

Implementation of Manufacturing Industrial Management System using Linear Programming Models

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Abstract

This paper intends to provide estimating the profit for Automobile Manufacturing Industry. Computerized decision support systems are needed for speedy computation increased productivity, technical support, quality support and competitive edge. Linear programming is the best known optimization model. It deals with the optimal allocation of resources among competing activities. Using special mathematical procedures, the LP approach applies a unique computerized search procedure to find the best solution. It uses Simplex method to solve the car production without insufficient resources. Big-M is also utilized for the order based model. However, Simplex method and Big-M method gets the results in decimal value. Integer programming is applied in order to get the results in integer values.

1. Introduction

Computerized information system supports business and organizational decision-making activities. This system is extensively used in business and management. It helps such managers solve problems they typically face in real world. A properly design of this system is an interactive software-based system intended to help decision makers with useful information from raw data, documents, knowledge, and/or business model to identify and solve problems and make decisions.

Linear programming helps in attaining the optimum use of productive resources. It can be able to improve the quality of required materials. It also helps in re-evaluation of basic plan for changing conditions. A solution that satisfies both the conditions of the problem and the given objective is termed an 'optimum solution'. A typical example is that of the manufacturer who must determine what combination of his available resources will enable him to manufacture his products in a way which not only satisfies his production schedule, but also

maximizes profit. This problem has as its basic conditions the limitations of the available resources and the requirements of the production schedule, and as its objective the desire of the manufacturer to maximize his gain.

Modeling a motor vehicle is a very complex task. Many hard-to-predict variables need to be considered during the process. The profitability of a car manufacturing industry depends largely on how well it uses its capacity. However, managing this operation is immensely difficult. The proposed system models a car manufacturer's work to flow over order-based model in order to fulfill the task of decision making process. This paper presents the computerized information system for car production. Linear programming is used to model the production in order to maximize profit and minimize cost. Section 2 presents the related work that uses other approaches for decision support system. Linear Programming, Simplex method, Big-M and Integer Programming models are described in Section 3. Section 4 describes Proposed System. Section 5 illustrates how System Implementation is used in this paper. Section 5 deals with Conclusion.

2. Related Work

Selinger [3] developed a dynamic programming model of a linear project. His work ignored to incorporate the cost as decision variable in the optimization process. As extension of the Selinger's work, Russel and Caselton [4] formalized a N-stage dynamic programming solution into two state variable to determine the minimum project duration. In the optimization process, the developed model ignored the activities costs as a decision variable.

Reda [5] developed a linear programming to identify minimum cost maintaining constant production rates and repetitive projects. This method can only be used for nontypical linear project and not applicable to industrial management systems. Most of the developed models assume the activities are accomplished serially. In reality, most industrial

management activities are accomplished concurrently while others accomplished serially.

Elmaghraby [6] considered completion schedules on an arbitrary set of milestone events by developing an efficient algorithm to determine the project schedule, which minimizes the sum of the total cost plus penalties for late completion. Another extension was by Moore [7] by using goal programming to consider multiple objectives, such as completion times, resources leveling and operation within a limited budget.

Senouci [1] presented a dynamic programming formulation for the scheduling of non sequential or nonserial activities to determine the project time-cost profile which determines possible project duration and their minimum project total cost. The formulation considers the effects of interruptions, minimum project direct cost, and minimum project duration.

3. Linear Programming

Linear programming (LP) is the general technique of optimum allocation of scarce or limited resources, labor, material, capital, energy, etc, to several competing activities, such as products, jobs, new equipment, projects, etc. In mathematics, LP problem involve the optimization of a linear objectives function, subject to linear equality and in equality constraints.

LP problems determine the way to achieve the best outcome (such as maximize profit or lowest cost) gives some list of requirements represented as linear equations.

LP is problem that can be expressed in canonical form:

$$\begin{aligned} &\text{Maximize } C^T x \\ &\text{Subject to } Ax \leq b \end{aligned}$$

X represents the vector of variables (to be determined), while 'C' and 'b' are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($C^T x$ is this Case). The equations $Ax \leq b$ are the constraints which specify a convex polyhedron over which the objective function is to be optimized.

LP can be applied to various fields of study. Most extensively it is used in business and economic situation, but can also be utilized for some engineering problems. Some industries use LP models in transportation, energy consumptions, telecommunications, and manufacturing. It has provided useful in modeling diverse types of problems in planning, routing, scheduling, assignment and design.

