The Government of the Union of Myanmar
Ministry of Education

Department of Higher Education (Lower Myanmar)
and
Department of Higher Education (Upper Myanmar)

Universities Research Journal

Vol. 1, No. 3 December, 2008.

Universities Research Journal 2008
Vol. 1, No. 3

Editorial Board

Prof. Dr Phway Phway, Head of the Department of Chemistry, University of East Yangon
Prof. Dr. Khin Myo Nwe, Head of the Department of Chemistry, University of Hinthada
Prof. Dr. Myint Myint Sein, Head of the Department of Chemistry, University of Mandalay
Prof. Dr. Aye Aye Wai, Head of the Department of Chemistry, University of Magway
Prof. Khin Hnin Lwin, Head of the Department of Chemistry, University of Monywa
Prof . Dr. Sein Sein Aung, Department of Chemistry, University of Panglong
Prof. Dr. Tha Zin, Head of Chemistry Department, University of Loikaw
Prof. San San Wai, Head of the Department of Chemistry, University of Lashio
Prof. Dr. Aye Myatt Mynn, Head of the Department of Physics, University of Mawlamyine
Prof. Dr. Than Hla, Head of the Department of Physics, Yangon University of Distance Education
Prof. Dr. Khin Mar Kyu, Head of the Department of Physics, University of Dagon
Prof. Dr. Thet Tun Aung, Head of the Department of Physics, University of Pathein
Prof. Dr. Daw Thein Win, Head of the Department of Physics, University of Sittway
Prof. Dr. Khin Soe Win, Head of the Department of Physics, University of Pyay
Prof. Dr. Myint Myint Moe, Head of the Department of Physics, University of Dawei
Associate Prof. Dr Ni Ni Zin, Head of the Department of Physics, University of East Yangon
Prof. Dr Tin Tin Win, Head of Department of Physics, University of West Yangon
Prof. Dr. Mya Mya Win, Head of the Department of Physics, University of Hinthada
Prof. Myint Yee, Head of the Department of Physics, University of Maubin
Prof. Dr. Myint Myint Tun, Head of the Department of Physics, University of Hpaan

Prof. Dr. Than Win, Head of the Department of Physics, University of Myeik
Prof. Swe Swe Yi, Head of the Department of Physics, University of Monywa
rof. Dr Thet Thet, Head of the Department of Physics, Mandalay University of Distance Education
Prof . Dr. May Yee Thein, Head of Physics Department, University of Panglong
Prof. Dr. Soe Soe Nwe, Head of the Department of Physics, University of Lashio
Prof. Dr. Khin May Oo, Head of Department of Physics, Myinchan Degree College

Contents

Production of Kaonic Nuclei K^-pp by $p(p, K^+)$ and $p(d, K^0)$ Reactions

Htar Win Htike[1], Mar Mar Htay[2] and Khin Swe Myint[3]

Abstract

Reactions $p(p, K^+)K^-pp$ and $d(p, K^+)K^-pp$ are promising means to populate kaonic nuclei and in this work, the differential cross sections of the above reactions have been investigated theoretically. Existence of kaonic nuclear bound states is of utmost significance in studying antikaon nucleon interactions. The present work would provide experimentalists with important information on setting up experiments to search for bound state kaonic nuclear systems. Green's function method have been employed to determine the cross sections and it is found that there is a remarkable peak structure in spectrum which indicates the formation of K^-pp kaonic nuclei. The optimum incident proton energy for $p(p, K^+)$ and $d(p, K^+)$ reactions to produce K^-pp is proposed to be 4GeV.

Key words: Kaonic nclei, Green's function method, antikon nucleon interactions.

Introduction

Theoretical Research of Kaonic Nuclear States

Recently, existence of exotic few-body nuclear systems involving an antikaon $\overline{K}$(K^- or $\overline{K}^0$) as a constituent have been predicted based on phenomenologically constructed $\overline{K}N$ interactions (Akaishi and Yamazaki, 2002). The predicted binding energies and level widths of the bound states in K^-ppn, K^-ppnn and K^- ^{8}Be are E_K=118, 114, 108 MeV and Γ = 21, 34, 20 MeV respectively (Yamazaki and Akaishi, 2006). These binding energies are very large and the corresponding states lie below the $\Sigma\pi$ emission threshold (~100 MeV). Because of the strong $\overline{K}N$ attraction they have enormously high nucleon densities ρ_{ave}~0.5fm^{-3}, about 3 times the normal nuclear density ρ_0~ 0.17 fm^{-3}. In deeply bound $\overline{K}$ states the core nuclei are largely compressed due to the strong attraction of a $\overline{K}$. Such compact nuclear systems are called "$\overline{K}$ nuclear clusters" or $\overline{K}$ bound states in non-existing nuclei.

