Analysis of Various ΛN Effective Interactions

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Abstract

The purpose of our research is to study the various models of effective hyperon-nucleon (YN) interactions in ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$, ${}^{12}_{\Lambda}B$, ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$. The corresponding Λ-core nucleus potentials from the effective YN interactions (YNG) are dependent on the Fermi momentum (k_F). Firstly, we fit the Fermi momentum (k_F) by reproducing the experimental binding energy of ${}^{5}_{\Lambda}He$. Secondly, we construct the Λ-core nucleus interaction of the above Λ-hypernuclei by folding the effective YNG interactions with density distribution of the core nucleus. And then we determined the k_F parameters of the core nuclei (${}^{9}B$, ${}^{10}B$, ${}^{11}B$, ${}^{11}C$, ${}^{15}O$). The effective YNG interactions are also dependent on the spin state. We have calculated the binding energy of above Λ-hypernuclei by taking into account only the even-state interactions.

Key words: YNG interactions, Fermi momentum, spin state.

Introduction

In parallel with the experimental development, theoretical investigations have been made to understand the nature of hyperon-nucleon interactions. The best testing ground for these YN interaction models is the experimental Λ-binding energies of various Λ hypernuclei. It is essential to derive YN effective interactions from free space YN interactions in order to determine Λ-binding energies. Our purpose is to study various YN effective interactions which are Fermi momentum k_F dependent. We are going to determine the Fermi momentum of finite nuclei by reproducing the Λ binding energies of ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$, ${}^{12}_{\Lambda}B$, ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$, where Λ-hypernuclei is considered as two-body system composed of Λ and a core nucleus. We will

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obtain hyperon-core nucleus (Λ-core) interaction by folding YN effective interactions with density distribution of the corresponding core-nucleus.

Effective Hyperon-Nucleon Interactions (YNG)

The one-boson-exchange models for YN interactions have been proposed by the Nijmegen group (Nagels et al., 1979; Maessen et al., 1992) and Jülich group (Holzenkamp et al., 1989; Reuber et al., 1992). There are three models, namely model-ND and model-NF having the hard-core and model-NS having the soft-core. Recently the revised versions of the soft core model have been proposed Yamamoto, which are NSC97d and NSC97e models. In Jülich group, there are two versions: model-A (JA) and model-B(JB). In the present work, we are going to employ the effective YN interactions of Yamamoto and Bando (Yamamoto *et al*., 1985, 1994; Pniewski *et al*., 1985, Yamamoto, 1998).

The central parts of these effective ΛN interactions are parameterized in three-range Gaussian form as

$$V_{\Lambda N}(r) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\exp\left(-\left(\frac{r}{\beta_i}\right)^2\right)$$

which is called YNG interactions. The parameters for models YNG-ND, -NF, -NS, -NSC97d, -NSC97e, -JA and -JB are given in (Yamamoto *et al*., 1994; Yamamoto, 1998). The effective YNG interaction is spin state dependent which has four combinations: singlet even state (^1E), triplet even state (^3E), singlet odd state (^1O) and triplet odd state (^3O).

Spin Averaged Value for ΛN Interactions in ${}^{5}_{\Lambda}\mathrm{He}$

Antisymmetrizations between baryons in Λ- hypernuclei is carried out by taking the average spin value of YN interactions. These spin averaged values are obtained by using the following scheme.

$$P^{+}_{\Lambda N}({}^{1}S_0, T = \tfrac{1}{2}) = \sqrt{\tfrac{1}{2}}\left\{\Lambda^{+}_{\uparrow}p^{+}_{\downarrow} - \Lambda^{+}_{\downarrow}p^{+}_{\uparrow}\right\} + \sqrt{\tfrac{1}{2}}\left\{\Lambda^{+}_{\uparrow}n^{+}_{\downarrow} - \Lambda^{+}_{\downarrow}n^{+}_{\uparrow}\right\}$$

$$P^{+}_{\Lambda N}({}^{3}S_1, T = \tfrac{1}{2}) = \Lambda^{+}_{\uparrow}p^{+}_{\uparrow} + \sqrt{\tfrac{1}{2}}\left\{\Lambda^{+}_{\uparrow}p^{+}_{\downarrow} + \Lambda^{+}_{\downarrow}p^{+}_{\uparrow}\right\} + \Lambda^{+}_{\downarrow}p^{+}_{\downarrow}$$

$$+ \Lambda^{+}_{\uparrow}n^{+}_{\uparrow} + \sqrt{\tfrac{1}{2}}\left\{\Lambda_{\uparrow}n_{\downarrow} + \Lambda_{\downarrow}n_{\uparrow}\right\} + \Lambda^{+}_{\downarrow}n^{+}_{\downarrow}$$

