New Contribution to the Nucleus-Nucleus Potentials for Heavy Ion Fusion Reactions

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Abstract

A modified Ngô (MNgô) potential is proposed for describing nucleus-nucleus interaction based on the Skyrme energy-density functional approach. Fusion barriers for a large number of fusion reactions from light to heavy systems can be described well with this potential. Our modified Ngô (MNgô) potential reproduces the experimental data nicely compared to its older version and other proximity versions.

Key words: Fusion barrier, nuclear potential, fusion cross section.

1. Introduction

The nucleus-nucleus potential is a key ingredient in the description of nuclear reactions. There are also simple parameterizations of the nucleus-nucleus potential, which are often used to describe various heavy-ion reactions.

In describing heavy-ion fusion, the potential around the fusion barrier radius R_B is most important and the presence of a pocket in the nuclear potential allows a simple conceptual criterion for fusion. Once the significant density overlapping occurs, a substantial loss of kinetic energy and angular momentum occurs from the relative motion to nuclear intrinsic degrees of freedom and all the flux passing the barrier lead to fusion. Generally, the primary ingredient in any nuclear reaction calculation is the nucleus-nucleus potential, consisting of the repulsive Coulomb interaction and an attractive nuclear part. Although the Coulomb term $V_C(r)$ is well-known, there are large ambiguities in the nuclear potential $V_N(r)$, and many attempts have been made to extract information on this quantity from experimental data for heavy-ion reactions.

Research on nucleus-nucleus interaction potential in heavy-ion fusion reactions has attracted a great deal of attention. Some of macroscopic potentials such as Wood-Saxon potential and many other proximity potentials without explicitly considering microscopic properties of nuclei like shell effects and the situation of nuclear interaction are widely used. Simultaneously, some potentials such as the entrance channel potential, and M3Y potential based on microscopic nucleon-nucleon interactions and/or realistic density distributions of nuclei were proposed for describing the nucleus-nucleus interaction microscopically. These models provided accurate information for heavy ion reactions at energies near barrier. But these microscopic potentials are slightly more complicated computationally than those macroscopic potentials, generally.

In this paper, we have performed a modified Ngô (MNgô) potential based on the Skyrme energy-density functional approach. Although a large number of experimental data are available for different reactions, we restrict ourselves to the 21 reactions only. We shall compare as many as four phenomenological potentials. This will include Ngô, a modified Ngô, Wood-Saxons and the modified Wood-Saxons potentials. We shall calculate the fusion cross sections by numerically solving the Schrödinger equation within these above potentials. The fusion barrier heights and positions are also evaluated. Section 2, deals with formalism in detail, Sec. 3 contains the results, and we conclude our results in Sec. 4.

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2. Formalism

Modified Ngo Potential

In earlier attempts, based on the microscopic picture of a nucleus and on the idea of energy density formalism, the potential from Ngô and collaborators enjoy special status. In this model, calculations of the ion-ion potential are performed within the framework of energy density formalism due to Bruckener et al., using a sudden approximation. The need of Hartree-Fock densities as input in this model limited its scope. This not only made calculations tedious, but it also hindered its application to heavier nuclei. The above-stated parameterization was improved Ngo and Ch. Ngô, by using a Fermi-density distribution for nuclear densities as

$$\rho_{n,p}(r)=\frac{\rho_{n,p}(0)}{1+\exp\left[\left(r-C_{n,p}\right)/0.55\right]} \tag{2.1}$$

where C represents the central radius of the distribution and is defined as (with b=1 fm),

$$C_i = R_i\left[1-\left(\frac{b}{R_i}\right)^2+---\right] \tag{2.2}$$

Here $\rho_{n,p}(0)$ is given by

$$\rho_n(0)=\frac{3}{4\pi}\frac{N}{A}\frac{1}{r_{0_n}^3};\ \rho_p(0)=\frac{3}{4\pi}\frac{Z}{A}\frac{1}{r_{0_p}^3} \tag{2.3}$$

