INVESTIGATION OF TRANSITION ENERGY FROM SUB-BARRIER TO DEEP SUB-BARRIER ENERGIES

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Abstract

The recent experimental data of heavy-ion fusion cross sections available up to deep-subbarrier energies in order to discuss the threshold incident energy for a deep-subbarrier fusion hindrance phenomenon have been studied. One-dimensional potential model with a Woods-Saxon internuclear potential had employed. Fitting the experimental data in two different energy regions with different Woods-Saxon potentials, we define the threshold energy as an intersection of the two fusion excitation functions. It is found that the threshold energies so extracted are in good agreement with the empirical systematics as well as with the values of the Krappe-Nix-Sierk (KNS) potential at the touching point. We also discuss the asymptotic energy shift of fusion cross sections with respect to the potential model calculations, and show that it decreases with decreasing energies in the deep-subbarrier region, although it takes a constant value at subbarrier energies.

Introduction

Fusion is a reaction in which two separate nuclei combine together to form a compound nucleus. Theoretically, it has been considered that because of a strong cancellation between the repulsive Coulomb interaction and an attractive short range nuclear interaction between the colliding nuclei, a potential barrier, also called Coulomb barrier, is formed, which has to be surmounted in order for fusion to take place [Hagino K et al].

Heavy-ion fusion reactions at low incident energies provided a good opportunity to study the quantum tunnelling phenomena of many-particle systems. Recently, fusion cross sections have been measured for the first time at deep sub-barrier energies for medium-heavy mass systems, such as ^{64}Ni+^{64}Ni, ^{58}Ni+^{58}Ni, ^{64}Ni+^{89}Y and it was pointed out that fusion cross sections show an unexpected

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behaviour at very low energies (deep sub-barrier energy region) with a much steeper falloff than obtained in conventional

coupled channels calculations or from Wong formula. This motivates us to investigate the asymptotic energy shift of fusion cross sections with respect to the single-channel calculations, which provides a measure of enhancement of sub-barrier fusion cross sections [Ichikawa T et al].

In this study, we investigate the onset (threshold energy) for the deep sub-barrier hindrance with the asymptotic energy shift of fusion cross sections using recent available experimental data.

Calculation of Fusion Cross Section

The Schrodinger equation in three dimensions with a potential given by

$$V(r) = V_N(r) + V_C(r) + V_l(r) \tag{1}$$

yields

$$[-\frac{\hbar^2}{2\mu}\nabla^2 + V(r) - E]\Psi(r) = 0 \tag{2}$$

where μ = reduce mass.

In the absence of the potential, we consider the plane wave $\psi(r) = \exp(i\boldsymbol{k}.\boldsymbol{r})$

$k = \sqrt{\frac{2\mu E}{\hbar^2}}$ = wave vector

It can be expanded in the complete set of Lengendre polynomials $P_l(\cos\theta)$ as an asymptotic form of

$$\psi(r) = e^{i\vec{k}.\vec{r}} \rightarrow \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)i^{\ell}\left(\frac{e^{-i(kr-\frac{l\pi}{2})}}{r} - \frac{e^{i(kr-\frac{l\pi}{2})}}{r}\right)P_l(\cos\theta),\ r \longrightarrow \infty \tag{3}$$

where θ = the angle between $\boldsymbol{r}$ and $\boldsymbol{k}$.

In the presence of the potential, replacing the plane waves with the corresponding Hankel functions obtained by Coulomb interaction,

$$\psi(r) = \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)i^{\ell}\left(\frac{H_{\ell}^{(-)}(kr)}{r} - S_{\ell}\frac{H_{\ell}^{(+)}(kr)}{r}\right)P_{\ell}(\cos\theta),\ r \longrightarrow \infty \tag{4}$$

where $H_l^{(+)}$ (kr) and $H_l^{(-)}$ (kr) are the outgoing and the incoming Coulomb wave function, respectively. S_l is called the *S*-matrix an is in general a complex quantity. Fusion reaction can be regarded as absorption of the incident flux and the difference of total radial flux between the incoming and the outgoing wave is evaluated from Eq. (3) as

$$J_{in} - J_{out} = \frac{\hbar\pi}{\mu k}\sum_{\ell}(2\ell+1)(1-|S_\ell|^2) \tag{5}$$

