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Analysis and Simulation of Sinusoidal PWM Technique for Sine Wave Inverter

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***Abstract* — A sinusoidal pulse width modulation technique of generating sine wave with minimized harmonics is implemented in this paper. The technique is essentially focused on sinusoidal-pulse-width modulation (SPWM) control of H-bridge DC-AC inverter stage connected in series between high voltage DC links (Vs) and output (V_O). It also provides the producing of SPWM signals to control the H-bridge inverter driving stage such as single phase or three phases. This technique is mathematically modelled and simulated in Matlab. Finally, the results are verified that the output sine wave is voltage regulated and ripple less. The proposed technique “generation of sinusoidal PWM by comparing a sinusoid with a triangular wave” aims to implement of a voltage regulated sine wave inverter with ripple free and glitch free output sine wave that can be operated electronic devices efficiently.**

***Keywords*— sine wave inverter, sinusoidal-pulse-width modulation, H-bridge, single-phase DC-AC inverter, Matlab simulink.**

I. INTRODUCTION

Energy crisis are of special attention now-a-days. A need for reasonable power rating inverter is required to smoothly operate electrical and electronic appliances.There are different topologies for implementing sine wave inverter as discussed in [1]. Sine wave inverter is widely used in many commercial and industrial applications including uninterruptable power supplies, induction heating, variable frequency drives, electrical vehicle drives and HVDC links. The design of an inverter using H-bridge topology [2] is shown in Fig. 1.

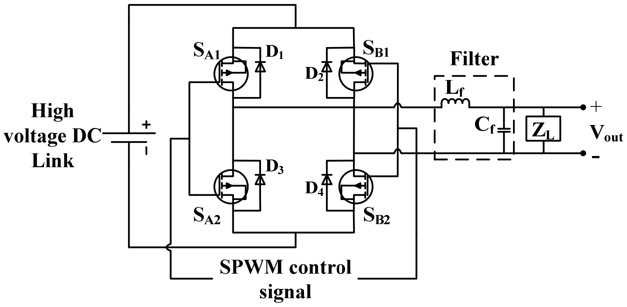

Fig. 1 Design of an H-bridge inverter

The sine wave inverter design is involved by using rectification and inversion modes which include bridge technique, PWM technique, converter or a transformer, output filter and a feedback loop for voltage regulation. There are many methods of generating PWM. Most common of these are comparing a sinusoid with a triangular wave.

II. DESCRIPTION OF SINUSOIDAL PULSE WIDTH MODULATION (SPWM)

The concept underlying SPWM is to build up the total waveform of the AC output voltage by means of multiple pulses, with the pulse widths distributed sinusoidally. The shaping of the output voltage waveform is generally achieved by having multiple pulses in each half-period. The theoretical waveform of a SPWM control is sketched in Fig. 2.

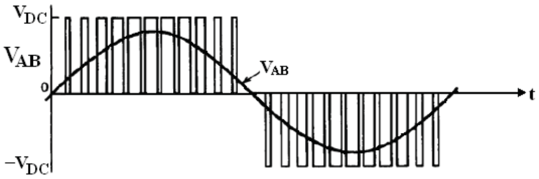

Fig. 2 Theoretical waveforms of SPWM control.

In each half-period, the pulse width is maximum width in the middle. From the center, the pulse widths decrease as cosine function towards either side. In the SPWM technique, voltage control is implemented by varying the widths of all the pulses, at the same time maintaining the cosine relationship. If a pure sine wave is required for the output voltage, it can be used the SPWM waveform whereas the total harmonic content is still very significant. The order of the harmonics in the SPWM waveform depends on the number of pulses per half-cycle employed. If SPWM is implemented in an inverter with a large number of pulses per half-cycle, the harmonic frequencies will be so large that for many applications, such as motor speed control, no separate filter may be needed on the output side. If a clean sine wave is required for the output voltage, this can be achieved by a filter consisting of only small values of L and C.

A. H-Bridge Inverter

The H-bridge DC-AC inverter is shown in Fig.1. The power supply is V_{DC} from DC-DC link voltage source converter. This inverter is composed of H-connection MOSFETs and LC output filter.