Ngô parameterized the nucleus- nucleus interaction potential in the spirit of proximity concept. The interaction potential can be divided into the geometrical factor and a universal function. The nuclear part of the parameterized potential is written as;

$$V_N(r)=\overline{R}\,\phi(r-C_P-C_T)\quad MeV \tag{2.4}$$

where $\overline{R}=\frac{C_P C_T}{C_P+C_T}$. Now the nuclear radius R_i reads as:

$$R_i=\frac{NR_{n_i}+ZR_{p_i}}{A_i},\ (i=P,\ T) \tag{2.5}$$

The equivalent sharp radius for protons and neutrons are given as;

$$R_{p_i}=r_{0_{p_i}}A_i^{1/3};\ R_{n_i}=r_{0_{n_i}}A_i^{1/3} \tag{2.6}$$

with $r_{0_{p_i}}=1.128$ fm; $\quad r_{0_{n_i}}=1.1375+1.875\times10^{-4}A_i$ fm.

The above radius formulas for the neutrons and protons take isotopic dependence into account. The universal function $\phi(s=r-C_P-C_T)(in\ MeV/\ fm)$ is noted by

$$\begin{aligned}\phi(s)&=-33+5.4(s-s_0)^2 \qquad for\ s\langle s_0\\ &=-33\exp\left[-\frac{1}{5}(s-s_0)^2\right] \quad for\ s\geq s_0\end{aligned} \tag{2.7}$$

and s_0 = -1.6 fm. We labeled this potential as Ngo.

In this contribution, we shall use that the nuclear part of the potential between two colliding ions can be quite accurately described by the approximate analytical form

$$\begin{aligned} V_N(r) &= -V_0 + \alpha(r - r_0)^2 \qquad for\ r \langle r_0 \\ &= -V_0 \exp\left[-\alpha(r - r_0)^2\right] \quad for\ r \geq r_0 \end{aligned} \tag{2.8}$$

In the framework of mean field theory, where, as before, *r* is the distance between the centers of mass of target and projectile and r_0 the distance for which the nuclear has its minimum. The distance r_0 and the depth of the potential V_0 are a direct result of our semi-classical calculation. These two quantities are parameterized in the following way as functions of the masses A_P, A_T and reduced isospins $I_i = (A_i - 2Z_i)/A_i$ of target and projectile nuclei

$$r_0(A_P, A_T) = r_0\left(A_P^{1/3} + A_T^{1/3}\right) + b \tag{2.9}$$

And $b \approx 1$,

$$V_0(A_P, A_T, I_P, I_T) = \upsilon_0\left[1 + \kappa(I_P + I_T)\right]\frac{A_P^{1/3} A_T^{1/3}}{A_P^{1/3} + A_T^{1/3}} . \tag{2.10}$$

The values of the five parameters (ν_0, κ, r_0, b and α) of our modified potentials are listed in table 2.1. In order to obtain the potential that the system of two colliding nuclei experiences it have to add the Coulomb potential to the nuclear part. For distances large compared to the sum of the radii of the equivalent sharp surfaces, this Coulomb potential is very well approximated by the Coulomb potential between two point charges, but as soon as the densities of the two nuclei interpenetrate this approximation will no longer give good results and it turns out that the difference

$$\Delta V_{Cout}(r) = V_{Cout}(r) - \frac{Z_P Z_T e^2}{r} \tag{2.11}$$

Will be, in the case of the superheavy elements considered here, of the order of 3 MeV already in the vicinity of the touching configuration (s = 0). We labeled this potential as MNgo in this research.

Modified Woods-Saxon Potential

Now let us discuss how to obtain the parameters of the MWS potential from the entrance-channel potential. The nucleus-nucleus potential reads

$$V(r) = V_N(r) + V_C(r) \tag{2.12}$$

where V_N and V_C are the nuclear interaction and the Coulomb interaction, respectively. We take $V_C(r) = e^2 Z_P Z_T / r$ and the nuclear interaction V_N to be Woods-Saxon form with five parameters determined by fitting the obtained entrance-channel potentials:

$$V_N(r) = \frac{V_0}{1 + \exp\left[(r - r_0)/a\right]} \tag{2.13}$$

Here *r* denotes the center-of-mass distance between the projectile nucleus of mass number A_P and the target nucleus of mass number A_T. There are many parameterizations in the

