Comparison of Quark-Like Model and Liquid Drop Model for Most of Stable Nuclides

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Abstract

In this paper, the nuclear binding energies of most nuclides in the integrated quark-like model (*QLM*) are compared with available experimental values and also with values from the liquid drop model (*LDM*). Compared to *LDM*, this *QLM* formula is not only simple to comprehend but also possesses the features of natural symmetry.

Key words: Nuclear binding energy, mass-energy equivalence, nucleon, quark confinement.

Introduction

One of the purposes of the nuclear physics is to introduce the proper mathematical models from which the properties and the behavior of nuclides can be explained. The mass-energy equivalence formulated by Einstein plays the central role in all nuclear reactions, ranging from radioactive decay to nuclear power reactors. In both the fission and fusion processes the binding energy per nucleon, *B.E*/*A*, is reduced (algebraically) and an equivalent amount of energy is liberated, which, in the former case, is utilized in nuclear reactors and, in the later case, in the H-bomb, if it be permissible to apply the term “utilization” to an, as yet, purely destructive process. Fusion processes are also though to occur on a large scale on the sun and other similar stars. Fusion reactions could explain the continued production of vast amounts of energy that the sun constantly pours out into space without apparent diminution.

The *B.E* per nucleon, *B.E*/*A*, sometimes called the binding fraction, for a given nuclide is obtained by dividing the total *B.E*. by the number of nucleons contained in that nucleus, as follows

$$B.E/A= [(ZM_H +(A-Z)m_n-M(A,Z))/A]931.48MeV. \qquad (1)$$

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where M_H is the atomic mass of ${}_{1}^{1}H$, m_n is the neutron mass. And $M(A,Z)$ is the atomic mass of the ${}_{Z}^{A}X$ atom.

In the liquid drop model, nucleons are not described individually. They are considered as averaged values, therefore this model has been successful in describing some properties of nuclei such as average binding energy per nucleon, whereas for other nuclear properties such as nuclear excited states, magic number and nuclear magnetic moments it has not so much to present.

The nucleons as free particles moving in a spherical potential and also the Pauli Exclusion Principle intensively limit the interaction between the nucleons. Such a consideration in the shell model provides orbits with approximate stable and defined energy levels. The fundamental assumption in a nuclear shell model is the independence of nucleon motion (free particles) regardless of the existence of strong attractive force between the nucleons. With these assumptions it is predicable that such a model is able to describe nuclear microscopic properties such as excited states energy, magic number and nuclear magnetic moments, but it is impotent to provide a nuclear binding energy formula.

Here the nuclear binding energy is obtained from the new perspective based upon a quark state model of nuclei. From this point of view, since each nucleon is made of three quarks, the binding energy of nuclei contains a volume term proportional to $3A$ (A is mass number). By considering the asymmetry in the number of up and down quarks and also coulomb correction, a new formula is presented that calculates the nuclear binding energy in terms of only N and Z numbers for most of the stable nuclides.

In this paper, it is attempted to compare a well-known liquid drop model with a new quark-like model for binding energy of most nuclides based upon intuitive assumptions that will be presented in the next section.

Formalism

Liquid Drop Model

On the basis of the liquid drop model, Weizsacker, and several others, have attempted to express the masses of nuclei in terms of nuclear characteristics in connection with their binding energy and stability. This formula is known as semi-empirical mass formula. One can express the binding energy *B.E* as the sum of a number of terms as below:

(i) Volume Energy

The binding energy is proportional to the number of nucleons. Since the nuclear volume is proportional to mass number *A*, as a first approximation, there is an attraction or volume energy (E_V), proportional to *A*.

(ii) Surface Energy

Obviously, the nucleus is assumed to resemble a spherical liquid drop of radius $R=r_0A^{1/3}$. As in case of a liquid drop, the nucleons on the surface of the nucleus are not surrounded by as many neighbors as those in the interior. In other words, the nucleons are less strongly bound on the surface, and consequently, the nucleon forces are unsaturated. Since the binding energy is proportional to the number of nucleons (i.e., *A*), and the number of nucleons is reduced on the surface, the binding energy is, therefore lowered by an amount which varies as the surface area of the nucleus. As the nuclear radius is proportional to $A^{1/3}$, the area of the nuclear surface is related to $A^{2/3}$. The energy term corresponding to this surface tension effect is called the surface energy (E_s).

