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# N-BODY SIMULATIONS OF HOMOGENEOUS GALAXIES

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## Abstract

N-body simulations of homogeneous galaxies have been made to explore the physical proposal of critical density and mass, angular momentum and so on. Fundamental properties of galactic dynamics and structure formation have also been studied. Mathematica codes and finite element method have been briefly explored. As the situation dictates, Mathematica software is used for some detailed computations and visualization of the results.

**Keywords:** homogeneous galaxies, galactic dynamics, finite element method.

## Introduction

Attempts have been made to investigate the structure formation in the $\gamma$ gravity f(R) model with N-body simulations. The $\gamma$ gravity model is a proposal which, unlike other viable f(R) models, not only changes the gravitational dynamics, but can in principle also have signatures at the background level that are different from those obtained in $\Lambda CDM$ (Cosmological constant, Cold Dark Matter). The aim of this paper is to study the nonlinear regime of the model in the case where, at late times, the background differs from $\Lambda CDM$. We quantify the signatures produced on the power spectrum, the halo mass function, and the density and velocity profiles. To appreciate the features of the model, have compared it to $\Lambda CDM$ and the Hu-Sawicki f(R) models. For the considered set of parameters it was found that the screening mechanism is ineffective, which gives rise to deviations in the halo mass function that disagree with observations. This does not rule out the model per se, but requires choices of parameters such that ($|f_{R0}|$) is much smaller, which would imply that its cosmic expansion history cannot be distinguished from $\Lambda CDM$ at the background level.

Since the discovery in 1998 that the Universe is speeding up instead of slowing down (as would be expected if gravity is always attractive), considerable effort has been devoted to understanding the physical mechanism behind this cosmic acceleration.(Peebles P.J.E,1970) The two main theoretical approaches considered in the literature to explain this phenomena are (1) to assume the existence of a new component with a sufficiently negative pressure $(p < -\rho/3)$, generically denoted dark energy; and (2) to consider that general relativity has to be modified at large scales, or more accurately, at low curvature (modified gravity). The simplest dark energy candidate is Einstein′s cosmological constant ($\Lambda$) with an equation of state $\omega_{DE} \equiv p_{DE}/\rho_{DE} = -1$. However, in spite of its very good accordance with current observations, $\Lambda$ has some theoretical difficulties such as its tiny value as compared with theoretical predictions of the vacuum energy density, the cosmic coincidence problem, and related fine-tuning. This situation has motivated the search for alternatives like modified-gravity theories. The simplest modified-gravity

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candidates are the so-called f(R)-theories, in which the Lagrangian density $L = R + f(R)$ is a nonlinear function of the Ricci scalar R.(Planck et al ,2013)

As is well known, metric f (R)-theories can be thought of as a special case of a scalar-tensor theory; a Brans-Dicke model with a coupling constant $\omega_{BD} = 0$. An accelerated expansion appears naturally in these theories. The very first inflationary model, proposed by Starobinsky more than three decades ago, is driven by a term of the type $f(R) = \alpha R^2$ $(\alpha > 0)$ and is still in excellent accordance with observations. More recently, the idea of an acceleration driven by late-time curvature has also been explored. These authors considered a theory in which $f(R) = -\alpha R^{-n}$ $(\mathrm{n} > 0$ and $\alpha > 0)$. However, these models do not have a regular matter dominated era and are incompatible with structure formation.(Amendola et al,2007)

To build a cosmologically viable f(R) theory, some stability conditions have to be satisfied (a) $f_{RR} = d^2 f / dR^2 > 0$ (no tachyons);(b) $1 + f_R \equiv 1 + df/dR > 0$ (the effective gravitational constant $\left(G_{eff} = G_N/(1+f_R)\right)$)does not change sign;(c) after inflation, $\lim_{R\to\infty} = f(R)/R = 0$ and $\lim_{R\to\infty} = f(R) = 0$ (General Relativity is recover at early time); (d) $|f(R)|$ is small in recent times, to satisfy solar system and galactic scale constraints. In addition to those conditions, there are some desirable characteristics that a viable cosmological model has to satisfy. It should have a radiation-dominated era at early times and a saddle point matter- dominated era phase followed by an accelerated expansion as a final attractor.