In evaluating Eq. (4), the radial flux has been integrated for all possible values of 'θ'. Divided by the incident flux, the fusion cross section is then given by

$$\sigma_F(E) = \frac{\pi}{k^2}\sum_{l}(2l+1)(1-|S_l|^2) \tag{6}$$

In heavy ion fusion reactions, incoming wave boundary condition (IWBC) is often applied with keeping the potential real. Under the incoming wave boundary condition, the wave number for the l-th partial wave, which is defined by

$$k_\ell(r) = \sqrt{\frac{2\mu}{\hbar^2}(E - V_0(r) - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2})}\,. \tag{7}$$

Then, Eq. (6) is transformed to

$$\sigma_F(E) = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)P_\ell(E) \tag{8}$$

where $P_l(E)$ is the penetrability for the *l*-wave scattering defined as

$$P_\ell(E) = 1 - |S_\ell|^2 \tag{9}$$

Threshold Energy of Fusion Cross Section at Deep Sub-barrier Energy Region

Behaviour of fusion cross section at this energy regions is illustrated well by the logarithmic derivatives *[L(E) = dln(σE)/dE]* and also possible in terms of an S factor. Historically, the S factor was introduced as a useful way of parameterzing cross sections for radioactive capture, and for light-ion fusion reactions [Ichikawa T et al]. It is defined in terms of the fusion cross section σ as,

$$S(E) = E\sigma(E)\exp(2\pi\eta) \tag{10}$$

where $\eta = Z_1 Z_2 e^2/(\hbar\nu)$ = Sommerfeld parameter, ν = beam velocity.

Fusion behaviours in deep sub-barrier energy region of interest have been analyzed by using heavy-ion fusion excitation functions in terms of the *S* factor by Jiang *et al.*,. It had been shown that the steep falloff in cross section observed in heavy-ion systems translates into the maximum of the S factor [Jiang C.L *et al*].

The relation between the S factor and logarithmic derivative (Eq. 3.1) of the low-energy fusion data can be related by the derivative of the S factor.

$$\frac{dS}{dE} = S(E)\left[L(E) - \frac{\pi\eta}{E}\right] \tag{11}$$

A maximum in the S factor implies that the derivative of S factor $dS/dE = 0$. This is fulfilled when the logarithmic derivative is

$$L_{CS}(E) = \frac{\pi\eta}{E} = \frac{\pi Z_1 Z_2 e^2}{E^{3/2}}\sqrt{\frac{m_N}{2}\frac{A_1 A_2}{A_1 + A_2}} \tag{12}$$

This function is the logarithmic derivative for a constant S factor. The logarithmic derivative *L(E)* extracted from the experimental data will intersect the curve $L_{cs}(E)$ exactly at the energy where the experimental S factor exhibits a maximum.

Jiang *et al.*, defined the maximum of experimental *S* factor or the energy at intersection point is the threshold energy E_s, used to characterize the unexpected steep falloff of the measured fusion cross sections. They also proposed a simple empirical formula [Ichikawa T *et al*].

$$E_S = 0.356\left[Z_1 Z_2 \sqrt{A_1 A_2/(A_1 + A_2)}\right]^{2/3} \tag{13}$$

In this study, the threshold energy E_s is determined by finding the intersection of calculated fusion cross section fitted to reproduce the data in the sub-barrier and deep sub-barrier energy region. In Fig. 1, the solid line is obtained by fitting the calculated fusion cross section with experimental data in the range of 10^{-2} *mb* to $10^{\mathbf{0}}$ *mb*. Then, fitting the data below 10^{-3} *mb* or lower energy region gives the dashed line. The energy at the intersection point of the two curves is defined the threshold energy or onset of sub-barrier hindrance.

Threshold energies for several systems are shown in Fig. 2. The empirical values proposed by Jiang, *et. al.*,(Eq. 13) is also shown by solid line for comparison. Stars are available experimental data extracted from maximum of astrophysical S-factor. All the results are summarized in Table 1.1. It can be seen that the calculated threshold energy is in good agreement with those estimated from the maximum of astrophysical *S*-factor.