Spin averaged value of ΛN pair in ${}^{5}_{\Lambda}\mathrm{He}$ is determined as follows;

core nucleus ${}^{4}\mathrm{He}$ in spin state is written as

$$\left|{}^{4}\mathrm{He}\right\rangle_{00} = p^{+}_{\uparrow}p^{+}_{\downarrow}n^{+}_{\uparrow}n^{+}_{\downarrow}\left|0\right\rangle$$

Where, $\left|0\right\rangle$ is a vacuum state.

The ground state of ${}^{5}_{\wedge}\mathrm{He}$ has total spin $J^{\pi} = \frac{1}{2}^{+}$ $(J_z = \frac{1}{2})$. Let us denote that state by

$$\left|I\right\rangle = \left|{}^{5}_{\Lambda}\mathrm{He}\right\rangle_{\frac{1}{2}\frac{1}{2}} \qquad (T = 0, T_z = 0)$$

$$\left|I\right\rangle = \Lambda^{+}_{\uparrow}\left|{}^{4}\mathrm{He}\right\rangle_{00}$$

$$\left|I\right\rangle = \Lambda^{+}_{\uparrow}\, p^{+}_{\uparrow}\, p^{+}_{\downarrow}\, n^{+}_{\uparrow}n^{+}_{\downarrow}\left|0\right\rangle$$

$$P_{\Lambda N}({}^{1}S_0)\left|I\right\rangle = -\sqrt{\tfrac{1}{2}}p^{+}_{\uparrow}n^{+}_{\uparrow}n^{+}_{\downarrow}\left|0\right\rangle - \sqrt{\tfrac{1}{2}}p^{+}_{\uparrow}p^{+}_{\downarrow}n^{+}_{\uparrow}\left|0\right\rangle$$

$$P_{\Lambda N}({}^{3}S_1)\left|I\right\rangle = p^{+}_{\downarrow}n^{+}_{\uparrow}n^{+}_{\downarrow}\left|0\right\rangle - \sqrt{\tfrac{1}{2}}p^{+}_{\uparrow}n^{+}_{\uparrow}n^{+}_{\downarrow}\left|0\right\rangle + p^{+}_{\uparrow}p^{+}_{\downarrow}n^{+}_{\downarrow}\left|0\right\rangle - \sqrt{\tfrac{1}{2}}p^{+}_{\uparrow}p^{+}_{\downarrow}n^{+}_{\uparrow}\left|0\right\rangle$$

Then, the averaged spin weight of ΛN interactions is

$$V_{\Lambda N-\Lambda N} = V_{\Lambda N-\Lambda N}({}^{1}S_{0}, T=\tfrac{1}{2}) + 3\, V_{\Lambda N-\Lambda N}({}^{3}S_{1}, T=\tfrac{1}{2})$$

Folding Potential of Hyperon-Nucleus Interactions

Hyperon-nucelus interaction is obtained by folding the effective YN interactions with density distribution of the core nucleus as

$$V_{\Lambda-\text{core}}(\vec{r}) = \int V_{\Lambda N}(\vec{r}-\vec{R})\rho(\vec{R})d\vec{R} \qquad (1)$$

The density distribution $\rho(\vec{R})$ for alpha is

$$\rho(\vec{R}) = \left(\frac{4\sigma}{3\pi}\right)^{\frac{3}{2}} \exp\left(-\left(\frac{4}{3}\sigma R^{2}\right)\right) \qquad (2)$$

where $\sigma = 0.521\,\text{fm}^{-2}$.