(iii) Coulomb Energy

The protons present in the nuclear volume experience a coulomb repulsion, (i.e. long range electrostatic force of repulsion) which tends to lower the binding energy. As each proton is repelled by $(Z-1)$ protons, the total coulomb energy is proportional to $(Z(Z-1)e^2)/R$. The effect of this coulomb repulsion on the binding energy is termed as the Coulomb energy (E_c).

(iv) Asymmetry Energy

We have seen that the maximum stability occurs when $Z= A/2$, i.e. the number of neutrons are equal in the most stable (light) nuclides. But when the number of neutrons exceeds, i.e. $(A - Z)>Z$, instability of the nuclide appears. This is known as the asymmetry or composition effect. Since the excess neutrons occupy the higher quantum states than the other nucleons, they contribute a smaller amount (per neutron) to the total binding energy. Obviously, with the introduction of asymmetry, the binding energy decreases. The lowering of binding energy, called the asymmetry or neutron excess energy (E_a) is proportional to the square of the neutron excess, $= (A - 2Z)^2$ and inversely to A.

(v) Pairing Energy

It has been observed that even-Z, even-N nuclides are the most abundant amongst the stable nuclides. Nuclides with odd-Z, odd-N are least the least stable, while nuclides with even-Z, odd-N or vice versa, are of intermediate stability. This may be attributed to the pairing of nucleons of the same type or pairing of the nucleon spins. In case of even-even nuclei, all the spins are paired, and so there is a positive contribution to the binding energy for nuclei with odd-Z, odd-N because of the presence of unpaired proton and unpaired proton and unpaired neutron spins. There is no contribution for nuclei with even-Z, odd-N or the reverse. This contribution to the binding energy, arising from the spin or odd-even effect is known as the pairing energy (E_δ).

The total binding energy of a nuclide of mass number A proton number Z is obtained by combining all the above energy terms, i.e.

$$B.E(A, Z) = E_v - E_s - E_c - E_a \pm E_\delta, \qquad (2)$$

$$B.E(A, Z) = a_v A - a_s A^{(2/3)} - a_c Z^2 A^{(-1/3)} - a_a (A - 2Z)^2 A^{-1} \pm E_\delta. \qquad (3)$$

The constants are empirical and can be determined from the experimental values of the masses (or binding energies). The currently accepted values expressed in unified atomic mass scale are $a_v = 0.016919$ u; $a_s = 0.019114$ u; $a_c = 0.0007626$ u; $a_a = 0.02544$ u; and $a_\delta = 0.036$ u.

Quark -Like Model

Let us consider the quark model of nuclei in which the atomic nucleons, instead of containing protons and neutrons is made of quark-gluon soup. Of course the structure of nucleon as are free and bound particle has been intensively investigated, but here we are considering such nucleons as constituents of nuclide. In this state matter is neither condensed nor free like a gas but is loosely bound.

Asymptotic freedom is held between quarks contained in nucleons, but within the nucleus due to longer distance between the quarks, strong nuclear force acts weakly and causes the quark-gluon soup formation. Nucleons are formed and exit from the nucleus via external means such as collisions from which energy is given to the nuclei similar to the jet formations due to quark confinement. In other words, in the context of nuclear quark model, nuclei are assumed to be made of quark-gluon soup instead nucleon. Within such new concept of nuclei, in order to calculate the nuclear binding energy let us make the following assumptions:

(i) Nuclear binding energy is of the order of about one percent of the remaining mass energy of the constituent quarks, namely, $m_q c^2$ where q stand for up and down quarks.

(ii) The binding energy depends upon the volume of the quark-gluon soup within the nuclei, therefore it is proportional to $3A$ where A is the mass number.