There are viable f(R) gravity theories that satisfy all the criteria above. However, there is generic difficulty from which all these "viable" f(R) theories suffer: the curvature singularity in cosmic evolution at a finite redshift. It can be shown that this type of singularity problem can be curved, for instant, by adding a high-curvature term proportional to $R^2$ to the density Lagrangian. Therefore, it is not possible to have cosmic acceleration with totally consistent f(R) theory modifying gravity at low curvatures.

The specific case of a viable f(R) theory called $\gamma$ gravity. Generally, in almost all viable f(R) theories, structure formation imposes such strong constraints on the parameters of the models that the effective equation of state parameter cannot be distinguished from that of a cosmological constant. In $\gamma$ gravity the steep dependence on the Ricci scalar R facilitates the agreement with structure formation. The parameter that controls the steepness in $\gamma$ gravity allows measurable deviation from $\Lambda CD\mathrm{M}$ at both linear perturbation and background levels, while still compatible with both current observations.(Appleby et al,2010) The main goal of this paper is to study the effects of $\gamma$ gravity on the structure formation at nonlinear scales for choices parameters where the model has observable signature on the background expansion history of our Universe.

## $\gamma$ -Gravity Review

We investigate spatially flat cosmological models in the context of gravity, a viable f (R) theory defined by the following ansatz:

$$f(R) = -\frac{\alpha R_*}{n}\gamma\left[\frac{1}{n},\left(\frac{R}{R_*}\right)^n\right] \qquad (1)$$

Where $\gamma(n,x)=\int_0^x t^{n-1}e^{-t}dt$ is the incomplete $\Gamma$−function and $\alpha$ , n and $R_*$ are free positive constants. In reality, $\gamma$ gravity can be thought of a simple generalization of exponential gravity.

$$f(R)=-\alpha R_*[1-e^{-\left(R/R_*\right)}], \quad (2)$$

Obtained by fixing n = 1 in Eq. (1). We emphasize that $\gamma$ gravity can satisfy all the stability and viability conditions. For fixed n, there is a minimum value ( $\gamma_{\min}$ ) of the parameter $\alpha$ such that for values $\alpha>\alpha_{\min}$ a late-time accelerated attractor is achieved. Consider this case throughout from Eq.(1) ,obtain the following derivatives:

$$f_R=-\alpha e^{-\left(R/R_*\right)^n} \quad (3)$$

$$f_{RR}=\frac{\alpha n}{R}\left(\frac{R}{R_*}\right)^n e^{-\left(\frac{R}{R_*}\right)^n} \quad (4)$$

Note from Eq.(3) that with increasing n, the steepness of the f (R) function increases. Higher n means smaller $|f_{R0}|$, and the departures from GR will be smaller accordingly. (O′ Dwyer et al, 2013)

Although there is no cosmological constant, f (0) = 0, it follows from Eq. (1) that GR with $\Lambda$ is recovered at high curvatures. Therefore, for $R>>R_*$ the models behave like $\Lambda CDM$. Since interested in phenomena that occurred after the beginning of the matter-dominated era, neglect radiation and write the effective cosmological constant (the cosmological constant of the reference $\Lambda CDM$ model) as

$$\tilde{\Lambda}=\frac{\alpha R_*}{2n}\Gamma(1/n)=3\tilde{H}_0^2(1-\tilde{\Omega}_{m0}) \quad (5)$$

