A Linear Programming Model for Optimum Resource Allocation

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Abstract

A linear programming model is built to explore the possibilities of optimizing farm returns, considering crop activities for selected crop region. To determine the optimum organization of crops, three districts in Ayeyarwady division, namely Pathein district, HinThaDa district and MyaungMya district, are chosen. As the selected crops, nine different crops which covered 2% of the total cultivated area of the division during the last year are included in the model. Due to the double cropping practice of the division, the crops in question are considered separately as the rainy crops and the winter crops. The rainy crop are AyeYarMin rice, MeeGout rice, peanut, sunflower and corn. As the winter crops, ShweThweYin rice, MaNawThuKha rice, peanut, sunflower, sorghum, block matpe, green gram, cow bean, and boquake bean are selected. The system will also suggest the farmers the best cropping patterns to earn the maximum profit based on his choice of crop combination.

1. Introduction

Agricultural planning problems are important from both social and economic points of view. They involve a complex interaction of nature and economics. Nowadays, the population of the world has been increasing with the steady rate. Due to the increase in population, there is a need of more production of food to meet the ever increasing demand and requirement of raw materials for various industries arising out of steady population growth.

For the fulfillment of above purposes, productivity growth in agriculture must be considered essential. One way of achieving high productivity is to increase the cultivated areas of crops. However, third world countries like India and others are losing land due to population growth and industrialization. Because of this factor, the production of crop per unit area must be increased by the proper utilization of available resources.

Planning of crops is the most crucial factor of agricultural planning. Crops' planning depends on several resources like the availability of land, water, labor, and capital. It also requires consideration of methods of irrigation, soil characteristics, cropping pattern, cropping intensity, social economic conditions, climate, and many other factors.

Our country, Myanmar, is an agricultural country. 58% of the populations of the country are peasants and farmers. The whole economic building of the country is based on the agricultural section. There is no industrial production that does not rely, directly or indirectly, on agriculture. Moreover, the population of the country is now seventy millions. For the ever increasing population, food security is the extremely important. So, a system for agricultural planning is required in order to raise the productivity of agricultural section.

Ayeyarwady division is located in the lower part of the country and has area of 13567 square miles and comprises five districts. The division is so rich in rivers and streams that it is very good for growing various crops especially rice. The division is responsible for the 31% of the rice production of the whole country. Thus, it is called “granary or the great rice pot” of Myanmar country.

Farmer' profit cannot be maximized without optimum cropping pattern which ensures efficient utilization of available resources. To maximize his profit, the farmer might choose among the available production alternatives, the most efficient method in the use of productive resources and the one which satisfies the previously- stated goals.

In the cases where the decision is related to the allocation of scarce resources, the farmer’s responsibility is to find efficient methods that can help him to make the right decision. To solve this problem, the mathematical programming models are the most recommended. The main objectives of the system in hand were (i) to develop a linear programming model which determines an optimum crop organization in the selected zones in Ayeyarwady Division.(ii) to estimate how much acreage of the principal crops should be grown in the chosen areas to earn maximum profit.

2. Background theory

Linear programming (LP) is the general technique of optimum allocation of 'scarce' or 'limited' resources such as labor, material, machine, capital, energy, etc. to several competing activities, such as products, services, jobs, new equipment, projects, etc. on the basis of a given criterion of optimality. The term 'limited' here is used to describe availability of scarce resources during planning period. The criterion of optimality generally is either performance, return on investment, utility, time, distance, etc.

The development of linear programming has been ranked among the most important scientific advances of the mid-20^{th} century. Today, it is a standard tool that has saved many thousands or millions of dollars for most companies or businesses of even moderate size in the various industrialized countries of the world; and its use in other sectors of society has been spreading rapidly.

Linear programming is a child of modern mathematics; it is now about 20 years old as an operational tool for problem solving, but its origins go back beyond this in the scientific literature of both mathematics and economics. The early applications were primarily those in military operations. But, now, it is being used for a wide range of problem from diet planning, oil refining, education, energy planning, pollution control, transportation planning and scheduling to research and development and almost in all functional areas of management production, finance, marketing and personnel.