The H-connection has four MOSFET switches and four diodes as shown in Fig. 1. D_1~D_4 are anti-parallel diodes to protect S_{A1}~S_{B2}. The main operation is to generate the driven signals of S_{A1}~S_{B2} by using SPWM, and then the output waveform V_O is desirable to be a regulated sinusoidal wave. In the effective positive-half-cycle, let S_{A1} and S_{B2} be turn on, and then the output V_O can be obtain the positive voltage value. Similarly, let S_{B1} and S_{A2} be turn on, as the negative value of V_O can be obtained. All about SPWM operation is shown in Fig.3. When the switches (S_{A1}~S_{B2}) be turn on or turn off in the short times, a large rush voltage is occurred. If the large voltage exceeds the safe operating value, then the switches could be damaged. So, the anti-parallel diodes are needed to protect the switches, i.e. reduce switching loss and rush voltage.

At the output terminal, there is a low-pass LC output filter. It is composed of filtering inductor L and capacitor C, which are connected with the load in series-parallel. Its main function is to reject high-frequency harmonic of V_O so as to obtain the lower value of total-harmonic-distortion (THD).

B. Generation of Sinusoidal PWM signal

In the sinusoidal PWM generation scheme, the PWM technique based on the comparison method of a sinusoid with a triangular wave is illustrated as shown in Fig. 3.

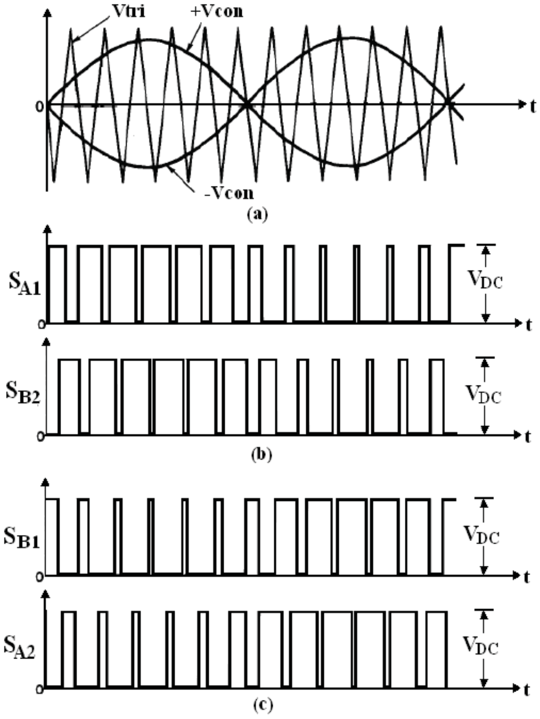

Fig. 3 Theoretical switching stage waveforms of SPWM control.

It is used two voltage waveforms: a sinusoidal (V_{con} or V_{ref}), which can be seen the reference voltage with the same frequency as the inverter output, and a high frequency voltage, which has a triangular waveform (V_{tri}). The output waveforms of switching MOSFETs, S_{A1}, S_{A2}, S_{B1}, and S_{B2} can be generated as Fig. 3 by combining $\pm V_{con}$ and triangular V_{tri}. The detailed operation of the SPWM control is as follows:

When $+V_{con} > V_{tri}$, S_{A1} is ON and S_{A2} is OFF.
When $+V_{con} < V_{tri}$, S_{A1} is OFF and S_{A2} is ON.
When $-V_{con} > V_{tri}$, S_{B1} is ON and S_{B2} is OFF.
When $-V_{con} < V_{tri}$, S_{B1} is OFF and S_{B2} is ON.

Based on the above operation, there are four kinds of combinations for output V_O as:

(1)S_{A1}, S_{B2} is ON: $V_O = +V_{DC}$.
(2)S_{B1}, S_{A2} is ON: $V_O = -V_{DC}$.
(3)S_{A1}, S_{B1} is ON: $V_O = 0$.
(4)S_{A2}, S_{B2} is ON: $V_O = 0$.

According to the above explanation, the circuit belongs to a single-phase voltage-mode inverter type. By using SPWM, the difference between output frequency (f_o) and harmonic frequencies are increased. Such a result will be helpful to output filter design. Besides, by using the SPWM control, the regulation capability can be enhanced for different desired outputs, including the different output voltage/frequency.