(iii) Due to the asymmetric distribution of up and down quarks and also the existence of the Coulomb force between them, one concludes that the binding energy depends upon terms such as $\left(\frac{N^2 - Z^2}{Z}\right)$.

These assumptions, plus intuitive physical curiosity led us to the following formula for the calculation for nuclear binding energy.

$$B.E(Z,N) = \left\{\left[3A - \left(\frac{(N^2 - Z^2) + \delta(N-Z)}{Z} + 3^2\right)\right] \times \frac{m_u c^2}{\alpha}, A \geq 5, \right. \qquad (4)$$

where $m_u c^2$=330 (MeV), α = 90 – 100 and δ stand for nuclear beta-stability line condition and is defined as follows:

$$\delta(N-Z) = \begin{cases} 0 \; for N \neq Z, \\ 1 \; for N = Z. \end{cases} \qquad (5)$$

Similar to the semi-empirical *LDM*, each term in Eq. (4) contains some physical insight. The presence of 3^n law with n=1, 2 shows the 3-quark constituent of nucleons and perfectly fits the experimental values of binding energy.

Results and Discussion

One has to calculate the binding energy per nucleon of each nuclei for given *Z* and *N* numbers and then compare it to the experimental values.

Several distinguishable features are found that should be addressed here. Figures (1), (2) and (3) give the binding energy per nucleon for most of the known stable nuclei. Careful consideration of Figs. (2) and (3) reveals the meaningful accuracy of our quark like model compared to liquid drop model with respect to the experimental data (Fig. (1)). Compared to liquid drop model in which five terms are presented for binding energy formula, our model consists of only two terms that depend only upon *N* and *Z,* indicating a more simple and comprehensive vision of the nuclide. Similar to *LDM*, deviations from the experimental data in this model come from two extreme sides namely, the lightest nuclides in which less than 3 quarks are needed to be formed, and from the heaviest ones in which the number of neutrons are much more than proton numbers.

In quark like model the 3^2 factor in the binding energy formula needs for justification and ignoring the surface term in this binding energy formula is interested, the problem that exists in other models too. Here at the surface of the nuclide the number of quark bonding is negligible compared to those of quarks in the core of the nuclide. So the surface term will be ignored. The binding energy per nucleon for quark like model is in excellent agreement with the experimental data available over the whole range of *A* number, with a similar jump for $A<10$ and similar drop for $A>150$, as shown in Fig. (1) and Fig. (3).

Conclusion

This quark-like model presents a new vision and a new picture of what we call nuclei. A new formula is presented that calculates the nuclear binding energy in terms of only *N* and *Z* numbers for most stable nuclides. The semi-empirical mass formula, based upon only liquid drop model contains at least five terms to be calculated, whereas in Quark like formula only two terms are calculated. We believe the results obtained from these above models are not only simple to understand but also more physical and relatively closer to the experimental data than other models. Other characteristics of nuclei are being studied in the framework of these models.

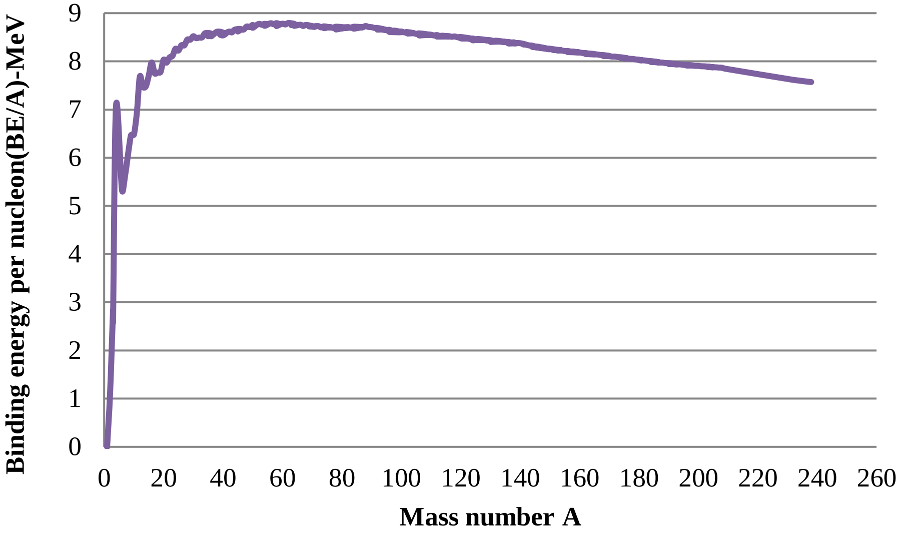

Fig. (1) Experimental Data of Nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei.