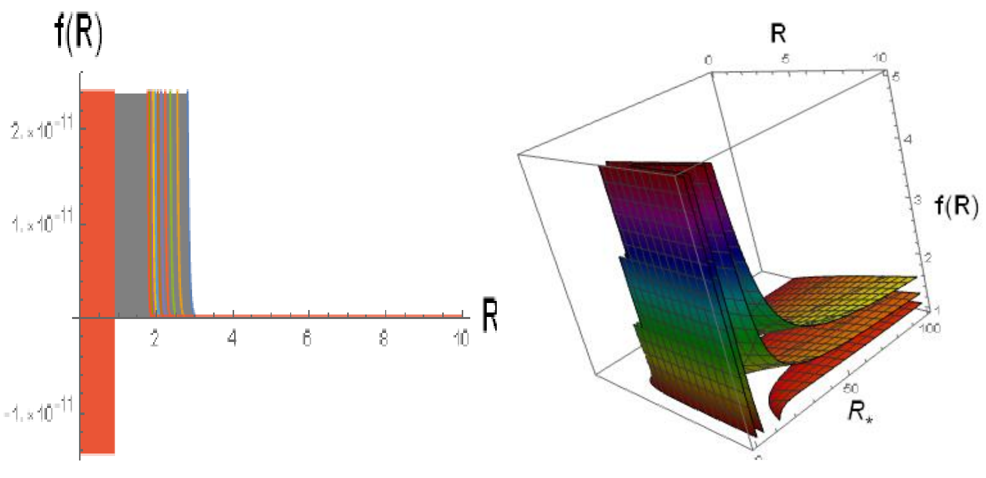


**Figure 1** The 2D and 3D profiles of gamma gravity f(R)

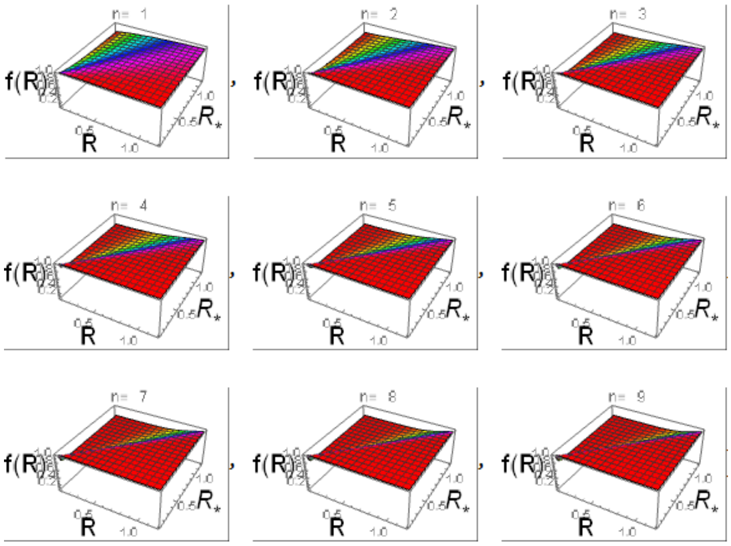


**Figure 2** 3-D snapshot profiles of gamma gravity f(R)

In the equation above, $\tilde{\Omega}_{m0}$ denotes the present value of the matter density parameter that a $\Lambda\, CDM$ model would have if it had the same matter density today $(\tilde{\rho}_{m0})$ as the modified gravity f(R) model. $\tilde{H}_0$ represents the Hubble constant in the reference $\Lambda\, CDM$ model. Therefore, $m^2 \equiv 8\pi G\tilde{\rho}_{m0}/3 = \tilde{\Omega}_{m0}H_0^2 = \Omega_{m0}H_0^2$, where $\Omega_{m0}$ and $H_0$ are the present value of the matter energy density parameter and Hubble parameter in the f(R) model, respectively. It is useful to rewrite $R_*$ as

$$\frac{R_*}{m^2} = \frac{6nd}{\alpha\Gamma(1/n)} \tag{6}$$

Where $$d = (1-\tilde{\Omega}_{m0})/\tilde{\Omega}_{m0}$$

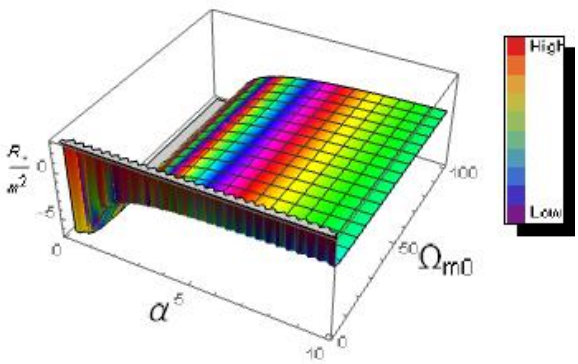


**Figure 3** Profile of $R_*/m^2$ in terms of $\alpha$ and $\Omega_{m0}$.

## N-Body Equations

f(R) models are equivalent to a scalar-tensor theory, where the first derivative of the f (R) function, $f_R$. (Brax et al.2008) This field propagates according the equation

$$\Upsilon f_R = \frac{\partial V_{eff}}{\partial f_R} = \frac{(1-f_R)R + 2f + \kappa^2 T}{3} \tag{7}$$