C. Analysis of PWM technique and Filter design

To confirm to our theoretical SPWM waveform of Fig. 3, where 40 individual pulses are used to synthesize one-half cycle of the AC output waveform, the pulse repetition frequency for a 50 Hz output will be 4 kHz. This is well within the range of realistic frequencies actually used in our presentation of the SPWM technique. Otherwise, in the description of the output waveform, there will be too large a number of closely spaced very narrow pulses, because of which the physical relationships that is needed to highlight relating to pulse width be obscured.

1) Mathematical Expression of PWM topology

The method used for the mathematical analysis of PWM is Laplace transformation. The switching frequency of the inverter is 4 kHz so the time period is 0.25 msec. There are 80 pulses in 0.02 sec which is equal to time period of sinusoid. Analysis is applied on the positive cycle of the sinusoid only. There is a delay of 0.25 msec between two consecutive pulses. Keeping all these factors in mind, the PWM of the inverter can be mathematically expressed as:

$$f_n(x) = \begin{cases} 1, & 0.25\times10^{-3}\,\mathrm{n} < x \le 0.25\times10^{-3}(\mathrm{n} + \sin(\omega\times0.25\times10^{-3}\,\mathrm{ns})) \\ 0, & 0.25\times10^{-3}(\mathrm{n} + \sin(\omega\times0.25\times10^{-3}\,\mathrm{ns})) < x \le 0.25\times10^{-3}(n+1) \end{cases} \tag{1}$$

Where $f_n(x)$ is the pulse at n and $0 \le \mathrm{n} \le 39$

and $$f(x) = \sum_{n=0}^{39} f_n(x) \tag{2}$$

Laplace transform of rectangular pulse is given in equations (3) and (4)

$$f(x) = \begin{cases} 1, & 0 < x \le p \\ 0, & x > p \end{cases} \rightarrow \mathcal{L} \rightarrow \frac{1}{s}\left(1 - e^{-ps}\right) \tag{3}$$

and Laplace transform of shifted pulse is given by

$$f(x)=\begin{cases} 1, & r<x\leq p \\ 0, & x>p \end{cases} \rightarrow \mathcal{L} \rightarrow \frac{1}{s}\left(1-e^{-ps}\right)e^{-rs} \tag{4}$$

The width of the pulse varies according to the (5)

$$p = 0.25\times10^{-3}\times\sin(2\pi\times50\times0.25\times n\times10^{-3})$$

$$p = 0.25\times10^{-3}\times\sin(0.0785\times n) \tag{5}$$

as mentioned before n is the number of pulse
So expression of Laplace transform of PWM can be given by:

$$X(s)=\sum_{n=0}^{39}\frac{1}{s}\left(1-e^{-0.25\times10^{-3}\times\sin(0.0785\times n)s}\right)e^{-0.25\times10^{-3}\,ns} \tag{6}$$

X(s) is the Laplace transform of the input. Desired output is the sine wave. So the system response can be formulated and system response is equal to the filter response. Filter response will be discussed in Filter design.

D. Filter design

In the filter design, output is known and input is analyzed and filter response is manipulated using (7)

$$H(s)=\frac{Y(s)}{X(s)} \tag{7}$$

1) Mathematical Model of Filter design

Laplace transform of a sine wave is given by:

$$Y(s)=\frac{\omega}{s^2+\omega^2} \tag{8}$$

By using equations (6), (7) and (8) H(s) is manipulated.

$$H(s)=\frac{\omega}{s^2+\omega^2}\times\sum_{n=0}^{39}\frac{e^{0.25\times10^{-3}\,ns}}{\frac{1}{s}\left(1-e^{-0.25\times10^{-3}\times\sin(0.0785\times n)s}\right)} \tag{9}$$

The Taylor series can be expressed as

$$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots, \qquad -\infty < x < \infty \tag{10}$$

Applying Taylor series on equation (9) we get

$$H(s)=\frac{\omega}{s^2+\omega^2}\times\sum_{n=0}^{39}\frac{1+0.25\times10^{-3}\,ns+\ldots}{\frac{1}{s}\left(0.25\times10^{-3}\times\sin(0.0785\times n)s+\ldots\right)} \tag{11}$$

Neglecting higher order terms and applying summation the result is obtained as follows:

$$H(s)=\frac{3000}{0.05s^2+10s+1000} \tag{12}$$

2) Physical realization of Filter

Transfer function of filter, resembles to the transfer function of 2nd order low pass filter. Physically realizable values closest to the transfer function are given in Table I.