LP is an important field of optimization for several reasons. Many practical problems in operations research can be expressed as LP problems. A number of algorithms for other types of optimization problems work by solving LP problems as sub-problems. Ideas from LP have inspired many of the central concepts of optimization theory, such as duality, decomposition and the importance of convexity and its generalizations

3.1. Simplex Method

Simplex algorithm is a popular algorithm for numerically solving linear programming problems. Consider a linear programming problem,

$$\begin{aligned} &\text{maximize } C^T x \\ &\text{subject to } Ax \leq b, x \geq 0 \end{aligned}$$

with $x = (x_1, \dots, x_n)$ the variables of the problem, $c = (c_1, \dots, c_n)$ a vector representing the linear form to optimize, A a rectangular p, n matrix and $b = (b_1, \dots, b_p)$ the linear constraints.

The simplex algorithm requires the linear programming problem to be in augmented form, so that the inequalities are replaced by equalities. The problem can then be written as follows in matrix form:

Maximize Z in:

$$\begin{bmatrix} 1 & -c^T & 0 \\ 0 & A & I \end{bmatrix} \begin{bmatrix} Z \\ x \\ x_s \end{bmatrix} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$x, x_s \geq 0$$

where $x_s = (x_{s,1}; \dots; x_{s,p})$ are the introduced slack variables from the augmentation process, ie the non-negative distances between the point and the hyperplanes representing the linear constraints A.

By definition, the vertices of the feasible region are each at the intersection of n hyperplanes (either from A or $x \geq 0$). This corresponds to n zeros in the $n+p$ variables of (x, x_s) . Thus 2 feasible vertices are adjacent when they share $n-1$ zeros in the variables of (x, x_s) . This is the interest of the augmented form notations : vertices and jumps along edges of the polytope are easy to write.

The simplex algorithm goes as follows :

- Start off by finding a feasible vertex. It will have at least n zeros in the variables of $(\mathbf{x}, \mathbf{x}_s)$, that will be called the n current **non-basic variables**.
- Write Z as an affine function of the n current basic variables. This is done by transvections to move the non-zero coefficients in the first line of the matrix above.
- Now we want to jump to an adjacent feasible vertex to increase Z 's value. That means increasing the value of one of the basic variables, while keeping all $n-1$ others to zero. Among the n candidates in the adjacent feasible vertices, we choose that of greatest positive increase rate in Z , also called **direction of highest gradient**.
 - The changing basic variable is therefore easily identified as the one with maximum positive coefficient in Z .
 - If there are already no more positive coefficients in Z affine expression, then the solution vertex has been found and the algorithm terminates.
- Next we need to compute the coordinates of the new vertex we jump to. That vertex will have one of the p current non-basic variables set to 0, that variable must be found among the p candidates. By convexity of the feasible polytope, the correct non-basic variable is the one that, when set to 0, **minimizes the change in the moving basic variable**. This is easily found by computing p ratios of coefficients and taking the lowest ratio. That new variable replaces the moving one in the n basic variables and the algorithm loops back to writing Z as a function of these.

3.2. Big-M Method

The Big-M Method for solving an LP problem can be summarized in the following steps:

Step 1: Express the LP problem in the standard form by adding surplus variables and artificial variables. Assign a zero coefficient to surplus variables and a very large positive number $+M$ (minimization case) and $-M$ (maximization case) to artificial variable in the objective function.

Step 2: The initial basic feasible solution is obtained by assigning zero value to original variables.

Step 3: Calculate the values of $c_j - z_j$ in last row of the simplex table and examine these values.

- (i) If all $c_j - z_j \geq 0$, then the current basic feasible solution is optimal.

(ii) If for a column, k , $c_k - z_k$ is most negative and all entries in this column are negative, then the problem has an unbounded optimal solution.

(iii) If one or more $c_j - z_j < 0$, then select the variable to enter into the basic with the large negative $c_j - z_j$ value. That is,

$$c_k - z_k = \text{Min}\{c_j - z_j; c_j - z_j < 0\}$$

The column to be entered is called key or pivot column.

Step 4: Determine the key row and key element in the same manner as discussed in the simplex algorithm of the maximization case.

3.3. Integer Programming

In the linear programming, each of the decision variable as well as slack and/or surplus variable is allowed to take any real or fractional value. However, there are certain practical problems in which the fractional value of the decision variables has no significance.