literature for V_0, r_0 and a. Commonly used are those due to the Woods-Saxon parameterization of the Akyiiz-Winther potential. We labeled this potential as WS in this research. In the framework of mean field theory,

$$V_0 = u_0\left[1+\kappa\left(I_P + I_T\right)\right]\frac{A_P^{1/3}A_T^{1/3}}{A_P^{1/3}+A_T^{1/3}} \qquad (2.14)$$

$$r_0 = r_0\left(A_P^{1/3}+A_T^{1/3}\right)+c \qquad (2.15)$$

$I_P = \left(N_P - Z_P\right)/A_P$ and $I_T = \left(N_T - Z_T\right)/A_T$ in Eqn. (2.14) are the isospin asymmetry of projectile and target nuclei respectively.

By varying five parameters u_0, κ, r_0, c and a of the MWS potential, the obtained optimal values of the parameters are listed in Table 2.2 and the minimal deviation calculated with these parameters is $s^2 = 0.0017$.

Table 2.1. Parameters of the modified Ngo potential.

$r_0(fm)$	$b(fm)$	$\upsilon_0(MeV)$	κ	$\alpha(fm^{-2})$
1.202	−2.4	−49.07	−0.4734	0.173

Table 2.2. Parameters of the modified Woods-Saxon potential.

$r_0(fm)$	$c(fm)$	$u_0(MeV)$	κ	$a(fm)$
1.27	−1.37	−44.16	−0.40	0.75

3. Results and Discussion

The basic concepts of the nuclear reaction are usually used in terms of an interaction that is a function of the distance *r* between the center of mass of the target and projectile and consists of a repulsive Coulomb term $V_C(r)$ and a short ranged attractive nuclear component $V_N(r)$. In Fig. (3.1) the nuclear part of the interaction potential V_N (MeV) is displayed as a function of internuclear distance *r* (fm) using MNgo, MWS, Ngo, WS versions of the proximity potentials for the reaction ^{16}O+^{208}Pb. From the figure, the MNgo and MWS potentials follow the Woods-Saxon-type distributions. We see that Ngo potential have a repulsive core.

In Fig. (3.2) all the potentials have acceptable shape: that is, attractive at long distances followed by repulsive at shorter distances. Interestingly, the deepest potential caused by the MNgo potential.

The presence of a pocket in the nucleus-nucleus potential allows a simple conceptual criterion for fusion. If the system enters the pocket, it is captured and fuses. Details of the dymamical processes forming the fully equilibrated compound nucleus are unnecessary, although the general picture of the processes that cause capture inside the barrier is fairly clear. Once significant density overlap occurs, nucleon-nucleon interactions become prolific, resulting in a substantial loss of kinetic energy and angular momentum from the relative motion, so that the nuclei can no longer escape from the potential well and all the flux passing the barrier leads to fusion.

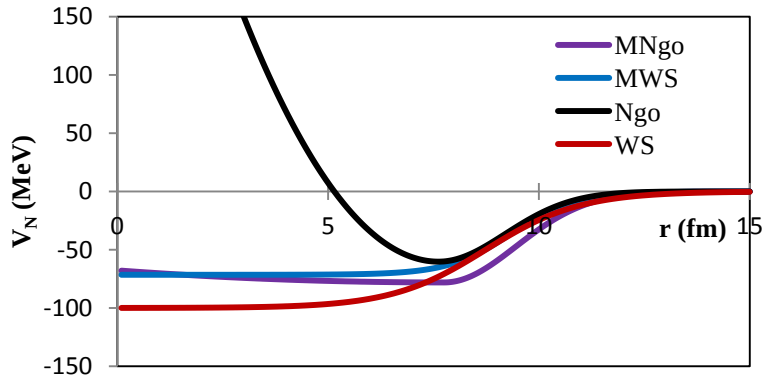


Fig. (3.1) The nuclear part of the interaction potential V_N (MeV) as a function of interaction distance r (fm) for the reaction of $^{16}O+^{208}Pb$ using different proximity potential.

