Optimization of Profit for Liquefied Petroleum Gas Extraction Plant by using Simplex and Genetic Algorithm

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Abstract

Linear Programming (LP) is a class of mathematical programming models concerned with the efficient allocation of limited resources to specified activities with the objective of meeting a desired goal. Linear Programming model is a simplified representation of a real-life system. This system is intended to formulate linear programming model of LPG (Liquefied Petroleum Gas) plant Minbu and calculate the maximum profit of objective function of LP model by using simplex method (maximization case) and Simple Genetic Algorithm. This system can search optimal solution for Linear Programming model of LPG plant. LP model can be solved by many methods: these are Graphical method, Two Phase method and Big- M method. But this system can solve LP problem using simplex method and simple genetic algorithm.
Keywords: Linear Programming Model, Genetic Algorithm, Optimization, LPG Extraction Plant

1. Introduction

For an increasingly large number of applications, there are many problems that can decide for them what they need to do in order to satisfy their design objectives. The application of evolutionary computation techniques for the solution of optimization problems is now the major area of research. Optimization problems reveal the fact that the formulation of engineering design problems involves linear terms for constraints and objective function but certain other problems involve nonlinear terms for them. In some problems, the terms are not explicit functions of the design variables. Unfortunately, there does not exit a single optimization algorithm which will work in all optimization problems equally efficiently. Some algorithms perform better on one problem, but may perform poorly on other problems. That is why the optimization literature contains a large number of algorithms, each suitable to solve a particular type of problem [1].

So, this system is intended to formulate the linear programming model to represent the problem and develops a computer-based procedure for optimal solution by using simplex method and genetic algorithm. It then shows the results and determines the optimal solution of LPG plant of Minbu as a case study.

2. Background System

LPG (Liquefied Petroleum Gas) plant produces three types of products propane, butane and naphtha. The production process contains three departments to produce these products [2].These are-

(1)Dehydration Department
(2)Fractionation Department
(3)Refrigeration Department

It gets raw material which is called associated gas from Minbu, Htout Shar Pin and Kan Ni gas terminal. In mathematics, LP problems involve the optimization of linear objective function, subject to linear equality and inequality constraints. It gives the best outcome or optimal solution (e.g. maximum profit, least effort, etc) by using a linear mathematical model. In LPG plant, linear programming model can be used to solve problems regarding to-

(1) Profit maximization
(2) Cost minimization
(3) Transportation

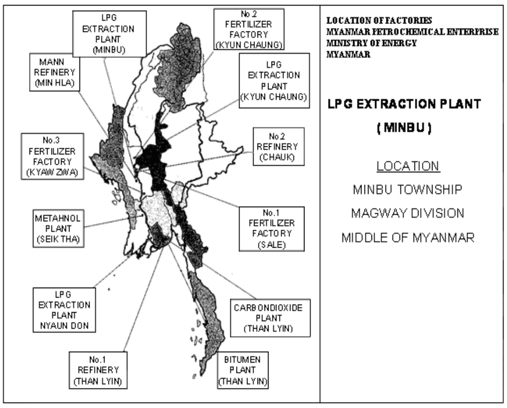

Figure 1. Location of LPG Plant

2.1. LP Model Formulation for the Plant

LPG Extraction Plant produces three types of product propane, butane and naphtha. Production department consists of three primary sections for these products dehydration, fractionation and refrigeration section as shown before. The selling prices of one unit (liter) of product propane, butane and naphtha respectively are kyat 50, 70 and 75. Other utilities factory cost for one unit (liter) of each product are kyat 5 for propane, 6 for butane and 10 for naphtha. The plant has only one of each of type of production section or department. Costs per hour to run each of the three sections is kyat 20 for dehydration, 30 for fractionation and 30 for refrigeration section.

If there are 240 liters of associated gas, production department can produce 39 liters of propane, 60 liters of butane and 10.8 liters of naphtha. Capacities (product unit per hour) for each part on section are shown in Table 1.

Table 1. Capacities of Production Department

	Propane	Butane	Naphtha
Dehydration	25 liter/hr	4 liter/hr	25 liter/hr
Fractionation	25 liter/hr	2 liter/hr	2 liter/hr
Refrigeration	4 liter/hr	3 liter/hr	4 liter/hr

Table 2. User defined data of LPG Plant

	Propane	Butane	Naphtha	Cost/hr (ks)
Dehydration	25 liter/hr	4 liter/hr	25 liter/hr	Ks 20
Fractionation	25 liter/hr	2 liter/hr	2 liter/hr	Ks 30
Refrigeration	4 liter/hr	3 liter/hr	4 liter/hr	Ks 30
Utilities cost per unit (liter)	Ks 5	Ks 6	Ks 10	
Selling Price per unit (liter)	Ks 50	Ks 70	Ks 75	
Associated Gas	39 liters	60 liters	10.8 liters	240 liters

Table 2 shows the user defined data of LPG plant in order to formulate the linear programming model.

