Mathematical Problem Solving

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Abstract

We need to understand difference between teaching of mathematics and mathematical problem solving. Mathematical problems solving is not a method of teaching mathematics. Mathematical problem solving is to understand the problems and to find appropriate method to solve that problem. As the problem solving involves higher order thinking skills, the students will benefit from developing skills, understandings, dispositions and values that young people are likely to need to effectively negotiate 21st century institutional environments. And have link between text book and real world.

Key Words: Problem solving, Critical thinking, Developing skill

Introduction

An Exercise is something that you are already supposed to know how to do. A Problem is something which when you start, you don't know what is going to happen! Exercises for someone are sometime problem for others depending on your background. For example 9 x 7 is an exercise to me but for a child that is learning multiplication it is a problem! Mathematics is a core subject in the curriculum, occupying more periods of teaching lessons than other subjects in the timetable.

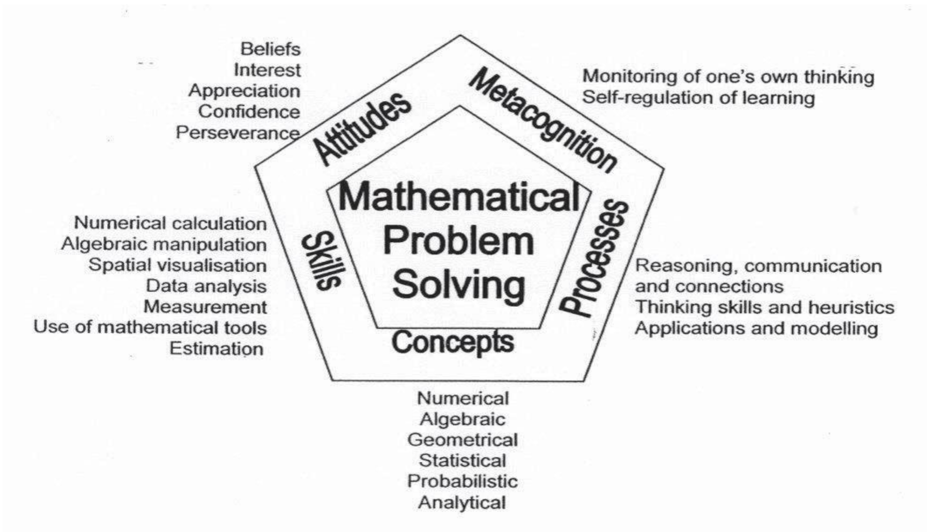
The main question is, Why is mathematics so important? What we cannot do in teaching Mathematics are, (i) Interconnected

(ii) Learning experiences in mathematics

(iii) Application of mathematics in the real world problem.

In this paper, I show interconnected with simple mathematics problem with Polya's Model.

Problem Solving Model (Polya's Model of Problem Solving (Polya, 1945))



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In Polya's model there are four simple step to solv the problem. They are

(i) Understand the Problem
(ii) Devices a Plan
(iii) Carry out the Plan
(iv) Check and Expand

Solving the simple problem

Problem 1(Russian Roulette Problem)

Two bullets are placed in two consecutive chambers of a 6-chamber revolver. The cylinder is then spun. Two persons play a safe version of Russian Roulette. The first person points the gun at his hand phone and pulls the trigger. The shot is blank. Suppose that you are the second person and it is now your turn to point the gun at your hand phone and pull the trigger. Should you pull the trigger or spin the cylinder another time before pulling the trigger?

You need to understand how revolver is work. And Draw a diagram, in the figure below assume that the red color chamber is load with bullet and white are blank.