Where $\kappa^2 = 8\pi G/c^4$ and T is the trace of energy-momentum tensor, $T = g_{\mu\nu}T^{\mu\nu}$.In the quasi-static limit, this equation becomes

$$\frac{1}{a^2}\nabla^2 f_R = \frac{R - R(a)}{3} - m^2 a^{-3}\delta_m \tag{8}$$

Where

$$\frac{R(a)}{m^2} = 3(a^{-3} + 4d) + \Delta_R(a), \tag{9}$$

and $\Delta_R(a) = x_2(a) + 12x_1(a)$.The Ricci scalar R in function of $f_R$ is given by inverting Eq. (3)

$$R = R_* \log\left(\frac{a}{|f_R|}\right)^{1/n} \tag{10}$$

The geodesic equation, needed to update the particle positions, reads

$$\ddot{x} + 2H\dot{x} = -\frac{1}{a^2}\nabla\left(\Phi - \frac{f_R}{2}\right), \tag{11}$$

Where $\Phi$ is the Newtonian potential, which the dynamics is given by the Poisson equation

$$\nabla^2\Phi = \frac{3m^2}{2}\frac{\delta_m}{a}, \tag{12}$$

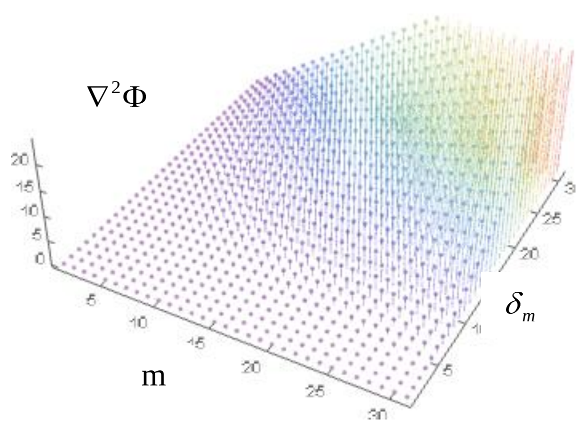


**Figure 4** Profile of $\nabla^2\Phi$ in terms of and $\delta_m$

When implementing these equations in the N-body code, need to rewrite them in code-units given by

$$\tilde{x} = \frac{x}{B_0}, \tilde{\Phi} = \frac{\Phi a^2}{(H_0 B_0)^2}$$
$$d\tilde{t} = \frac{H_0 dt}{a^2}, \nabla_{code} = B_0 . \nabla \tag{13}$$

Here $B_0$ is the size of the simulation box. In terms of $\tilde{f}_R = -a^2 f_R$, the evolution equation becomes

$$\frac{d^2\tilde{x}}{d\tilde{t}^2} = -\nabla_{code}\Phi - \frac{1}{2(B_0 H_0)^2}\nabla_{code}\tilde{f}_R \tag{14}$$

$$\nabla^2_{code}\Phi = \frac{3}{2}\Omega_{m0} a \delta_m \tag{15}$$

$$\nabla^2_{code}\tilde{f}_R = \Omega_{m0}(H_0 B_0)^2 a^4 \times \left\{ -\frac{R_*}{3m^2}\log\left(\frac{aa^2}{\tilde{f}_R}\right)^{1/n} + \left[a^{-3} + 4d + \frac{\Delta_R(a)}{3}\right] + a^{-3}\delta_m \right\} \tag{16}$$

These are the only equations need to implement and solve in the N-body code. For comparison it also needs the linearized field equation. Simulations with this equation compared to the full $f_R$ equation is a good measure of the amount of screening that takes place in the model. The linearized $f_R$ equation is simply

$$\frac{1}{a^2}\nabla^2 \delta f_R = m_\phi^2(a)\delta f_R - m^2 a^{-3}\delta_m \tag{17}$$

Where $\delta f_R = f_R - f_R(a)$ and $m_\phi^2(a) = \frac{1}{3 f_{RR}(a)}$ .In code units, taking $u = -\frac{\delta f_R a^2}{2(H_0 B_0)^2}$ , we obtain

$$\nabla^2_{code} u = \left[m_\phi(a) a B_0\right]^2 + \delta_m \frac{\Omega_{m0} a}{2} \tag{18}$$

and the geodesic equation becomes

$$\frac{d^2\tilde{x}}{d\tilde{t}^2} = -\nabla_{code}\Phi - \nabla_{code} u \tag{19}$$