There are certain decision problems where decision variables make sense only if they have integer values in the solution. It can be classified into three categories:

- Pure (all) integer programming problems in which all decision variables are required to have integer values.
- Mixed integer programming problems in which some, but not all, of the decision variables are required to have integer values
- Zero-one integer programming problems in which all decision variables must have integer values of 0 and 1.

In this system, Pure integer programming is used to optimized the production model.

3.3.1 Gomory's All Integer Cutting Plane Method

It begins by solving the LP problem without taking into consideration the integer value requirement of the decision variables. If the solution so obtained is an integer, the current solution is the optimal solution to the given ILP problem. If some of the basic variables do not have non-negative integer value, an additional linear constraint called the Gomory constraint (or cut) is generated. After having generated a linear constraint (or cutting plane), it is added to the bottom of the optimal simplex table so that the solution no longer remains feasible. The new problem is then solved by using the dual simplex method.

Steps of Gomory's All Integer Programming Algorithm

Step 1 Initialization: Formulate the standard integer problem. If there are non-integer coefficients in the constraint equations, convert them into integer coefficients. Solve it by simplex method, ignoring the integer requirement of variables.

Step 2 Test the optimality: (a) Examine the optimal solution. If all basic variables (i.e. $x_{Bi} = b_i \geq 0$) have integer values, the integer optimal solution has been derived and the procedure should be terminated. The current optimal solution obtained in Step 1 is the optimal basic feasible solution to the integer linear programming.

(b) If one or more basic variables with integer requirements have non-integer solution values, then go to Step 3.

Step 3 Generate cutting plane: Choose a row r corresponding to a variable x_r , which has the largest fraction value f_r and generate the cutting.

Step 4 Obtain the new solution: Add the cutting plane generated in Step 3 to the bottom of the optimal simplex table as obtained in Step 3. Find a new optimal solution by using the dual simplex method, i.e. choose a variable to enter into the new solution having the smallest ratio: $\{(c_j - z_j) / y_{ij} ; y_{ij} < 0\}$ and return to Step 2. The process is repeated until all basic variables with integer requirements assume non-negative integer values.

4. Proposed System

This paper presents an Implementation of Manufacturing Industrial Management System using Linear Programming Models. Production model is generated based on resource constraints, costs and profits. Linear programming model (LP) is used to generate the optimized model. LP can be applied to various fields of study. Simplex method is used to solve the problem for the cases where there is no order constraint. For the order based production, Big M method is used. Integer Programming is added to produce integer results. Figure 1 presents the proposed system architecture.

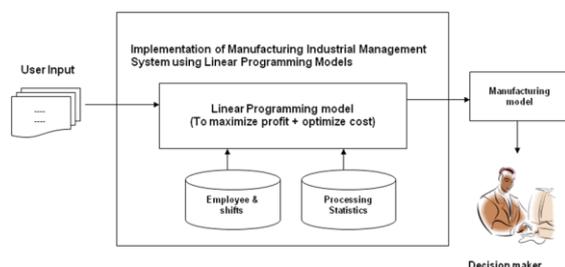


Figure 1. Proposed System Design

4.1 General Model for Car Production

There needs to be a balance between the total amount of production and its constraints. Simplex Method of Linear Programming Model is used to solve the problem.

It has the following steps

- Set up the initial solution
- Test for feasibility
- New Solution

Big M method is used to handle the order base model.

- Add slack, artificial variables
- Process as in Simplex Method

In case of results are in decimal forms, we need to apply them into integer programming model and it has the following steps –

- Finding the Cutting edge
- Generate New Solution

4.2. Standard Form

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following three parts. A linear function to be maximized

e.g maximized $C_1x_1 + C_2x_2$

Problem constraints of the following form

e.g $a_{11}x_1 + a_{12}x_2 \leq b_1$

$a_{21}x_1 + a_{22}x_2 \leq b_2$

$a_{31}x_1 + a_{32}x_2 \leq b_3$

Non-negative variables , $x_1 \geq 0, x_2 \geq 0$

The problem is usually expressed in matrix form

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

4.2 Modeling

A standard mathematical modeling technique called LP is applicable. The tree components of the system are as follows

Decision Variables

x_1 =units of Jeep to produce

x_2 =units of light truck to produce

x_3 =units of Saloon to produce

Result Variables

Total profit= Z

The objective is to maximize total profit

E.g $Z = 1000000 x_1 + 1200000 x_2 + 1300000 x_3$

Uncontrolled Variable Constraints

Labor constraint: $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \leq b_1$

Cost Constraints: $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \leq b_2$

Time Constraints: $a_{31}X_1 + a_{32}X_2 + a_{33}X_3 \leq b_3$
 Results for $x_1, x_2, x_3 \geq 0$ (in units)

4.3. Big-M Implementation

In handling orders, there are more constraints of basic constraints. In each order, there is one more constraint like $x_i \geq 5$. Simplex method does not work for the order processes like $x_i \geq 5$. BigM Method is used to solve this problem. It is a method of removing artificial variables from the basis. It assigns coefficients to artificial variables, undesirable from the objective function point of view.