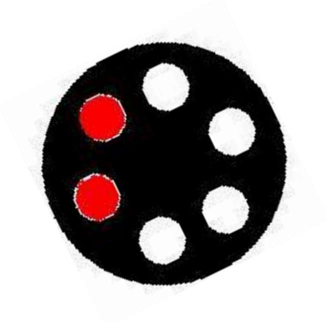

Figure 1. Revolver Chamber

Let the chambers be number from 1 to 6 and let the chamber spin such that 1 is followed by 2 followed by 3 and so on until 6 is followed by 1. We may assume that the bullets are placed in chambers 1 and 2. If the first person gets a blank, then he would have triggered either chambers 3, 4, 5 or 6. If you are the second person, do not spin, then you would have the following chambers as possibilities: 4, 5, 6 or 1. There are thus only 4 outcomes in the sample space, and only one (chamber 1) is bad. Thus, the probability of destroying your phone is ¼. Since the probability of destroying your phone if you spin is clearly $\frac{2}{6}=\frac{1}{3}$, it is better not to spin the cylinder.

(i) Understand the problem

Write down the heuristics you used to understand the problem.
Act it out and Draw a diagrams - of the 6 chambers in the revolver and two consecutive loaded chambers.
Restate the problem in another way – Which is less likely to destroy your phone?

(ii) Devise a plan

(a) Write down the key concepts that might be involved in solving the problem.
Probability, sample space.

(b) Write each plan concisely and clearly.

Plan(1)Aim for sub-goals – calculate the probability of destroying the phone if the cylinder is spun again, and the probability if the cylinder is not spun.

Plan(2)Aim for subgoals – to find the probability if the cylinder is not spun, first find the sample space, i.e. the list of chambers that can be hit by the hammer of the gun.

(iii) Carry out the plan

In this paper details and step by step calculation are left out.

(iv) Check and Expand

(a) Write down how you checked your solution.
By explaining the solution to a group member.
By setting up an experiment to simulate the problem and trying out a suitable number of iterations. A simple paper disc can be made and spun.

(b) Write down a sketch of any alternative solution(s) that you can think of.
Generalize the problem to m consecutive bullets in n chambers to obtain a general solution. Then see if the solution holds true for m = 2 and n = 6.

(c) Give at least one adaptation, extension or generalization of the problem.

Adaptation1:
Two bullets are placed in two consecutive chambers of a 6-chamber revolver. Two persons play a safe version of Russian Roulette. The first person points the gun at his hand phone and pulls the trigger. The gun fires and the first hand phone is destroyed. Suppose you are the second person and it is now your turn to point the gun at your hand phone and pull the trigger. Should you pull the trigger or spin the cylinder another time before pulling the trigger?

Adaptation2:
Two bullets are placed in two chambers of a 6-chamber revolver such that there is exactly one empty chamber between them. The cylinder is then spun. Two persons play a safe version of Russian Roulette. The first person points the gun at his hand phone and pulls the trigger. The shot is blank. Suppose you are the second person and it is now your turn to point the gun at your hand phone and pull the trigger. Should you pull the trigger or spin the cylinder another time before pulling the trigger.

Generalizations:

m bullets are placed in consecutive chambers of an n-chamber. The cylinder is then spun. Two persons play a safe version of Russian Roulette. The first person points the gun at his hand phone and pulls the trigger. The shot is blank. Suppose you are the second person and it is now your turn to point the gun at your hand phone and pull the trigger. Should you pull the trigger or spin the cylinder another time before pulling the trigger?

Problem 2(Number of Square)

How many subsquare can you find in 7 x 7 Array?

Consider the r×r, where r = 1, 2, …, 7, squares in the 7×7 square. Starting with the bottom-leftmost r×r square, we can move one step to the right to get another r×r square. Continuing one step at a time to the right, we get exactly (8 – r) such squares with their 'base' on the first horizontal line. Next, we move up one step and consider the leftmost r×r square with the base on the second horizontal line. As before, we move one step at a time to the right and obtain exactly (8 – r) such squares with their base on the second horizontal line. We now observe that we can continue to move up one step at a time to get (8 – r) squares with the base on the same horizontal line. In all, we can use only the first (8 – r) horizontal lines as base lines.