We have

$$m_\phi^2(a) a^2 B_0^2 = \frac{a^2 (H_0 B_0)^2}{3\alpha n}\frac{R(a)}{H_0^2}\left[\frac{R_*}{R(a)}\right]^n e^{[R(a)/R]^n} \tag{20}$$

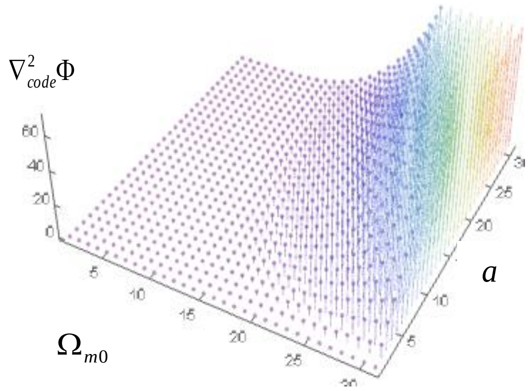


**Figure 5** Profile of the $\nabla^2_{code}\Phi$ in terms of $\Omega_{m0}$ and $a$.

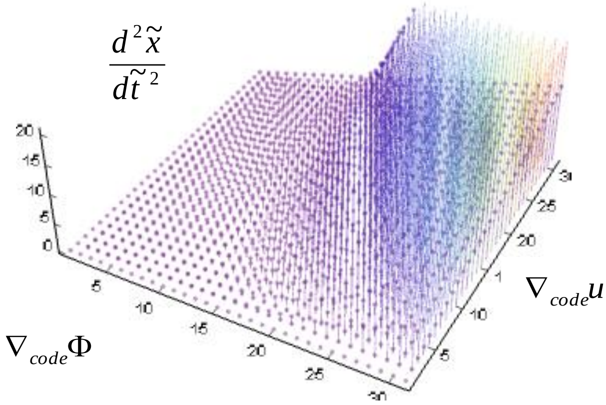


**Figure 6** Profile of the $\frac{d^2\tilde{x}}{d\tilde{t}^2}$ in terms of $\nabla_{code}\Phi$ and $\nabla_{code}u$.

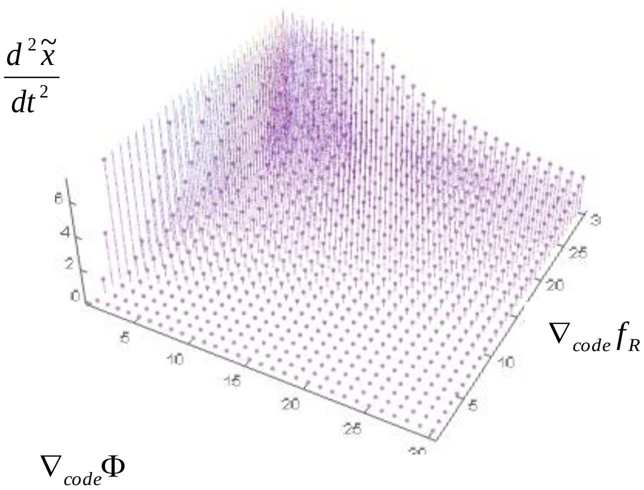


**Figure 7** Profile of the $\frac{d^2\tilde{x}}{d\tilde{t}^2}$ in terms of $\nabla_{code}\Phi$ and $\nabla_{code}f_R$.

## Result and Conclusions

We have investigated the nonlinear evolution in the $\gamma$ gravity, the f(R) theory of gravity that is a viable alternative to $\Lambda$ CDM. In the models under investigation one uses a screening mechanism to suppress the deviations from General Relativity at small and large cosmological scales. Specifically, this is what we called the chameleon screening mechanism. As a result of this screening mechanism, the strongest signatures in these models are expected to occur at the nonlinear regime of structure formation. Therefore, to unveil the imprints of such theories at astrophysical scales, we ran several cosmological N-body simulations. Originally, programmes are ran on highs specially supercomputers. In this paper, we make use of the simple iteration preceding such as Listpointplot3D, BesselJ(2Dand 3D),Contourplot3D, Revolutionplot3D and so on. The interesting point is such that the resulting normalizations are in agreement with the done by the high capacity computers.

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