Linear programming uses a mathematical model to describe the problem of concern. The adjective *linear* means all the mathematical functions in this model are required to be linear functions. The word *programming* does not refer here to computer programming; rather, it is essentially a synonym for *planning*. This word is used to specify a sort of planning that involves the economic allocation of limited resources by adopting a particular course of action of strategy amongst various alternative strategies to achieve the desired objective.

2.2 Assumptions of linear programming

Certainty: The value assigned to each parameter of a linear programming model is assumed to be a known constant.

Additivity: The value of the objective function for the given values of decision variables and the total sum of resources used, must be equal to the sum of the contributions (profit or cost) earned from each decision variable and the sum of the resources used by each decision variable respectively.

Divisibility: Decision variables in a linear programming model are allowed to have any values, including non-integer values that satisfy the functional and non-negativity constraints. Thus, these variables are not restricted to just integer values. Since each decision variables represents the level of some activity, it is being assumed that the activities can be run at fractional levels.

Linearity: All relationships in the LP model (i.e. in both objective function and constraints) must be linear. In other words, for any decision variable, the amount of particular resource say *i* used and its contribution to the cost in objective function must be proportional to its amount.

2.3 General mathematical model of LP

The general linear programming problem with n decision variables and m constraints can be stated in the following form:

Find the values of decision variables $x_1, x_2, ..., x_n$ so as to

$$\text{Optimize (Max or Min) } Z = c_1 x_1 + c_2 x_2 + ... + c_n x_n$$

subject to the linear constraints

$$\begin{array}{l} a_{11} x_1 + a_{12} x_2 + ... + a_{1n} x_n \;\; (\le, =, \ge)\; b_1 \\ a_{21} x_1 + a_{22} x_2 + ... + a_{2n} x_n \;\; (\le, =, \ge)\; b_2 \\ \vdots \\ a_{m1} x_1 + a_{m2} x_2 + ... + a_{mn} x_n \;\; (\le, =, \ge)\; b_m \end{array}$$

and $\quad x_1, x_2, ..., x_n \ge 0$ (non-negativity constraints)

Here, the c_j's are coefficients representing the pre unit contribution of decision variable x_j, to the value of objective function. The a_{ij}'s are called the technological coefficients or input-output coefficients. These represent the amount of i^{th} resource consumed per unit of variable x_j. In the given constraints, the a_{ij}'s can be positive, negative or zero. The b_i represents the total availability of the i^{th} resource. The term resource is used in a very general sense to include any numerical value associated with the right hand side of a constraint. It is assumed that $b_i \ge 0$ for all i. However, if any $b_i < 0$, then both sides of constraint i can be multiplied by -1 to make $b_i > 0$ and reverse the inequality of the constraint.

2.4 Linear program in standard form

The use of the simplex method to solve an LP problem requires that the problem be converted into its standard form. Except nonnegativity constraints, all other constraints must be equalities.

If $\sum_{j=1}^{n} a_{ij}x_j \le b_i$, then introduce new variable s (slack variable): $\sum_{j=1}^{n} a_{ij}x_{ij} + s_i = b_i$ and $s_i \ge 0$. ($i=1,2,\ldots,m$).

If $\sum_{j=1}^{n} a_{ij}x_j \ge b_i$, then add surplus (negative slack)variables and artificial variables not to violate non-negativity conditions of surplus variable (i.e. $s_i \geq 0$) and set:

$$\sum_{j=1}^{n} a_{ij}x_j - s_i + A_i = b_i$$

and $x_j, s_i, A_i \geq 0$. ($i=1,2,\ldots,m$).

Initially, $s_1, s_1, \ldots, s_m$ basic variables, $x_1, x_1, \ldots, x_n$ non-basic variables.