1. Demonstrator, Dr, Department of Physics, Mandalay University
2. Assistant Lecturer, Dr, Department of Physics, Mandalay University
3. Pro-Rector, Dr, Mandalay University

The basic ingredient for this new family of nuclear states is I=0 $K^{-}p$, which is identified as the known $\Lambda(1405)$ or Λ^{*} with a binding energy of B_K = 27 MeV and a width of Γ = 40 MeV. The lightest system of $\overline{K}$ nuclear clusters is $K^{-}pp$ (and its isospin partner $\overline{K}^{0}pn$), which was predicted to have B_K = 48 MeV and Γ = 61 MeV. It is vitally important to study such exotic objects from both theoretical and experimental sides. Experiments stimulated by these theoretical investigations have been performed at KEK and FINUDA to search for the kaonic deeply bound nuclear states.

These experimental evidences are not sufficient to obtain most reliable information on kaonic nuclear states yet. Thus many experimental researches have been awaited to be able to settle the controversial state of kaonic nuclear study. There are various reactions to populate $\overline{K}$ nuclear clusters, however, all of these cannot be carried out experimentally because some of these may have very low reaction cross sections. Therefore, theoretical calculations for feasible reactions producing kaonic nuclear bound states are urgently required.

Various $\overline{K}$ bound systems can be populated by the following reactions such as $p(p,K^{+})ppK^{-}$, $d(p,K^{0}p)ppK^{-}$, ${}^{3}He(K^{-},\pi^{-}p)ppK^{-}$, ${}^{3}He(\pi^{+},K^{+})ppK^{-}$, $d(p,K^{0})K^{-}ppp$, $d(p,K^{+})K^{-}ppn$ ${}^{3}He(p,K^{+})K^{-}ppn$, ${}^{3}He(p,K^{0})K^{-}ppn$. We are going to investigate the simplest reactions $p(p,K^{+})ppK^{-}$ and $d(p, K^{0}p)ppK^{-}$ to produce the lightest kaonic nuclear system .

Formulation of Production Cross Section

Transition matrix element for the above reaction can be expressed in terms of relative and center of mass momenta of initial state and final state,

$$T_{fi} = \frac{\hbar c}{\sqrt{2E_1}} \left[\mathbf{k}_1 \mathbf{k}_2 \Phi_f^n \middle| T \middle| \mathbf{k}_0, 0\right]. \tag{1}$$

After performing the necessary calculations, the above equation becomes

$$\left|T_{fi}\right| = \frac{\hbar c}{\sqrt{2E}} \delta(\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}_2)\left(\frac{2\pi}{L}\right) \int d\mathbf{q}' \left\langle \Phi_f^{(n)} \middle| \mathbf{q}' \right\rangle \left\langle \widetilde{\mathbf{k}}', \mathbf{q}' \middle| t \middle| \widetilde{\mathbf{k}} \right\rangle \tag{2}$$

where $\widetilde{\mathbf{k}} = \dfrac{M_p \mathbf{k}_0 - M_p 0}{M_p + M_p}$,

$$\widetilde{k}' = \frac{M_{ppK^-} k_1 - m k_2}{M_{ppK^-} + m} \text{ and}$$

$$q' = \frac{M_{\Lambda^*} q_p - M_p q_{\Lambda^*}}{M_{\Lambda^*} + M_p}.$$

Transition matrix element T_{fi} and transition rate W_{fi} are related by

$$W_{fi} = \frac{2\pi}{\hbar}\sum_n \delta(E_f^{(n)} - E_i)\,|T_{fi}|^2\,\rho(E), \tag{3}$$