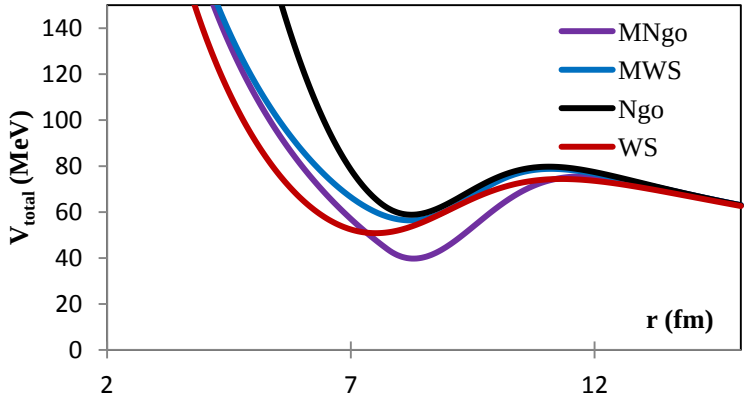


Fig. (3.2) The nucleus-nucleus interaction potential V_N (MeV) as a function of interaction distance r (fm) for the reaction of $^{16}O+^{208}Pb$ using different proximity potential.

Since the fusion happens at a distance greater than the touching configuration of the colliding pair, the previous form of the Coulomb potential is justified. One can extract the barrier height V_B and the barrier position R_B using the following conditions

$$\frac{dV(r)}{dr}\Big|_{r=R_B} = 0, \quad \text{and} \quad \frac{d^2V(r)}{dr^2}\Big|_{r=R_B} \leq 0 \qquad (3.1)$$

We display, in Fig. (), the percentage different of the fusion barrier heights $\Delta V_B(\%)$ defined as

$$\Delta V_B(\%) = \frac{V_B^{theor} - V_B^{expt}}{V_B^{expt}} \times 100 \qquad (3.2)$$

verses asymmetry parameter $A_s = \left(\frac{N-Z}{N+Z}\right)$.