2.5 Related works

Saad and Marino (2000) designed a micro-irrigation system with tapered manifold lines in the downhill condition, manimizing the annuals equivalent cost of the hydraulic network and energy annual cost assuring the maximum variation in the pressure head previously established by linear programming. The model proved to be efficient in the design of the irrigation systems in terms of emission uniformity desired.

B.Kareem and A.A. Aderoba formulated a linear programming to analyse the matainance related data of a cocoa processing industry in kure,Ondo State of Nigeria. The data were collected, lassified and analysed statistically.The data analysed includes maintenance budget, maintenance cycle, production capacity and waiting time of production facilities in case of failure. Data were analysed based on manpower cost, machine depreciation cost and the spare part cost, which were assumed to be proportion to the number/magnitude of the breakdowns.

One of the most successful techniques in the design and analysis of approximation algorithms for combinatorial optimization problems has been to first solve a relaxation of the problem, and then to round the optimal solution to the relaxation to obtain a near-optimal solution for the original problem. Although the relaxation used varies from problem to problem, linear programming relaxations have the provided the basis of approximation algorithms for a wide variety of problems.

3. System implementation

Linear programming technique was applied to build a model to calculate the best crop acreage which yields the maximum profit based on the farmers' choice of nine different kinds of crops theirs available resources and to suggest the optimum cropping pattern. The model is solved by using simplex method, a remarkably efficient method of linear programming.

The objective of the model was to maximize total net income (gross margin).Algebraically the model is summarized below:

3.1 Basic assumptions:

(a) The economic objective of the producer is to maximize profit, i.e. gross margin.
(b) Crops covering up-to 2% or above of the total cropped area were included in the model.
(c) Crops are considered as rainy and winter crops.
(d) Total production of various commodities is limited by the resources availability in the District.
(e) An acre of production can be substituted for an acre of other type of production.
(f) Farm labor supply does not pose limitation on crop production.

3.2 Mathematical presentation of the model

The Linear programming model of the following form is used as an analytical tool to explore the possibilities of optimizing farm returns.

The objective function was to maximize profit,

Maximize $Z \text{ (profit)} = \Sigma_{i=1}^{m} \Sigma_{j=1}^{n} C_{ij} X_{ij}$

Subject to the constraints

1. Land availability within each region

$$a_{ij}X_{ij} \leq L_j \quad \text{for all j} \quad (1)$$

2. Capital availability for each region

$$k_{ij} X_{ij} \leq K_j \quad \text{for all j} \quad (2)$$

3. Water availability for each region

$$w_{ij} X_{ij} \leq W_j \quad \text{for all j} \quad (3)$$

4. Limit of acreage for each crop

$$a_{ij} X_{ij} \leq Max_i \quad \text{for all i} \quad (4)$$

Non-negativity constraint

$$X_{ij} \geq 0 \quad i = 1,2\ldots,m. \ j = 1,2,\ldots,n.$$

Where,
Z = (Selling price *Yield – Cost price) x (Acreage)
i = 1, rice (MaNawThuKha, ShweThweYin)
i = 2, rice (AyeYarMin, MeeGout)
i = 3, peanut
i = 4, sunflower
i = 5, sorghum
i = 6, block matpe
i = 7, green gram

i = 8, cow bean
i = 9, boquake bean
j = 1, Pathein District
j = 2, HinThaDa District
j = 3, MyaungMya District
C_{ij} = net return per acre of crop i in region j
X_{ij} = number of acreage to be allocated to plant crop i in region *j*
a_{ij} = amount of land needed per unit of crop i in region j
L_j = total land availability in j^{th} region
k_{ij} = amount of capital required per acre of crop i in region j
K_j = total amount of capital availability for j^{th} region
w_{ij} = water consumption of crop i in j^{th} region
W_j = total amount of water available for j^{th} region
Max_i = maximum acreage of crop i in j^{th} region
m = no. of selected crops
n = no. of selected regions

3.3 General overview of simplex method

Step 1. Formulate the mathematical model and construct the initial simplex tableau.
Add slack variables to represent unused resources thus eliminating inequality constraint. Construct the simplex table that allows to evaluate various combination of resources to determine which mix will most improve the solution.