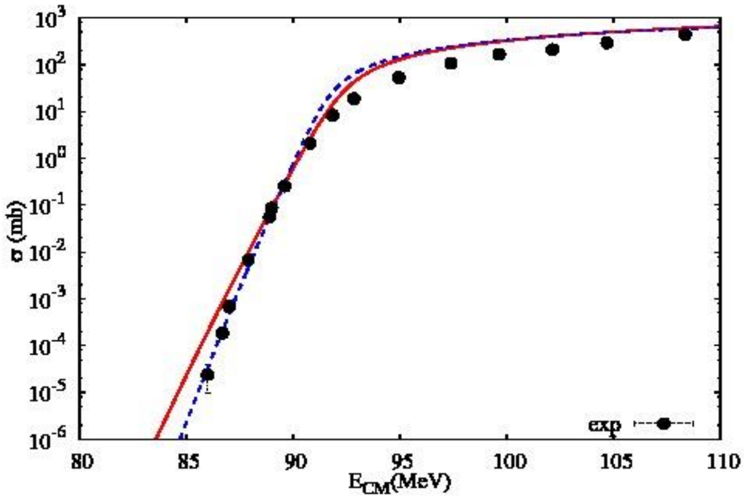

Fig. 1 Illustration of the determination of threshold energy by intersection point of the calculated fusion cross section fitted in the sub-barrier and deep sub-barrier energy region for ^{64}Ni+^{64}Ni.

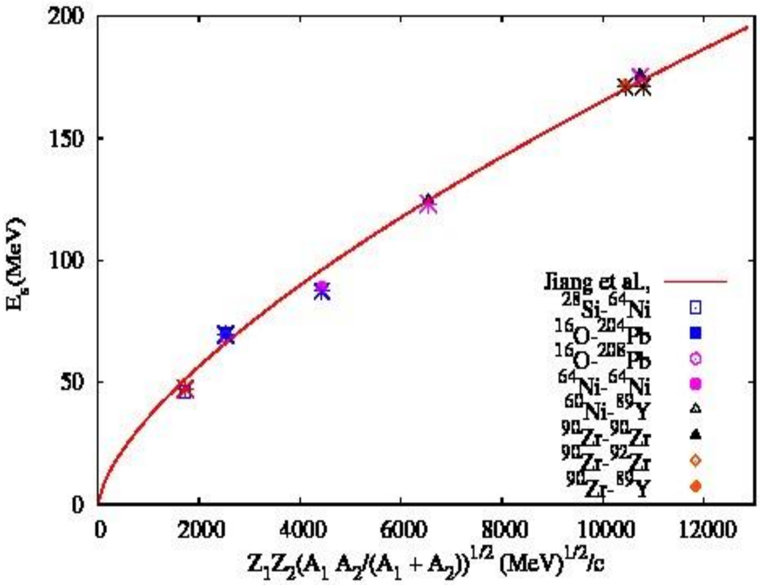

Fig. 2 The energy E_s where the intersection point of two curves as a function of the parameter, $\xi = Z_1Z_2\ (A_1A_2/A_1+A_2)^{1/2}$. The solid line is calculated with the empirical formula $E_s = 0.356\ \{Z_1Z_2\ (A_1A_2/(A_1+A_2))^{1/2}\ \}^{2/3}$ *MeV*. Stars are available experimental data extracted from maximum of S-factor.

Table 1.1 The experimental threshold energy E_s, empirical energy ($E_s = 0.356 \{Z_1Z_2 (A_1A_2/(A_1+A_2))^{1/2}\}^{2/3}$ *MeV* by Jiang *et al*), our calculated threshold energy E_s and $\xi = Z_1Z_2 (A_1A_2/A_1+A_2)^{1/2}$ are summarized.

system	E_s (exp)	E_s (emp)	E_s (our work)	ξ
$^{28}Si+^{64}Ni$	47.3±0.9	51.3	46.2	1730
$^{16}O+^{208}Pb$	69.6	66.1	71.1	2529
$^{64}Ni+^{64}Ni$	87.3±0.9	96.1	88.9	4435
$^{60}Ni+^{89}Y$	123±1.2	124.5	124.5	6537
$^{90}Zr+^{89}Y$	171±1.7	170.3	171.8	10436
$^{90}Zr+^{90}Zr$	175±1.8	173.2	176.1	10733
$^{90}Zr+^{92}Zr$	171±1.7	173.9	171.7	10792

Summary and Conclusion

In summary, we have studied the energy dependence of heavy-ion fusion cross sections at deep sub-barrier energies using the recent experimental data. To this end, we employed a one-dimensional potential model. In order to see at which energy the deep sub-barrier hindrance takes place, we estimated the threshold energy with a two-slope fit procedure. That is, we defined the threshold energy as an intersect of two fusion excitation functions, which fit the experimental fusion cross sections either in the sub-barrier energy region or in the deep sub-barrier energy region. We have shown that the threshold energies so defined are in good agreement with those estimated from the maximum of astrophysical S-factor.

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