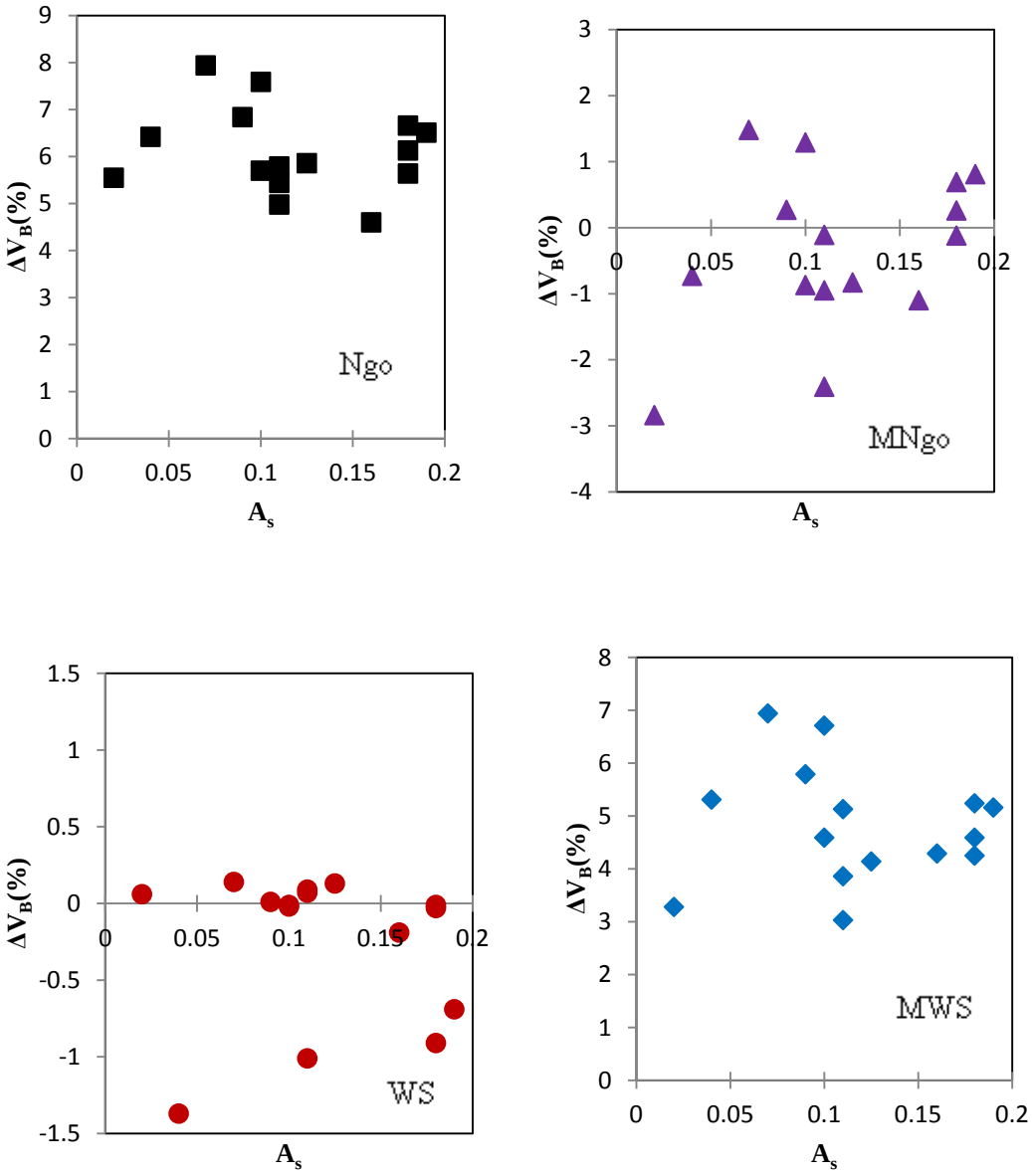


Fig. (3.3) The percentage deviation $\Delta V_B(\%)$ as a function of symmetry parameter A_s in Fig. (a) and the product of charges $Z_P Z_T$ in Fig.(b) using MWS potential.

We calculate the percentage deviation $\Delta V_B(\%)$ in Fig.(3.3) as a function of a symmetry parameter A_s and charges product $Z_P Z_T$. The experimental values are taken from the Ref. [Dutt, I. et al.,]. Interestingly, we see that Ngo potential and MWS potential are fail to reproduce the barrier heights satisfactorily, whereas WS potential gives better results compared to MNgo potential. We can report the discrepancy of the order 2% between the results of MNgo potential and experimental data.We noted that WS potential reproduces the experimental fusion barrier height within 1%. In the previous sub-section, we mentioned that the interaction potential WS potential occurs at larger distances than other MNgo potential.

From Fig. (3.3), it has been seen that the MWS potential and MNgo potential follow the Woods-Saxon-type distributions. The difference between the parameter sets used in MWS potential, Ngo potential, MNgo potential and Woods-Saxon (WS) potential is that the sets of fixed parameters are used in MWS potential, Ngo potential and MNgo potential while that of free parameter are used in WS potential. Therefore, WS potential gets better results compared to other Woods-Saxon-type parameterizations.

The theoretical fusion barrier heights using different phenomenological potentials are displayed as a function of experimentally extracted values in Fig. (3.4). The lines are the fits over the points. These fitted equations show the derivation from the experimental data. MNgo potential and WS potential are in close agreement with the experimental data, whereas Ngo potential and MWS potential fail to come closer to experimental data.