2.2. Decision Variables

Let x_1 be number of units of propane to be produced per hour.

Let x_2 be number of units of butane to be produced per hour.

Let x_3 be number of units of naphtha to be produced per hour.

2.3. Calculation of Profit for use in Objective function

Profit of one unit of propane

50-5-(20/25+30/25+30/4) = 35.5

Profit of one unit of butane

70-6-(20/4+30/2+30/3) = 34

Profit of one unit of naphtha

75-10-(20/25+30/2+30/4) = 41.7

The result of this can vary depending on the user input data entry. These decision variables are required for the formulation of linear programming model of the LPG extraction plant.

3. Simplex Method

Formulating a mathematical model, if there are n related decisions to be made, they are represented as decision variables (say $x_1, x_2, \ldots, x_n$). The measure of performance (e.g. Profit) is expressed as a mathematical function of decision variables (e.g. $P = 3x_1+2x_2+\ldots+5x_n$) is called the objective function. Any restrictions on the values that assigned to these decision variables are expressed by means of inequality or equality (e.g. $x_1+3x_2+2x_3$ =<10), are called constraints. It is a numerical method for optimizing many dimensional unconstrained problems. It is the more general class of search algorithms. It is concerned only with finding the single optimal point of LP problem that must be converted into simplex standard form. All the constraints should be expressed as equations by adding slack (s) or surplus (-s) or artificial variables (A). The right hand side of each constraint should be made non-negative [3].

3.1. Simplex Method Procedure

Step1: Formulation of the mathematical model
Step2: Set up the initial solution
Step3: Test for optimality
Step4: Select the variable to enter the basic
Step5: Test for feasibility
Step6: Finding the new solution
Step7: Repeat the procedure

3.1.1. Solving LP Model of LPG Plant by using Simplex Method

Subject to constraints

$$4x_1 + 25x_2 + 4x_3 \leq 100$$
$$2x_1 + 25x_2 + 25x_3 \leq 50$$
$$3x_1 + 4x_2 + 3x_3 \leq 12$$
$$39x_1+60x_2 + 10.8x_3 \leq 240$$
$$x_1, x_2, x_3 \geq 0$$

Objective function

$$\text{Max Profit } Z = 35.5x_1 + 34x_2 + 41.7x_3$$

This is the linear programming model of LPG plant that have been formulated depending on the users' input entry. Depending on the users' input entry, the model changes day by day. The constraints in this model change day by day. This LP model must be changed to standard form in order to solve with simplex method.

3.1.2. Standard Form

Subject to constraints

$$4x_1 + 25x_2 + 4x_3 + s_1 = 100$$
$$2x_1 + 25x_2 + 25x_3 + s_2 = 50$$
$$3x_1 + 4x_2 + 3x_3 + s_3 = 12$$
$$39x_1+60x_2 + 10.8x_3 + s_4 = 240$$

Objective Function

$$\text{Max Profit } Z = 35.5x_1 + 34x_2 + 41.7x_3 + 0s_1+ 0s_2+0s_3+ 0s_4$$

Table3. Initial Simplex Table

CB	BV	XB	X_1	X_2	X_3	S_1	S_2	S_3	S_4	Min ratio XB/x_3
		c_j	35.5	34	41.7	0	0	0	0	
0	S_1	100	4	25	4	1	0	0	0	100/4 = 25
0	S_2	15	2	25	25	0	1	0	0	50/25 = 2
0	S_3	12	3	4	3	0	0	1	0	12/3 = 4
0	S_4	240	39	60	10.8	0	0	0	1	240/10.8 = 22.2
Z=0		Z_J	0	0	0	0	0	0	0	
		$c_j - z_j$	35.5	34	41.7	0	0	0	0	

In table 3, we must first initialize the input data into the simplex table to solve the LP model of LPG plant. CB represents coefficient of basic variables, BV represents basic variables and XB represents the values of variables at the right hand side of the equation.