which is the *Fermi's Golden Rule*. W_{fi} is the transition rate, the delta term $\delta(E_f^{(n)} - E_i)$ guarantees the conservation of energy. The differential cross section can be written as

$$d\sigma = W_{fi}\frac{L^3}{v_0} \tag{4}$$

where, v_0 is the velocity of incident proton, $v_0 = \hbar k_0 c^2/E_0$ and v_0/L^3 is incident flux. So we have the differential cross section for the reaction by inserting the value of transition rate as follows:

$$d^6\sigma = \frac{L^3}{v_0}\frac{2\pi}{\hbar}\sum_n \delta(E_f^{(n)} - E_i)\left(\frac{L}{2\pi}\right)\left(\frac{L}{2\pi}\right)^3 d\mathbf{k}_2\,|T_{fi}|^2 \tag{5}$$

where $d^6\sigma$ is represented the differential cross section of the reaction with momentum of out going particle K^+ in the range k_1 and k_1+dk_1 and that of $\Lambda^* p$ in the range k_2 and k_2+dk_2. In order to include both bound states and continuum final states, we employ Green's function method (Morimatsu and Yazaki, 1985) in our calculation.

The energy conservation term $\delta(E_i - E_f^{(n)})$ can be written as

$$\sum_n \delta(E_i - E_f^{(n)}) = \sum_n \frac{-1}{\pi}\,\mathrm{Im}\left\langle \Phi_f^{(n)} \right| \frac{1}{E - H_{\Lambda^* p} + i\varepsilon} \left| \Phi_f^{(n)} \right\rangle \tag{6}$$

The equation of differential cross section in terms of coordinate representation becomes

$$d^3\sigma = (2\pi)^4 \frac{E_0}{k_0 2E_1} dk_1 \left(\frac{-1}{\pi}\right) \times \mathrm{Im}\left[dr\, dr' \langle \tilde{k}| t^* |\tilde{k}' r\rangle \langle r| \frac{1}{E - H_{\Lambda^* p} + i\varepsilon} |r'\rangle \langle \tilde{k}' r'| t |\tilde{k}\rangle \right] \tag{7}$$

where $\mathbf{r}$ is the relative coordinate of Λ^* and proton. ($\mathbf{r}_{\Lambda^* p}$). The term, $\langle r| \frac{1}{E - H_{\Lambda^* p} + i\varepsilon} |r'\rangle$, is solved by using Green's function method.

$$\frac{d^3\sigma}{dE_1 d^2\Omega_1} = \sigma_0 \frac{k_1 E_0}{2k_0}\left(\frac{-1}{\pi}\right)$$

$$\times \mathrm{Im}\left[\frac{2\mu}{\hbar^2} 4\pi \sum_{\ell=0}^{\infty} \iint dr dr' (2\ell+1)(-i)^\ell i^\ell e^{-\frac{r}{\beta}} e^{-\frac{r'}{\beta}} \times j_\ell(Qr)\, j_\ell(Qr') \frac{U_\ell^+(r_>) U_\ell^0(r_<)}{W} \right] \tag{8}$$

This is the final equation for mathematical formulation of differential cross section. $U_\ell^0(r_<)$ and $U_\ell^+(r_>)$ are the stationary and out-going solutions of the Schrödinger equation

$$\left\{ k^2 + \frac{d}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \tilde{V}(r) \right\} G(r, r') = 0 . \tag{9}$$

$U_\ell^0(0) = r^{\ell+1}$ and $U_\ell^+(r) = kr\, h_\ell^+(kr)$ are boundary conditions of these solutions at origin and at the asymptotic region respectively.

Results and Discussions

Energy Dependence of Reaction Cross Section

Since the threshold energy of above reaction is 2.4 GeV, we input the various kinetic energies of incident proton, KE_p, from 2.4 GeV to 7.0 GeV with 0.5 GeV incremental intervals.

For a particular value of the projectile proton kinetic energy, there are many possible ways of sharing the energy between the outgoing kaon K^+ and K^-pp (or Λ^*p) system. Thus we calculated the maximum total energies of outgoing K^+ (E_1^{max}) for each input proton energy (KE_p). By decreasing 5.0 GeV to this E_1^{max} about 300 times, the corresponding relative energies of K^-pp system, (E_{Λ^*p}) are obtained. Therefore we obtained 300 differential cross sections with respect to the relative energies of K^-pp system for one input kinetic energy of incident proton. From these values, the maximum differential cross section is found.