5. System Implementation

This system is implemented with Java programming language. Jdk 1.5 is used to develop the system. Car production is used as case study for decision support system. Data are collected from the Car Production Industrial Zone, Yangon. User makes order entry and problem statements are stored as amount of time necessary for each department. This system can be run in two mode, order based and non-order base system. Big-M method is used for order based system and Simplex method is used in non-order based system. Number of car types is not fixed in this system. Possible car types are stored in system's database and new car types can also be added to this system. Following scenario shows the processing steps by simplex method and big M methods according to the example of Section 4.2.

Subject to the Constraints:

Budget Constraint

$$6,500,000 X_1 + 4,200,000 X_2 + 3,500,000 X_3 \leq 70,000,000$$

Labor Constraint

$$7 X_1 + 6 X_2 + 7 X_3 \leq 76$$

Time Constraints

$$15 X_1 + 14 X_2 + 12 X_3 \leq 164$$

Maximize Z

$$470,000X_1 + 420,000X_2 + 350,000X_3$$

Initial solution is as shown in Table 1, where last three zeros (000) are removed to save the placeholder.

Table 1: Initial Solution of Simplex Method.

Cj	250	240	243	0	0	0
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CB	Vb	sv	x1	x2	x3	s1	s2	s3	MR
0	S1	7000	650	420	350	1	0	0	10.76
0	s2	76	7	6	7	0	1	0	10.85
0	s3	164	15	14	12	0	0	1	10.93
Z=0		Zj	0	0	0	0	0	0	
		Cj - Zj	47	42	35	0	0	0	

Table 2 shows the final solution of above problem by simplex method.

Table 2: Final Solution by Simplex Method

		Cj							
		250	240	243	0	0	0		
CB	Vb	sv	x1	x2	x3	s1	s2	s3	MR
47	x1	10.361	1	0	0.89	0	-0.442	0	-
42	x2	0.46	0	1	2.2	0	0.68	0	-
0	s3	0.645	0	0	5.45	0	-2.89	1	-
Z=5050		Zj	47	42	-	-	-	-	-
		Cj - Zj	0	0	-	-	-	-	-

After applying the above solutions to the integer programming, the results would be

$$X_1 = 10$$

$$X_2 = 1$$

$$Z = 47,0000 \times 10 + 42,0000 \times 1 = 4,700,000 + 420,000$$

$$= 5,120,000$$

Therefore,

$$\text{Total Number of Jeep} = 10$$

$$\text{Total Number of Light Truck} = 1$$

$$\text{Profit} = 5,120,000$$

When decision has to be done on order, Big M method is used. For the following problem

$$\text{Maximize } Z = 10x_1 + 9x_2 + 20x_3$$

Subject to constraints

$$120 x_1 + 90 x_2 + 135 x_3 \leq 1800$$

$$9 x_1 + 10 x_2 + 12 x_3 \leq 100$$

$$10 x_1 + 8x_2 + 11x_3 \leq 120$$

$$x_1 \geq 5$$

$$x_2 \geq 4$$

The standard form will be as follows:

$$\text{Maximize } Z = 10x_1 + 9x_2 + 20x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 - MA_1 - MA_2$$

$$120x_1 + 90x_2 + 135x_3 + s_1 = 1800$$

$$9x_1 + 10x_2 + 12x_3 + s_2 = 100$$

$$10x_1 + 8x_2 + 11x_3 + s_3 = 120$$

$$x_1 - s_4 + A_1 = 5$$

$$x_2 - s_5 + A_2 = 4$$

Table 3 shows the initial solution by Big M method, where slack variables and artificial variables are added to the solution.