Thus, the number of r×r squares = $(8 – r) \times (8 – r) = (8 – r)^2$.

Thus, the number of squares in the 7×7 square = $7^2 + 6^2 + \dots + 1^2 = 140$.

Simple Figure of 4 x 4 array and can observe that 4 x 4 arrays contain 4 x 4 square, 3 x 3 square, 2 x 2 square and 1 x 1 square.

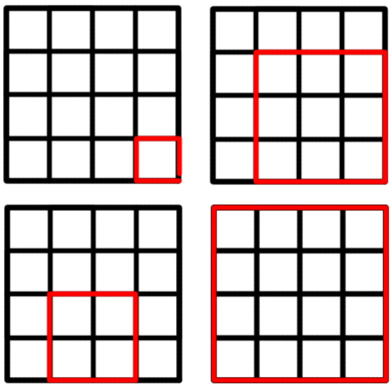

Figure 2. 4 x 4 Square and its Subsquare

(i) Understand the problem

Write down the heuristics you used to understand the problem. Draw a diagram - see that the squares to be counted are of various sizes.

(ii) Devise a plan

(a) Write down the key concepts that might be involved in solving the problem. Counting principles.

(b) Write out each plan concisely and clearly.Plan 1Consider a simpler problem (smaller numbers), Look for patterns - 1×1 square, 2×2 square, and 3×3 square.Plan 2Divide into cases - Classify the squares in the 7×7 square and count for each case; take the sum.

(iii) Carry Out the Plan

In this paper details and step by step calculation are left out.

(iv) Check and Expand

(a) Write down how you checked your solution.
Verify the pattern of the answer for the 4×4 square by counting out all the squares 'by hand'.

(b) Write down a sketch of any alternative solution(s) that you can think of.
By counting the number of squares that can be formed by using each of the points as the top left vertex of the square. However, this needs some messy notation.

(c) Give at least one adaptation, extension or generalisation of the problem.

Adaptation:
How many squares are there in a 7×7 square which does not contain the top-leftmost square?

Generalisation:
How many squares are there in an n×n square?
How many squares are there in an m×n rectangle?

Extension:
How many cubes are there in a 7×7×7 cube? (Perhaps this problem can be solved by similarly classifying and listing.)
How many rectangles are there in an 7×7 square? (A square is a special rectangle. There will be more rectangles in a 7×7 square than number of squares. However, perhaps the counting in this problem may be more complicated than the counting of squares.)
How many rectangles are there in an m×n rectangle?

As it turns out it's just a simply sum of squares problem. So for any size square you can just use the equation for the sum is $n^2 + (n-1)^2 + (n-2)^2 + \ldots + 1^2$, can be express in the form

$$n^2 + (n-1)^2 + (n-2)^2 + \ldots + 1^2 = \Sigma n^2$$

Let sum (S_n) of squares, for n squares.

$$S_n = \frac{1}{6} n(n+1)(2n+1)$$

By mathematical indction we can easiy show that

$$n^2 + (n—1)^2 + (n—2)^2 + . . . + 1^2 = \Sigma n^2 = \frac{1}{6} n(n+1)(2n+1)$$

Now we can get simple mathmatics formula for number of square in n x n squares is

$$\frac{1}{6} n (n+1) (2n+1)$$

Conclusion

As we see in above two problems, Problem 1 is Probability problem but it can be solve without piar knowledge of Probability. As we see in Problem 2, its Sequene and Series, also involve mathematical induction. We can find the answer of Problem 2 by using method of problem solving. With the knowledge of Mathematics, we can extend to find formula for (n x n) square, (n x m) rectangular and we can explain this problem will tell us about Series and Sequences. In real world, sum of series is some time diffcult and need find simple formula. To find the formula we need to use mathematical induction. By mathematical problem solving student can understand how to solve the ploblems, some part of Series and Sequences, mathematical induction problem and teaching method. Students can get critical thinking and also know how to solve problems and find way to get simple mathematic formula.

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