Table 4. Testing Optimal Solution

CB	BV	XB	X_1	X_2	X_3	S_1	S_2	S_3	S_4	Min ratio XB/x_1
		C_J	35.5	34	41.7	0	0	0	0	
0	S_1	92	92/25	21	0	1	-4/25	0	0	25
41.7	x_3	2	2/25	1	1	0	1/25	0	0	25
0	S_3	6	69/25	1	0	0	-3/25	1	0	2.17
0	S_4	218.4	38.136	49.2	0	0	-54/25	0	1	5.73
Z=83.4		Z_J	3.336	41.7	41.7	0	1.688	0	0	
		cj - zj	32.164	-7.7	0	0	-1.688	0	0	

In table 4, we test for optimality by using the result of c_j-z_j. If the value of result is negative then the solution of Z is optimal. If the value of result is positive then we must repeat the procedure of simplex method until the value becomes negative.

Table 5. Final Solution

CB	BV	XB	X_1	X_2	X_3	S_1	S_2	S_3	S_4	Min ratio XB/x_1
		C_J	35.5	34	41.7	0	0	0	0	
0	S_1	84	0	59/3	0	1	0	-4/3	0	25
41.7	x_3	42/23	0	67/69	1	0	1/23	-2/69	0	25
35.5	x_1	50/23	1	25/69	0	0	-1/23	25/69	0	2.17
0	S_4	135.5	0	35.3	0	0	1.226	-13.82	1	5.73
Z=153.32		Z_J	35.5	53.35	41.7	0	0.27	11.65	0	
		cj -zj	0	-19.35	0	0	-0.27	-11.65	0	

In table 5, all of the values of c_j-z_j are negative. So we can stop the repeat procedure and see the optimal solution. In this optimal solution, the value of Z = 153.32 and corresponding values of x_1, x_2 and x_3 are 35.5, 0 and 41.7.

4. Genetic Algorithm

A genetic algorithm is a computer algorithm that searches for a good solution to a problem among a large number of possible solutions. These search algorithms are based on the mechanism of natural selection and natural genetics. Genetic algorithms maintain a population of some feasible solutions for a given problem. This population undergoes evolution in a form of natural selection and natural genetics. All information required for creation of appearance and behavioral features of living organism is contained in its chromosomes. Reproduction generally involves two parents, and the chromosomes of the offspring are generated from portions chromosomes taken from the parents. In this ways, the offspring inherit a combination of characteristics from their parents. Gas attempts to use a similar method of inheritance to solve various problems [4].

Five components required by GA are defined by the system designer. In this system, system designer defined a way of encoding solution to the problem as binary form.

1. A way of encoding solution to the problem on chromosome or candidate solution.
2. An evaluation function which returns a rating for each chromosome given to it.
3. A way of initializing the population of chromosomes.
4. Operators that may be applied to parents when they reproduce to alter their genetic composition. Standard operators are mutation and crossover.
5. Parameter settings for the algorithm.

5. Implementation of Genetic Algorithm

Below is an outline of how a GA uses these components to search for optimal solutions:

1. Create an initial population of chromosomes.
2. Evaluate each of the chromosomes in the initial population.
3. Select chromosomes that will have their information passed onto the next generation.

4. Crossover the selected chromosomes to produce new offspring chromosomes.
5. Mutate the genes of the offspring chromosomes.
6. Repeat steps 3 through 5 until a new population of chromosomes has been created.
7. Evaluate each of the chromosomes in the new population.
8. Repeat steps 3 through 7 until some termination condition has been met. This termination condition can be based simply on the number of generations or it can be based upon more complex criteria.

In the calculation of GA for the LP model of LPG plant, new parameters for the algorithm are needed. As an example, generation size is 2, population size is 5, and crossover and mutation probability is 0.8 and 0.2 respectively. Step by step calculations of GA are shown in table 6 and table 7.The process of GA will repeat until the end of specified generation number and among all of these results it chooses the best optimal solution for the LPG plant.

Table 6. Generation Number 1

Gen no:	Pop no:	'x1' Bit string	'x1' value	'x2' Bit string	'x2' value	'x3' Bit string	'x3' value	Fitness
1	1	011	3	000	0	001	1	148.2
1	2	001	1	001	1	001	1	111.2
1	3	000	0	000	0	010	2	83.4
1	4	011	3	000	0	000	0	106.5
1	5	011	3	000	0	001	1	148.2

In table 6, it will repeat the procedure depending on the population number. In this GA calculation, a way of encoding chromosome is changing decimal value of x_1, x_2 and x_3 to binary form and substituting these values in objective function to find the best solution for Z.