Table 3: Initial Solution by Big M method

		Cj	10	9	20	0	0	0	0	0	0	0	-M	-M	
CB	Vb	sv	x1	x2	x3	s1	s2	s3	s4	s5	A1	A2	MR		
0	s1	1800	120	90	135	1	0	0	0	0	0	0	15		
0	s2	100	9	10	12	0	1	0	0	0	0	0	100/9		
0	s3	120	10	8	11	0	0	1	0	0	0	0	12		
-M	A1	5	1	0	0	0	0	0	-1	0	1	0	5		
-M	A2	4	0	1	0	0	0	0	0	-1	0	1	Inf		
Z=0	Zj	-M	-M	0	0	0	0	0	M	M	-M	-M			
	Cj	10													
	Zj	-M	9+M	20	0	0	0	0	M	M	0	0			

Table 4 is the final solution by Big M method.

Table 4: the final solution by Big M method

		Cj	10	9	20	0	0	0	0	0	-M	-M	M
CB	Vb	sv	x1	x2	x3	s1	s2	s3	s4	s5	A1	A2	R
0	S1	67	1.2	0	0	1	8	0	75	05	75	05	-
20	X3	0	0	0	1	0	0.0	0.7	0.8	0.7	0.8	-	-
0	S3	24.	25	0	0	0	0.8	1.7	1.1	1.7	1.1	-	-
10	x1	5	1	0	0	0	0	0	-1	0	1	0	-

9	x2	4	0	1	0	0	0	0	0	0	-1	0	1	-
Z=86	Zj	10	9	20	0	0	0	0	0	0	-	-M	-M	-
	Cj	-Zj	0	0	0	0	0	0	0	0	-1	-	-	-

After applying the above solutions to the integer programming, the results would be

$$X_1 = 5$$

$$X_2 = 4$$

$$Z = 5 \cdot 10 + 9 \cdot 4 = 50 + 36 = 86$$

Therefore,

$$\text{Total Number of Jeep} = 5$$

$$\text{Total Number of Light Truck} = 2$$

$$\text{Profit} = 122, (00000) \text{ K.}$$

6. Conclusion

The perceived benefits of using Linear Programming in production environment, improved communication cost reduction, increased productivity, time saving and improved customer and employee satisfaction. By using this system, managers can track the various business operations. This system provides special analysis of profitability and can control the overall production operation. The system enable the compression of time to decide. By using the optimization model, the alternatives can be generated automatically by changing the uncontrollable variable. New products come from time to time so this system is adaptable with new products.

7. References

- [1] Senouci A.B. and Eldin N.N., "A time-cost trade-off algorithm for nonserial linear projects", Canadian J. of Engineering, Vol.23, (1996), pp.134-149.
- [2] Birrell G.S., "Construction Planning-Beyond the Critical Path", J. of the Construction Division, Vol. 106, N° Co3, (1980), pp 389-407.
- [3] Selinger S., "Construction planning for linear projects", ASCE Journal of the Construction Division, Vol. 106, 2, (1980), pp.195-205.
- [4] Russel A.D. and Caselton W.F., "Extensions to linear scheduling optimization", ASCE Journal of Construction Engineering and Management, Vol.114, 1, (1988), pp.36-52.

- [5] Reda R.M., "RPM: repetitive project modeling", ASCE Journal of Construction Engineering and Management, 116, 2, (1990), pp. 316-330.
- [6] Elmaghraby S.E. and Pulat P.S., "Optimal Project Compression with Due-Dated Events", Naval Research Logistics Quarterly, Vol.26, N°2, (1997), pp. 331-348.
- [7] Moore L.J., Taylor III B.W., Clayton E.R., and Lee S.M., "Analysis of a Multi-Criteria Project Crashing Model," American Institute of Industrial Engineering Trans., Vol. 10, N°2, (1998), pp. 163-169.
- [8] Smith L.A., "Comparing Commercially Available CPM/PERT Computer Programs", Journal of Industrial Engineering, Vol. 10, N°4, (1997).
- [9] Schrage L.E., User's manual for LINDO; Scientific Press, Palo Alto, California, (1981).
- [10] Al Serraj Z.M., "Formal development of line-of-balance technique", ASCE Journal of Construction Engineering and Management, Vol.116, 4, (1990), pp. 689-704.
- [11] Savin D., Alkass S., and Fazio P., "Construction resource leveling using neural networks", Canadian J. of Engineering, Vol.23, (1996), pp.917-925.
- [12] Siemens N., "A Simple CPM Time-Cost Trade-off Algorithm," Management Science, Vol.17, N°6, (1997), pp.B354-363.