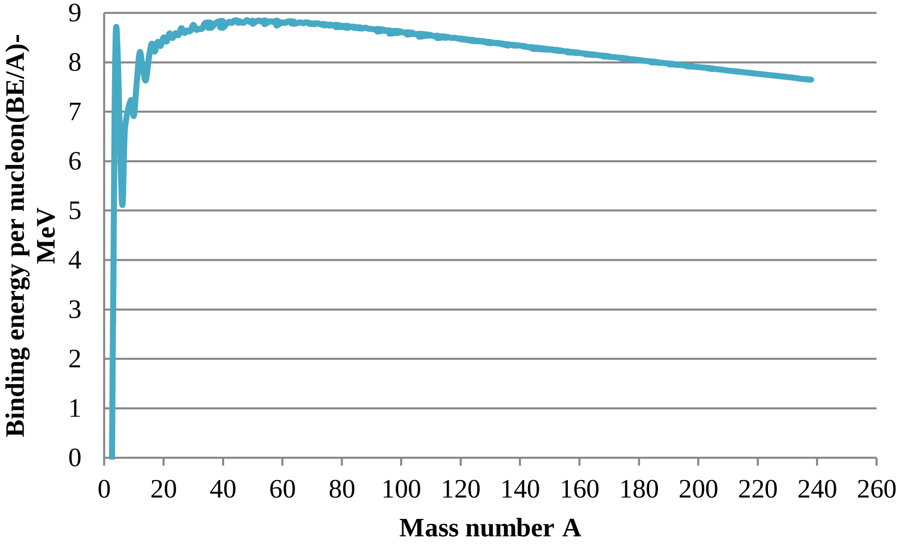

Fig. (2) Liquid drop model data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei.

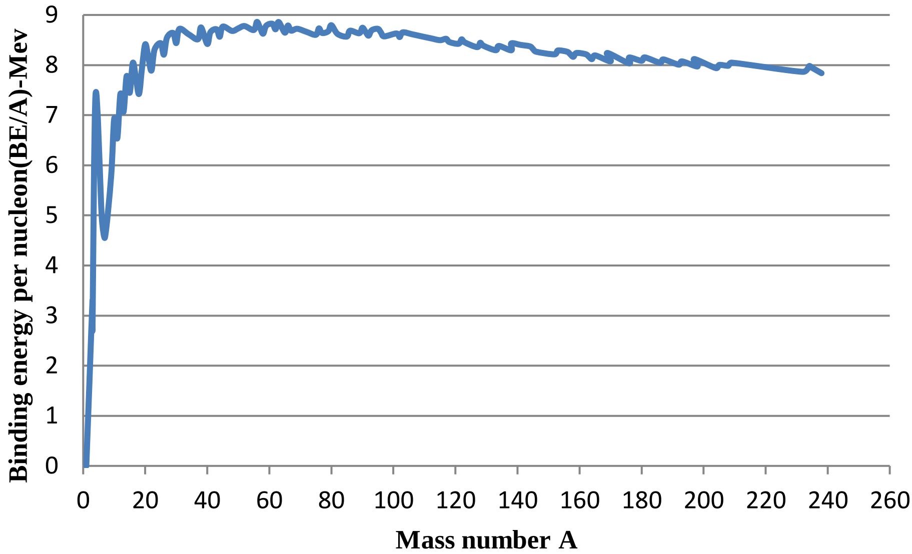

Fig. (3) Quark like model data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei.

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