TABLE I
REALIZABLE VALUES OF FILTER

Resistor	Variable load
Inductor	800 µH
Capacitor	400µF

III. ANALYSIS OF SINUSOIDAL PULSE WIDTH MODULATION TECHNIQUE

In this analysis, the Pulse Width Modulation (PWM) Technique comparing a sinusoid with a triangular wave is used the single-phase H-bridge inverter circuit model described in Fig.1 for this purpose. To analyse of the single-phase H-bridge inverter circuit model, the full-bridge topology is considered as the superposition of two half-bridge topology. A superposition of two half-bridge inverters is exactly the same and functions in the same manner as the full-bridge shown in Fig. 4.

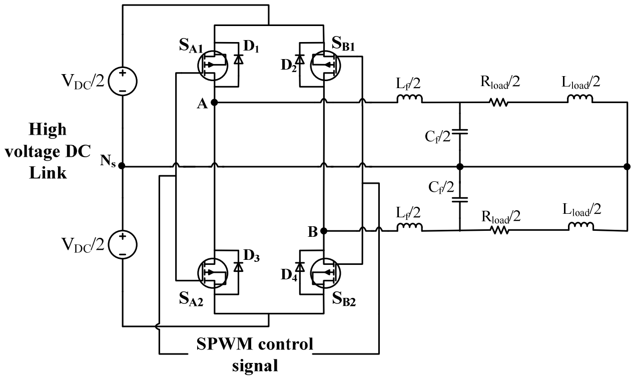

Fig. 4 A full-bridge is shown as a superposition of two half-bridge.

By applying Kirchoff's currents law and voltage law to the superposition of two half-bridge inverter topology, the following current and voltage equations can be driven as follows:

$$\begin{aligned} \frac{dV_{LAB}}{dt} &= \frac{2}{C_f}\left(i_{iAB}-i_{LAB}\right) \\ \frac{di_{iAB}}{dt} &= \frac{2}{L_f}\left(V_{iAB}-V_{LAB}\right) \\ \frac{di_{LAB}}{dt} &= -\frac{R_{load}}{L_{load}}i_{LAB}+\frac{2}{L_{load}}V_{LAB} \end{aligned} \tag{12}$$

Therefore, the above equations can be rewritten into a matrix form, respectively:

$$\begin{aligned} \frac{dV_L}{dt} &= \frac{2}{C_f}I_i-\frac{2}{C_f}I_L \\ \frac{dI_i}{dt} &= -\frac{2}{L_f}V_L+\frac{2}{L_f}V_i \\ \frac{dI_L}{dt} &= \frac{2}{L_{load}}V_L-\frac{R_{load}}{L_{load}}I_L \end{aligned} \tag{13}$$

where, $V_L=[V_{LAB}]$, $I_i=[i_{iAB}]$, $V_i=[V_{iAB}]$, $I_L=[i_{LAB}]$

Finally, the given plant model can be expressed as the following continuous-time state space equation

$$\dot{X}(t) = AX(t)+Bu(t), \tag{14}$$

where, $$X=\begin{bmatrix} V_L \\ I_i \\ I_L \end{bmatrix},\quad A=\begin{bmatrix} 0 & \frac{2}{C_f} & -\frac{2}{C_f} \\ -\frac{2}{L_f} & 0 & 0 \\ \frac{2}{L_{load}} & 0 & -\frac{R_{load}}{L_{load}} \end{bmatrix},\quad B=\begin{bmatrix} 0 \\ \frac{2}{L_f} \\ 0 \end{bmatrix},\quad u=[V_i]$$

Note that load line to line voltage V_L, inverter output current I_i, and the load current I_L are the state variables of the system, and the inverter output line-to-line voltage V_i is the control input (u).