Table 7. Generation Number 2

Gen no:	Pop no:	'x1' Bit string	'x1' value	'x2' Bit string	'x2' value	'x3' Bit string	'x3' value	Fitness
1	1	000	0	001	1	001	1	75.7
1	2	010	2	001	1	000	0	105
1	3	001	1	001	1	001	1	111.2
1	4	000	0	000	0	001	1	41.7
1	5	011	3	000	0	000	0	106.5

In table 7, it will repeat the procedure as before and stop the calculation because this is the end of generation number 2. As the final result, the best optimal solution is found in generation number 1 and population number 5. The best solution is chosen depending on the most maximum value of profit Z. And the values of x_1, x_2 and x_3 are corresponding values that present in the row of maximum value of Z.

6. System Design

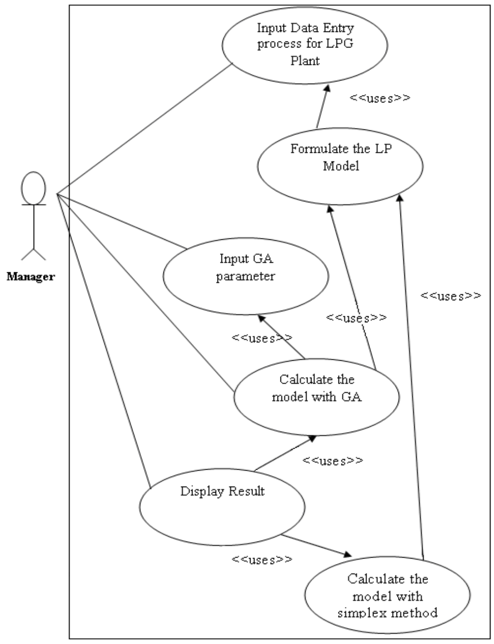

Figure 2. Use Case Diagram

In this system design as figure 2 shows, user can enter data entry for LPG plant and formulate the LP model. And then, user can calculate the optimal solution by using simplex method and GA method. In simplex result, the best optimal solution calculated from the model that has been formulated will show to the user and the user can save the result for the later use in comparison of results from the GA.

7. Implementation and Experimental result

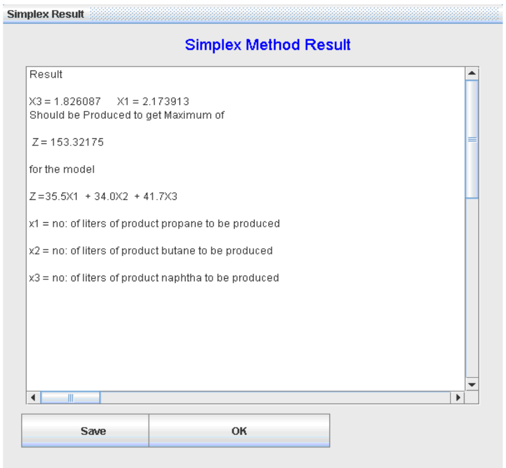

Figure 3. Simplex Solution

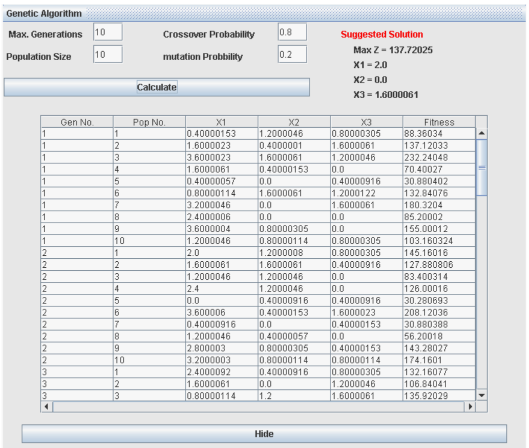

Figure 4. GA Solution

In table 8, profit of the simplex method is better than GA method. These testing results can be changed depending on user's input data.

Table 8. Comparison Result of Simplex and GA

Method	x_1	x_2	x_3	Max Profit Z
Simplex	2.173913	0	1.826087	153.32175
GA	2.0	0	1.6000061	137.72025

8. Conclusion and Further Extension

This system formulates the LP model depending on the user requirements and then calculates the model with two methods to get optimal solution. This system only provides the profit maximization LP problem of the LPG plant. It can be extended to the cost minimization and transportation problem of the LPG plant.

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