We use the effective YNG interaction of Nijmegen models and Jülich models which are in Gaussian form,

$$V_{\Lambda N}(r) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\exp\left(-\left(\frac{r}{\beta_i}\right)^2\right) \qquad (3)$$

where a_i, b_i, c_i, β_i = parameters from G – matrix, k_F = Fermi momentum

the potential between Λ-α is

$$V_{\Lambda=\alpha}(\vec{r}) = \left(\frac{4\sigma}{3\pi}\right)^{\frac{3}{2}} \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F{}^2)\left[\frac{\pi}{\frac{1}{\beta_i{}^2}+\frac{4\sigma}{3}}\right]^{\frac{3}{2}} e^{-\left[\frac{1}{\beta_i{}^2}-\frac{1}{\beta_i{}^4\left(\frac{1}{\beta_i{}^2}+\frac{4\sigma}{3}\right)}\right]r^2} \qquad (4)$$

The density distribution $\rho(\vec{R})$ for other core nuclei is

$$\rho(\vec{R}) = \rho_0\left[1+\alpha\left(\frac{R}{d}\right)^2\right]\exp\left(-\left(\frac{R}{d}\right)^2\right) \qquad (5)$$

The parameters of $\rho(\vec{R})$ for corresponding core nuclei are given in table(1).

Table (1) The parameters of $\rho(\vec{R})$ for core nuclei

Core nuclei	α (fm)	d (fm)
${}^{10}_{\Lambda}B$	0.837	1.71
${}^{11}_{\Lambda}B$	0.837	1.71
${}^{12}_{\Lambda}B$	0.811	1.69
${}^{12}_{\Lambda}C$	1.067	1.687
${}^{16}_{\Lambda}O$	1.517	1.805

The respective interaction between Λ and core nuclei is

$$V_{\Lambda-core}(\vec{r}) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\rho_0 \left[\frac{\pi}{\frac{1}{\beta_i^2}+\frac{1}{d^2}}\right]^{\frac{3}{2}} e^{-\left[\frac{1}{\beta_i^2}-\frac{1}{\beta_i^4(\frac{1}{\beta_i^2}+\frac{1}{d^2})}\right]r^2} \left[1+\frac{\alpha}{d^2}\left[\frac{3}{2(\frac{1}{\beta_i^2}+\frac{1}{d^2})}+\frac{r^2}{\beta_i^4(\frac{1}{\beta_i^2}+\frac{1}{d^2})^2}\right]\right] \qquad (6)$$

Results and Discussions

Λ-Binding Energy Dependence on Fermi Momentum

We investigate theΛ-binding energies of hypernuclei, namely ${}^{5}_{\Lambda}He$, ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$, ${}^{12}_{\Lambda}B$, ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$ by using Λ-nucleus folding potential based on effective YN interaction within the frame work of two-body calculation. In order to investigate the Fermi momentum dependence on Λ hypernuclear binding energy, we determine the ${}^{5}_{\Lambda}He$ binding energies by varying the Fermi momentum and the calculated binding energies for the already mentioned potential models are plotted as a function of k_F in Fig. 1. From these calculations it is found that approximately 1.0 fm^{-1} for ${}^{5}_{\Lambda}He$ hypernuclei. When we compare seven interactions at a particular k_F for ${}^{5}_{\Lambda}He$, one finds that the Jülich interaction JA gives rise to very weak Λ-binding in comparison with JB and other YNG interactions.

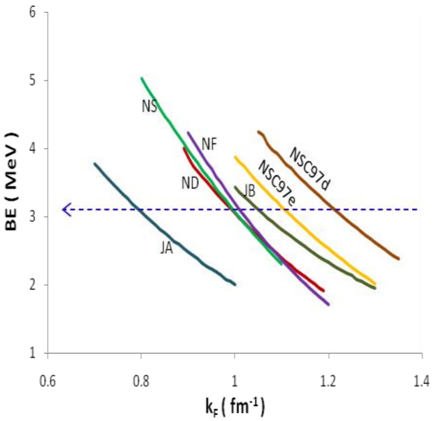

Fig (1) Binding energy vs Fermi momentum for ${}^{5}_{\Lambda}He$ with seven potential models.

${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$, ${}^{12}_{\Lambda}B$ Hypernuclei

We determined k_F values of ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}B$ which reproduces the experimental binding energy. The calculated binding energies of ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}B$ are plotted as a function of k_F as shown in Fig 2. When we compare the seven interactions at a particular k_F for ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}B$, one finds that the Jülich interaction JA gives rise to very weak Λ-binding in comparison with JB and that is still weaker than the other YNG interactions derived from the Nijmegen models.