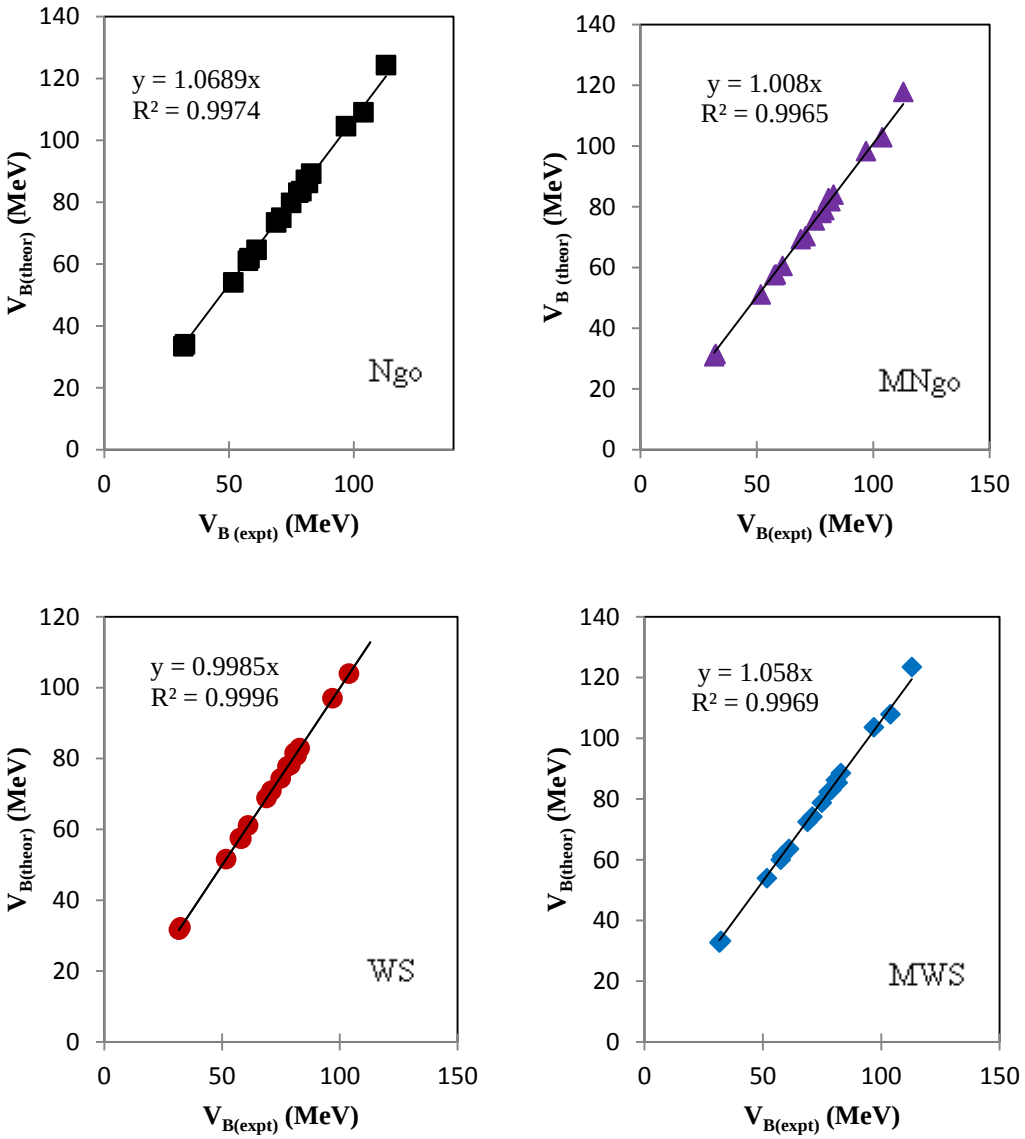
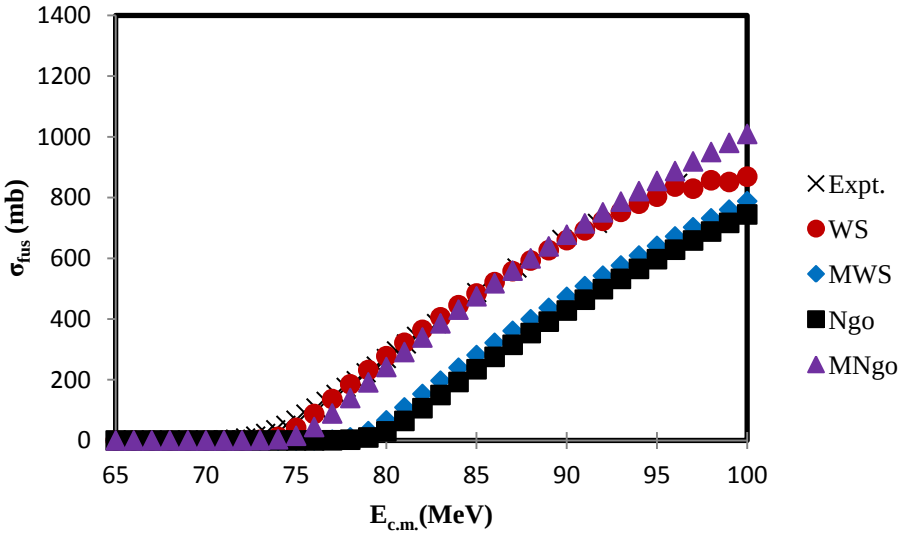