A. *System parameters for the single-phase H-bridge inverter Simulation*

The system configuration sine-triangle PWM inverter simulink model is given follow:

- MOSFETs: IRF3250, Max ratings: V_{CES} = 600, I_C = 80 A
- DC-link voltage: V_{DC} = 400V
- Fundamental frequency: f = 50 Hz
- PWM (carrier) frequency: f_{tri} = 4 kHz
- Modulation index: m = 0.8
- Output filter: L_f = 800µH and C_f = 400µF
- Load: L_{load} = 2mH and R_{load} = 5 Ω

B. *Simulation Steps*

1) Initialize system parameters using Matlab
2) Build Simulink Model
   - Generate carrier wave (V_{tri}) and control signal ($V_{control}$) based on modulation index (m)
   - Compare Vtri to Vcontrol to get V_{iAn},V_{iBn}
   - Generate the inverter output voltage V_{iAB} for control input (u)
   - Build state-space model
   - Send data to Workspace
3) Plot simulation results using Matlab

C. *Simulink model*

The simulink model for single phase H-bridge inverter is build according to the system parameters shown as in Fig. 5.

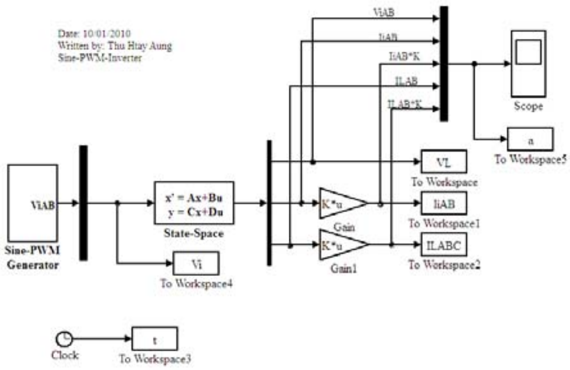

Fig. 5. Simulink Model for Overall system.

D. *Simulation Results*

Using Matlab /Simulink, this model is simulated to the circuit model described in Fig. 4 and then plots the waveforms of V_{iAB}, V_{LAB}, I_{iAB}, I_{LAB}.

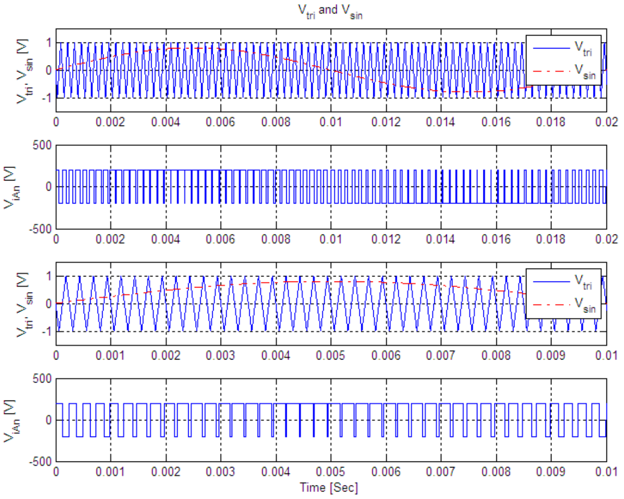

Fig. 6. Waveforms of carrier wave, control signal, and inverter output line to neutral voltage.

As shown in Fig. 6, the inverter output voltage is determined in the following:

- When $V_{control} > V_{tri} = V_{DC}/2$
- When $V_{control} < V_{tri} = -V_{DC}/2$

The triangular carrier waveform has fixed amplitude. The amplitude of the reference sine wave is usually made adjustable. Modulation index (m) is defined the ratio of amplitude of the reference wave and carrier wave as follows:

$$m=\frac{V_{control}}{Vtri}$$

The variation of the reference wave amplitude, keeping the carrier amplitude fixed, is the usual means employed to adjust the modulation index. The adjustment of the modulation index gives us a convenient way to adjusting the AC output voltage of the inverter. Since the inverter output frequency is the same as the reference sine wave, the inverter output frequency is adjustable by adjustment of the reference wave frequency.

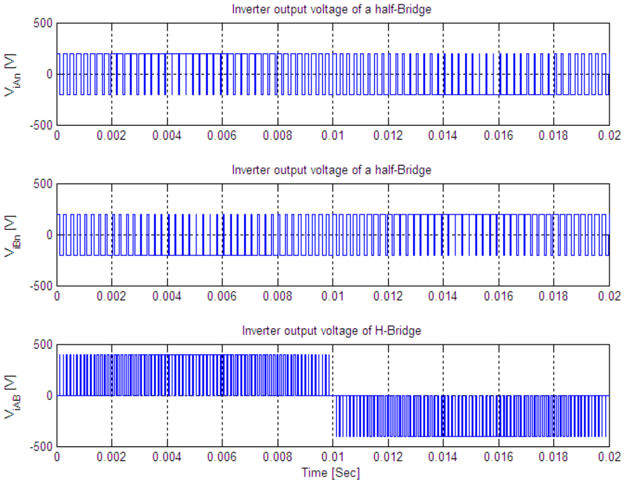

Fig. 7. Simulation results of inverter output line to line voltage per cycle.