Step 2. Find the sacrifice and imporvement rows.
Values in the sacrifice rows indicate what will be lost in per-unit profit by making a chanhe in resource allocation mix. Values in the sacrifice rows indicate what will be gained in per-unit profit by making a chanhe in resource allocation mix.

Step 3. Apply the entry criteria.
Find the entering variable. The entering variable is defined as the current non-basic variable that will almost the objective if its value is increased from 0.

Step 4. Apply the exit criteria.
Using the current tableau's exchange coefficient in entering variable column, calculate the following exchane ratio for each row as:

solution value/exchange coefficient

Find the lowest nonzero and nonnegative value. The basic varialbe in this row becomes the exiting varialbe.

Step 5. Construct the new simplex tableau.
To construct the new tableau, replace the exiting varialbe in the basic mix column with the new entering varialbe. Change the unit profit or unit loss column with the value for new entering variable. Compute the new row values to obtain the new set of exchange coefficients applicable to each basic varialbe.

Step 6. Repeat the step 2 to step 5 until the solution can be no longer improved.

4. Experimental result

A cooperative firm operates its own firm of 100 acres and has the fund 30,000,000 kyats .The output of the farm is limited both by the usable acreage and irrigation water and by the capital available of the firm.
Furthermore, the firm has a maximum quota for the total acreage that can be devoted to each of the crops set by the Ministry of Agriculture and Irrigation.

Table 1. Maximum quota for each crop

Crop	maximum quota (Acres)	water consumption (ac-ft)
MaNawThuKha rice	20	70
ShweThweYin rice	20	70
Peanut	15	0
Sunflower	10	0
Sorghum	8	0
Block matpe	10	0
Green gram	8	0
Cow bean	7	0
Boquake bean	10	0

The total available water of the firm is 2300 ac-ft.
The firm is considering for planting nine winter crops for the upcoming season.
The estimated yield per acre and selling price of each crop during the last year and cost per acre are given in the following table.

Table 2. Estimated yield and cost per acre and selling price of each crop

Crop	Estimated yield (Tin/acre)	selling price (K/acre)	Cost (K/acre)
MaNawThuKha	95	2,000	120,000
ShweThweYin	90	2,500	120,000
Peanut	55	9,000	140,000
Sunflower	31	5,000	75,000
Sorghum	80	7,000	360,000
Block matpe	19	6,000	54,000
Green gram	18	7,000	75,000
Cow bean	17	5,000	35,000
Boquake bean	16	5,000	30,000

The firm wishes to know how many acres of each crop should be planted at its land to maximize the expected profit.

4.1 Optimum solution

Optimal cropping pattern resulting from the gross margin driven LP model for the firm are presented in Table 3.

Table 3. Optimum cropping pattern

Crop	acreage	profit (kyats)
MaNawThuKha	20	1,400,000
ShweThweYin	12.85	1,349,250
Peanut	15	5,325,000
Sunflower	10	800,000
Sorghum	8	1,600,000
Block matpe	10	600,000
Green gram	8	408,000
Cow bean	7	350,000
Boquake bean	9.15	457,500
Total	100	12,289,750

In the optimum solution, all total available acreage of the firm, 100 acres, is efficiently used by each crop in accordant with the available water and capital resources and limited acreage. The firm investment is 30,000,000 kyats and it has the maximum profit 12,289,750 kyats. The water availability of the firm is 2300 ac-ft and the optimum cropping pattern uses 2299.5 ac-ft efficiently.

5. Conclusion

Linear programming is a powerful technique for dealing with the problems of allocating limited resources among competing activities as well as other problems having a similar mathematical formulation.
In agriculture, it has been used since long. Linear programming models were specified to solve many agricultural problems.

This system will built linear programming model to determine the most efficient pattern of agricultural production for the most commercial crops in the selected regions. It can be used by the individual farmers and cooperative farm who wish to know the optimum cropping pattern and how to allocate their resources.

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