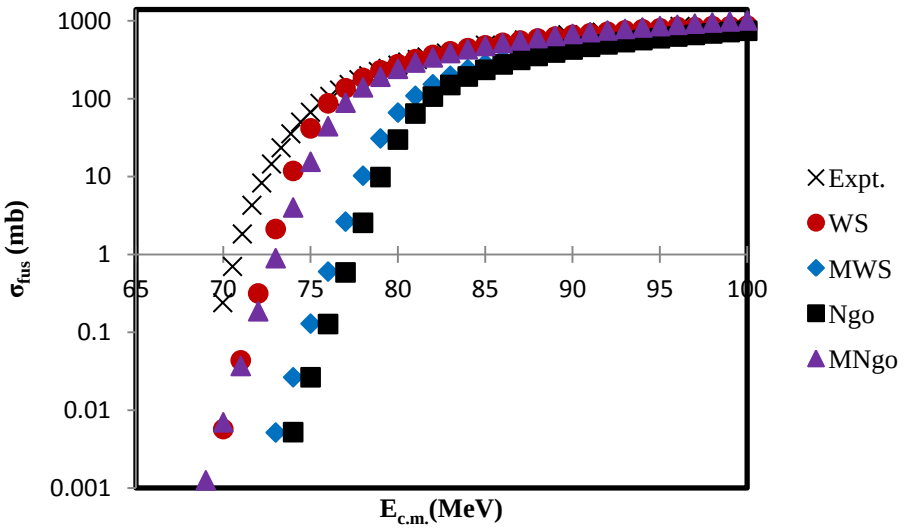
Fig. (3.4) Comparison of theoretical fusion barrier heights $V_{B(theor)}$ (MeV) using different proximity potentials with experimental values $V_{B(expt)}$ (MeV). The solid lines represent the straight line least squares fit created over different points.

As can be seen in the figures, the fusion cross sections at energies above the barrier upper panel of figures (3.5.a) and (3.6.a) are nicely explained by the MNgo potential and Woods-Saxon potential. But Ngo potential and MWS potential fail to come closer to the experimental data. MNgo potential provides satisfactory results in both deformed and spherical systems, ie., $^{16}O+^{208}Pb$ and $^{16}O+^{154}Sm$ systems respectively. This may be due to the fact that MNgo potential includes the microscopic information of the colliding systems.

When we analyze the fusion cross section at energies below the barrier lower panel of figures (3.5.b) and (3.6.b), all of the parameterizations provide suitable results for spherical system $^{16}O+^{208}Pb$, ie., they give the same slopes as that of experimental data as well asfor deformed system$^{16}O+^{154}Sm$.WS potential reproduces the fusion cross sections better than that of the MNgo potential. Inclusion of suitable coupling effects will improve the calculated results.