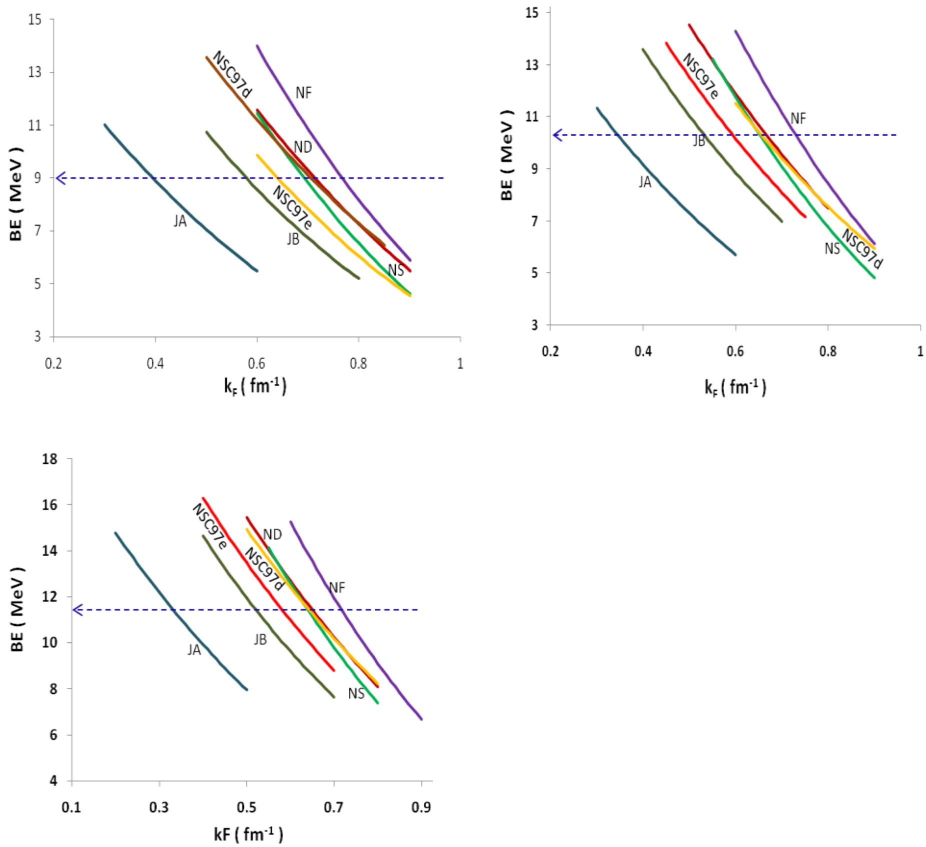

Fig (2) Binding energy vs Fermi momentum for ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}B$ with seven potential models.

${}^{12}_{\Lambda}C$ Hypernuclei

We investigate the Λ-binding energy dependence upon Fermi momentum in ${}^{12}_{\Lambda}C$ hypernuclei. The binding energy versus k_F curve for

${}^{12}_{\Lambda}C$ is shown in Fig 3(a). For ${}^{12}_{\Lambda}C$, the Fermi momentum is found that approximately 0.9 fm^{-1}. When we compare among these interactions at a particular k_F for ${}^{12}_{\Lambda}C$, one finds that the Jülich interaction JA gives rise to very weak Λ-binding in comparison with JB and that is still weaker than the other YNG interactions which feature is the same as that of ${}^{5}_{\Lambda}He$, ${}^{10,11,12}_{\Lambda}B$.

${}^{16}_{\Lambda}O$ Hypernuclei

In order to investigate the binding energy of ${}^{16}_{\Lambda}O$ we calculated the binding energies by varying the Fermi momentum for the seven potential models. The result is depicted in Fig 3(b). When we compare these interactions at a particular k_F for ${}^{16}_{\Lambda}O$, one finds that the Jülich interaction JA gives rise to very weak Λ-binding in comparison with JB and that JB is still weaker than the other YNG interaction. Since statistical weight of triplet even-state is three times that of singlet even state, we seen that NSC97d, NSC97e and NS are much less repulsive than the ND, NF, JA and JB. In the long range attractive region, ND and NF are more attractive than the other models as shown in Fig 4.