The simulation results of two half-bridge inverters' output voltage and the full-bridge inverter output voltage are shown in Fig. 7. It can be see that the output voltages are achieved by means of multiple pulses train according to the theoretical analysis of Sinusoidal Pulse Width Modulation (SPWM) technique.

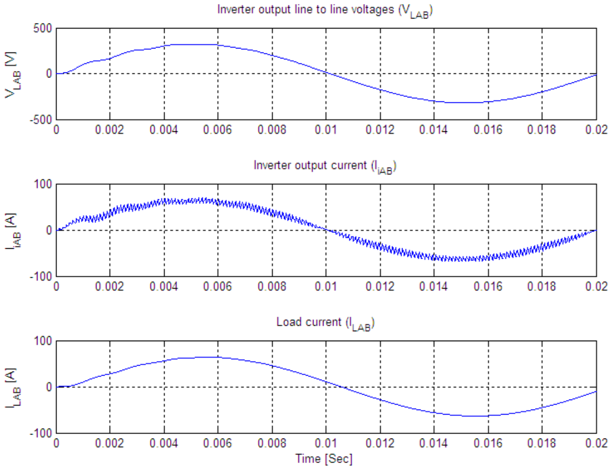

Fig. 8. Simulation results of load line to line voltage (V_{LAB}), current (I_{LAB}), and inverter output current (I_{iAB}) per cycle.

As shown in Fig. 8, the load voltage and current of H-bridge inverter circuit are regulated and desirable sine wave. According to the analysis of filter design, it is also clear that the inverter output current ripples can be cancelled by using the low pass LC filter.

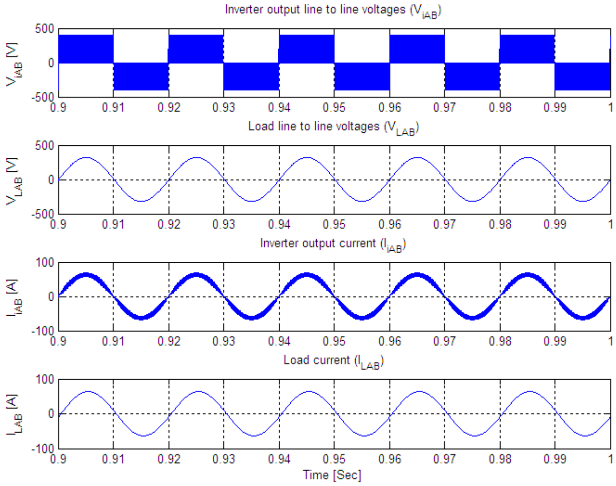

Fig. 9. Waveforms of inverter output voltage (V_{iAB}), inverter output current (I_{iAB}), load line to line voltage (V_{LAB}), and current (I_{LAB}).

Waveforms of inverter output voltage (V_{iAB}), inverter output current (I_{iAB}), and load line to line voltage (V_{LAB}), and current (I_{LAB}) are shown in Fig. 9.The inverter output voltage waveforms can be seeing as the sinusoidal pulse width modulation waveforms and its output current is sinusoidal sine wave with ripples. The simulation results are proved that load line to line voltage is regulated and current is ripple less.

IV. CONCLUSIONS

The theoretical analysis is implementable in microcontroller based sinusoidal pulse width modulation (SPWM) control scheme. According to the mathematically analysis of sinusoidal PWM technique, the sinusoidal pulses including in one cycle of sine wave with desired fundamental output frequency can be calculated. The proposed technique is also provided the generation of sinusoidal pulse width modulation by using microcontroller. The simulation results of the sinusoidal PWM technique proves that output sine wave is voltage regulated, ripple less and glitch free for the use of sine wave inverter, such as, transformer less UPS and DC-DC step up converter. Moreover, this simulink model can be also used in three-phase inverter topology.

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