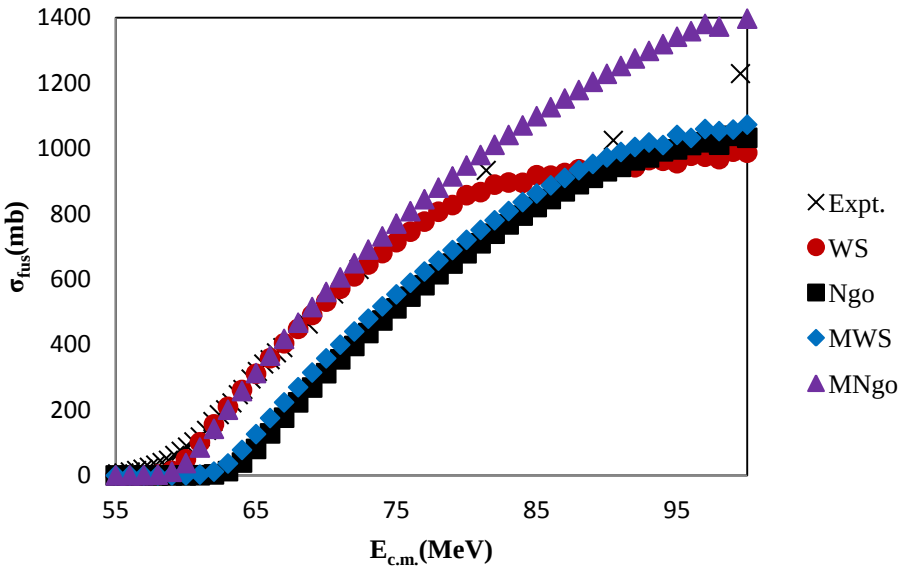


(a)

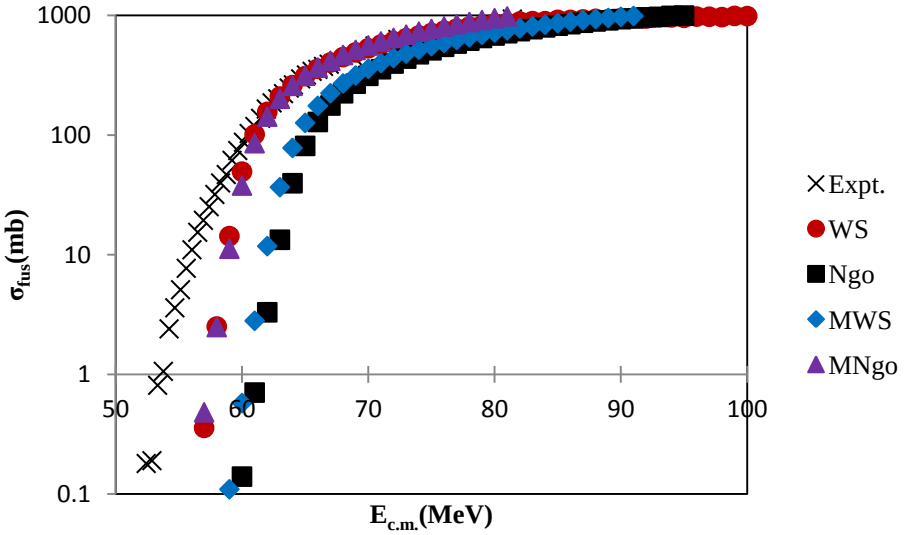


(b)

Fig. (3.5) The fusion cross section obtained by numerically solving the Schrödinger equation for the reactions $^{16}O+^{208}Pb$ as a function of center-of-mass energy E_{cm}. (a) in the linear scale, while (b) in the logarithmic scale.



(a)



(b)

Fig. (3.6) The fusion cross section obtained by numerically solving the Schrödinger equation for the reactions $^{16}O+^{154}Sm$ as a function of center-of-mass energy E_{cm}. (a) in the linear scale, while (b) in the logarithmic scale.

4.Conclusion

For the present study we propose new version of Ngo potential. A detail study reveals that all the potentials have acceptable shape: that is, attractive at long distances followed by repulsive at shorter distances. Interestingly, the deepest potential caused by the MNgo potential. The discrepancy is of the order 2% between the results of MNgo potential and experimental data.

Acknowledgement

We would like to express thanks to Rector, Dr Maung Maung, Lashio University, for his encouragement to do this research. Thanks are also due to Dr. Htun Hlaing, Pro-Rector of Lashio University for constant encouragement to do the research on every meeting.

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