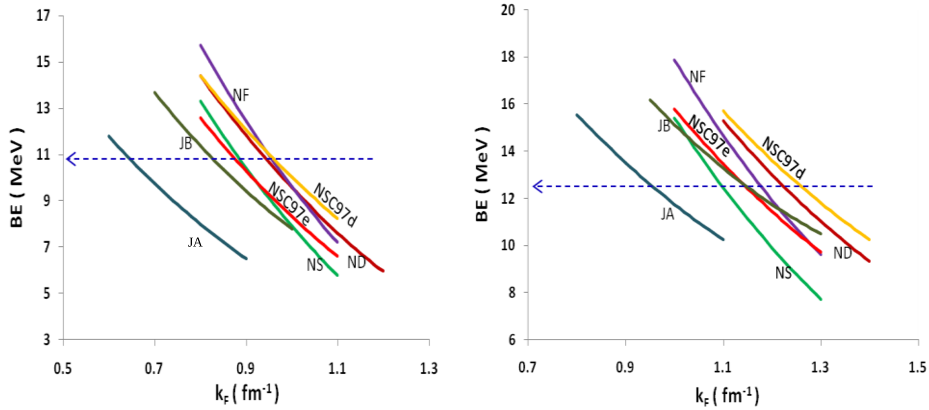

Fig 3(a,b) Binding energy vs Fermi momentum for ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$ with seven potential models.

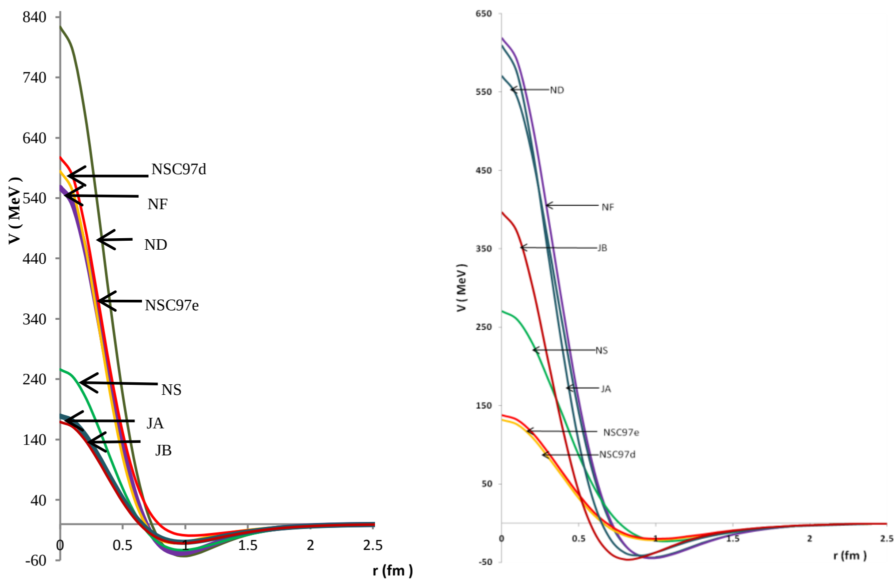

Fig (4) Effective YNG interactions for singletand triplet even-states.

Conclusion

In this paper, we have calculated the Λ-binding energies of ${}^{5}_{\Lambda}He$, ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$, ${}^{12}_{\Lambda}B$, ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$ with a folding potential. And then we determined the k_F

parameters of the core nuclei in order to reproduce the experimental binding energies of the above Λ-hypernuclei. The k_F values of ${}^{5}_{\Lambda}He$, ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$ are found to be approximately 1.0 fm^{-1}. From nuclear matter calculation, the value of k_F is approximately 1.35 fm^{-1}. We conclude that our calculated k_F values of ${}^{5}_{\Lambda}He$, ${}^{12}_{\Lambda}C$ and ${}^{16}_{\Lambda}O$ are reasonable. But the k_F values of ${}^{10}_{\Lambda}B$, ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}B$ hypernuclei are much smaller than the value of k_F for nuclear matter. We find that the Jülich interaction JA gives rise to very weak Λ-binding in comparison with other YNG interactions.

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