after 8th year.

We find that the weighted average cost in 9 years of MINICOMP is \$875.86 and weighted average cost in 8 years of CHIPCOMP is \$840.11, it is advisable to purchase CHIPCOMP.

Replacement of Items that Fails Suddenly

Some types of replacement policies dealing with suddenly fail

In the second section, we considered replacement of items that deteriorate with time resulting in increasing maintenance and operation costs. However, there are many real life situations in which items do not deteriorate with time but fail suddenly. A system usually consists of a large number of low cost items that are increasingly liable to failure with age. Sometimes, the failure of an item may cause a complete breakdown of the system. The costs of failure, in such a case will be quite higher than the cost of the item itself.

It is, therefore, quite important to know, in advance, as to when the failure is likely to take place so that the item can be replaced before it actually fails. Rigorous inspection may be required to detect imminent failures and once they are detected, preventive replacement may be quite economical. Quite often, however, it may not be possible to predict the time of failure by direct inspection. In such cases, the time of failure can be predicted from the probability distribution of failure time obtained from past experience.

Two types of replacement policies are considered when dealing with such situations;

(i) Individual replacement policy in which an item is replaced immediately after it fails.

(ii) Group replacement policy in which all items are replaced, at the end of an optimal time period, irrespective of whether they have failed or not, with a provision that if any item fails before the optimal time, it may be individually replaced.

Group replacement policy

Quite often a system consists of a large number of identical, low cost items which are more and more likely to fail with time. It may be economical to replace all such items at fixed intervals. Such a policy of replacement is called group replacement policy and is particularly suitable when the cost of individual item is comparatively small. An important example is replacing the street light bulbs.

Thus under this policy, we replace all items at fixed interval 't' whether they have failed or not.

The problem is to determine the optimum group replacement time interval.

Let N = The total number of items in the system,

N_t = Number of items that fail during time t,

C(t) = The total cost of group replacement after a time t, so that average cost per unit time is $\frac{C(t)}{t}$,

C_1 = Cost of replacing an item when all the items in that group are replaced simultaneously,

C_2 = Cost of replacing an individual item on failure.

Then, clearly $C(t)=C_1N+C_2[N_1+N_2+\ldots+N_{t-1}]$.

Therefore, the average cost per unit time $=\frac{C(t)}{t}=F(t)=\frac{C_1N+C_2\left[N_1+N_2+...+N_{t-1}\right]}{t}$. (13)

Now optimum group replacement time t' will be that period which minimizes the average cost per unit time.

The condition for minimum F(t) is $\Delta F(t-1)<0<\Delta F(t)$. (14)

$$\text{Now } \Delta F(t)=F(t+1)-F(t)=\frac{C(t+1)}{t+1}-\frac{C(t)}{t}$$

$$=\frac{C(t)+C_2N_t}{t+1}-\frac{C(t)}{t}=\frac{t\left\{C(t)+C_2N_t\right\}-C(t)\left(t+1\right)}{t\left(t+1\right)}$$

$$=\frac{tC_2N_t-C(t)}{t\left(t+1\right)}=\frac{C_2N_t-\frac{C(t)}{t}}{\left(t+1\right)}, \tag{15}$$

which must be greater than zero for minimum F(t).

That is, $\frac{C_2N_t-\frac{C(t)}{t}}{\left(t+1\right)}>0 \text{ or } C_2N_t>\frac{C(t)}{t}$.

(16)

Next, $\Delta F(t-1) = \dfrac{C_2 N_{t-1} - \dfrac{C(t-1)}{t-1}}{(t-1+1)} = \dfrac{C_2 N_{t-1} - \dfrac{C(t-1)}{t-1}}{t}$,

which must be less than zero for minimum F(t).

$$\text{That is,} \quad \frac{C_2 N_{t-1} - \dfrac{C(t-1)}{t-1}}{t} < 0 \quad \text{(or)} \quad C_2 N_{t-1} < \frac{C(t-1)}{t-1}. \tag{17}$$

From (16) and (17), we get the following group replacement policy:

(a) Group replacement should be made at the end of t^{th} period if the cost of individual replacement for the t^{th} period is more than the average cost per unit time through the end of t periods.

(b) Group replacement should not be made at the end of t^{th} period if the cost of individual replacement for the t^{th} period is less than the average cost per unit time through the end of t periods.

Example

The following failure rates table have been observed for a certain type of light bulbs in an installation with 1,000 bulbs.

End of week	1	2	3	4	5	6
Probability of failure to date	0.09	0.25	0.49	0.85	0.97	1.00

The cost of replacing and individual failed bulb is 3 Kyats. If all the bulbs are replaced in the same operation, it can be done by only 0.70 Kyats for a bulb. It is proposed to replace all the bulbs at fixed intervals, whether or not they have burnt out, and to continue replacing burnt out bulbs as they fail. (a) We determine the best interval between group replacements. (b) Also we establish if the policy, as determined is superior to the policy of replacing bulbs as and when they fail, there being nothing like group replacement. (c) We can also decide at what group replacement price per bulb, would a policy strictly individual replacement become preferable to the adopted policy.

Assume that all the bulbs failing during a week might fail at any time of the week and that the group replacements are made only at the end of a week. Let P_i be the probability that a new light bulb fails during the i^{th} week of its life. Then we get the following probability distribution of the lives of the bulbs: P_1=0.09, P_2=0.25–0.09=0.16, P_3=0.49–0.25=0.24, P_4=0.85–0.49=0.36, P_5=0.97–0.85=0.12, P_6=1.00–0.97=0.03.

Since the sum of all probabilities is unity, all probabilities higher than P_6 must be zero. Thus $P_7 = P_8 = P_9 = \ldots = 0$. Thus all light bulbs are sure to burn out by the 6^{th} week. From this distribution, we observe that 9 percent of the bulbs are expected to burn out during the first week of their life. In the lot of 1,000 bulbs, therefore, 90 bulbs are expected to fail in the first week. Similarly, 16% (160 bulbs) of this lot are expected to fail during the second week, 240 bulbs in the third week, 360 in the fourth week, 120 in the fifth week and the remaining 30 in the sixth week. Of the 90 bulbs that would be replaced in the first week, 9 percent (8 bulbs) would fail in the first week of their life, that is to say, in week 2, 16 percent (14 bulbs) would fail in week 3, and so on. Similarly, of the total replacements numbering 168 (160 of the original lot plus 8 out of the 90) replacement in the first week during the second week, 9 percent will fail during the third week, 16 percent during the fourth week and so on.

These number of failures (and hence number of replacements) in different weeks are calculated in Table 4 below;

Table 4

Week (i)	Expected number of failures (N_i)
0	$N_0 = N_0 = 1{,}000$
1	$N_1 = N_0p_1 = 1{,}000 \times 0.09 = 90$
2	$N_2 = N_0p_2 + N_1p_1 = 1{,}000 \times 0.16 + 90 \times 0.09 = 168$
3	$N_3 = N_0p_3 + N_1p_2 + N_2p_1 = 1{,}000 \times 0.24 + 90 \times 0.16 + 168 \times 00.09 = 269$
4	$N_4 = N_0p_4 + N_1p_3 + N_2p_2 + N_3p$ $= 1{,}000 \times 0.36 + 90 \times 0.24 + 168 \times 0.16 + 269 \times 0.09 = 432$
5	$N_5 = N_0p_5 + N_1p_4 + N_2p_3 + N_3p_2 + N_4p_1$ $= 1{,}000 \times 0.12 + 90 \times 0.36 + 168 \times 0.24 + 269 \times 0.16 + 432 \times 0.09 = 274$
6	$N_6 = N_0p_6 + N_1p_5 + N_2p_4 + N_3p_3 + N_4p_2\ N_5p_1$ $= 1{,}000 \times 0.03 + 90 \times 0.12 + 168 \times 0.36 + 269 \times 0.24 + 432 \times 0.16 + 274 \times 0.09$ $= 260$
7	$N_7 = 0 + N_1p_6 + N_2p_5 + N_3p_4 + N_4p_3 + N_5p_2 + N_6p_1$ $= 90 \times 0.03 + 168 \times 0.12 + 269 \times 0.36 + 432 \times 0.24 + 274 \times 0.16 + 260 \times 0.09$ $= 291$

Thus we find that the number of bulbs failing each week increases till the 4th week, then decreases and again increases from 7th week. Thus, N_i will continue to oscillate till the system attains a steady state. Now we can determine the total and the average weekly cost associated with the policy of replacing bulbs every week, every two weeks, … and so on as per the group replacement policy.

(a) Determination of optimal group replacement interval is described in following Table 5.

Table 5

Up to week	Total cost of group replacement	Average cost per week
1	$1{,}000 \times 0.70 + 90 \times 3 = 970$	970.00
2	$1{,}000 \times 0.70 + 3(90+168) = 1{,}474$	**737.00**
3	$1{,}000 \times 0.70 + 3(90+168+269) = 2{,}281$	760.33

As the average minimum cost is in the 2nd week, it is optimal to have a group replacement after every two weeks.

(b) When the policy of "replacing the bulbs as and when they fail" is adopted, there are no group replacements and it becomes an individual replacement policy. Since it is assumed that a bulb can fail at any time during the week, it is necessary to first determine the average (mean or expected) life a light bulb.

The average (expected) life of light bulbs is

$$\sum_{i=1}^{6} ip_i = 1\times 0.09 + 2\times 0.16 + 3\times 0.24 + 4\times 0.36 + 5\times 0.12 + 6\times 0.03 = 3.35.$$

The average number of failures per week is $\frac{1,000}{3.35} = 299$. The cost of individual replacement of bulbs per week is $3\times 299 = 897$ Kyats.

Since the cost of group replacements per week is 737 Kyats and that of individual replacements is 897 Kyats per week, it is advisable to adopt the policy of group replacements.

(c) Let x Kyats be the group replacement price per bulb. Then,

$$897 \text{ Kyats} < \frac{1,000x + 3(90+168)}{2} \text{ (or) } x > 1.02 \text{ Kyats.}$$

Therefore, when the group replacement price per bulb exceeds Kyats 1.02, the policy of strictly individual replacements becomes more economical.

Conclusion

In this paper, we present some replacement policies of items to determine the best replacement strategies. In the first policy of replacement, items are deteriorated with time. In our daily life, some industrials and military equipments are deteriorated with time. The increasing repair and maintenance cost necessitate the replacement of the equipment. In second replacement policy, items do not deteriorate with time but fail suddenly. The failure of an item may cause a complete breakdown of the system. The cost of failure, in such a case will be quite higher the cost of the item itself. It is, therefore, quite important to know, in advance, as to when the failure is likely to take place so that the item can be replaced before if actually fails. In such case, the time of failure can be predicted from the probability distribution of failure time. The objective of replacement policies is to minimize the sum of the cost of item, the cost of replacing the item and the cost associated with failure of item.

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