Table (1) shows the list of various differential cross sections with respect to the total energies of Λ^*p system (E_{Λ^*p}), for various kinetic energies of projectile proton, KE_p = 2.5 to 7.0 GeV. It is found that the maximum differential cross section ($d^3\sigma/dE_1/d^2\Omega_1$ = 0.21 μb/MeV/sr) is obtained at the projectile proton energy 4.0 GeV, which can be clearly seen in Figure (1).

Figure (2) shows the variations of differential cross sections with respect to the relative energies of Λ^*p at this incident energy 4.0 GeV. The maximum differential cross section ($d\sigma/dE_1/d^2\Omega_1$ = 0.21 μb/MeV/sr) is observed at E_{Λ^*p}= –64 MeV. This value gives the binding energy of a bound state K^-pp system. Then the observed decay width (-41.59 MeV) is also determined at FWHM of the peak of Figure (2). The differential cross section at (E_{Λ^*p}>0) corresponds to the free emission of Λ^* particle.

Differential Cross Section with Various p-p Interaction Ranges

We have known that the range of nucleon-nucleon interaction is proportional to the reciprocal mass of exchanged particle or mass of boson, ($1/m_B$). So we have analyzed the various differential cross sections with respect to the various boson masses m_B = 1020, 770 and 500 MeV/c^2 for φ meson, ρ meson, and K meson respectively which are shown in Table (2).

We have found that the binding energies of K^-pp system are the same (64 MeV) but cross sections are different. It can be clearly seen in Figure (3). The maximum differential cross section, 0.26 μb/MeV/sr, is obtained at m_B = 1020 MeV/c^2. These bosons are exchanged to mediate proton-proton interactions and the largest boson which means the shortest range interaction give the maximum differential cross section. It is the same as we expected from the beginning that the reaction is likely to realize at large momentum transfer or at short range interaction.

Λ^*p Size Effect on Reaction Cross Section

In order to investigate the Λ^*p size effect on the reaction cross section we constructed the different Λ^*p potentials as follows. The various Λ^*p potentials giving the same binding energy but different root mean square sizes are phenomenologically determined by varying the strengths and range para-meters. We demonstrated this effect by employing the potential parameter sets which are shown in Table (3).

Then the various bound-state peaks for various root mean square (rms) sizes from $R_{\Lambda^* p}$ = 0.80 fm to 2.33 fm are compared. Figure (4) shows that the bound-state peak decreases dramatically, when we increase the rms size from $R_{\Lambda^* p}$ =1.05 fm to 1.46 and 2.33 fm. It also shows that with a denser Λ^*p system ($R_{\Lambda^* p}$=0.93 and 0.80 fm) the peak height increases.

Concluding Remark

We concluded that the projectile energy 4 GeV gives the maxi-mum differential cross section for the reaction p(p,K^+)K^-pp. This is a significant information for experimentalist. We have found that differential cross section depends upon the p-p interaction range. The smaller the range of p-p interaction, the larger the value of differential cross section of this reaction. It means that mass of boson which is exchanged between two protons plays an important role in determining the differential cross section. And it also depends on the root mean square (size) distance of Λ^*p system. The shorter the root mean square distance of Λ^*p system, the higher the peak of the spectral shape of differential cross section. We can therefore extract information concerning the p-p interaction and Λ^*p interaction from the experimental data by considering these theoretical aspects.

Table (1) The list of various differential cross sections with respect to the relative energies of $\Lambda^{*}p$ ($E_{\Lambda^{*}p}$), for various KE_p.

KE_p (GeV)	$E_{\Lambda^{*}p}$ (MeV)	$d\sigma/dE_1/d^2\Omega_1$ (μb/MeV/sr)
2.5	-64.73	0.1564
3.0	-64.90	0.1941
3.5	-65.18	0.2097
4.0	-64.04	0.2142
4.5	-65.20	0.2117
5.0	-64.89	0.2057
5.5	-64.62	0.1973
6.0	-64.30	0.1877
6.5	-63.90	0.1773
7.0	-65.07	0.1667

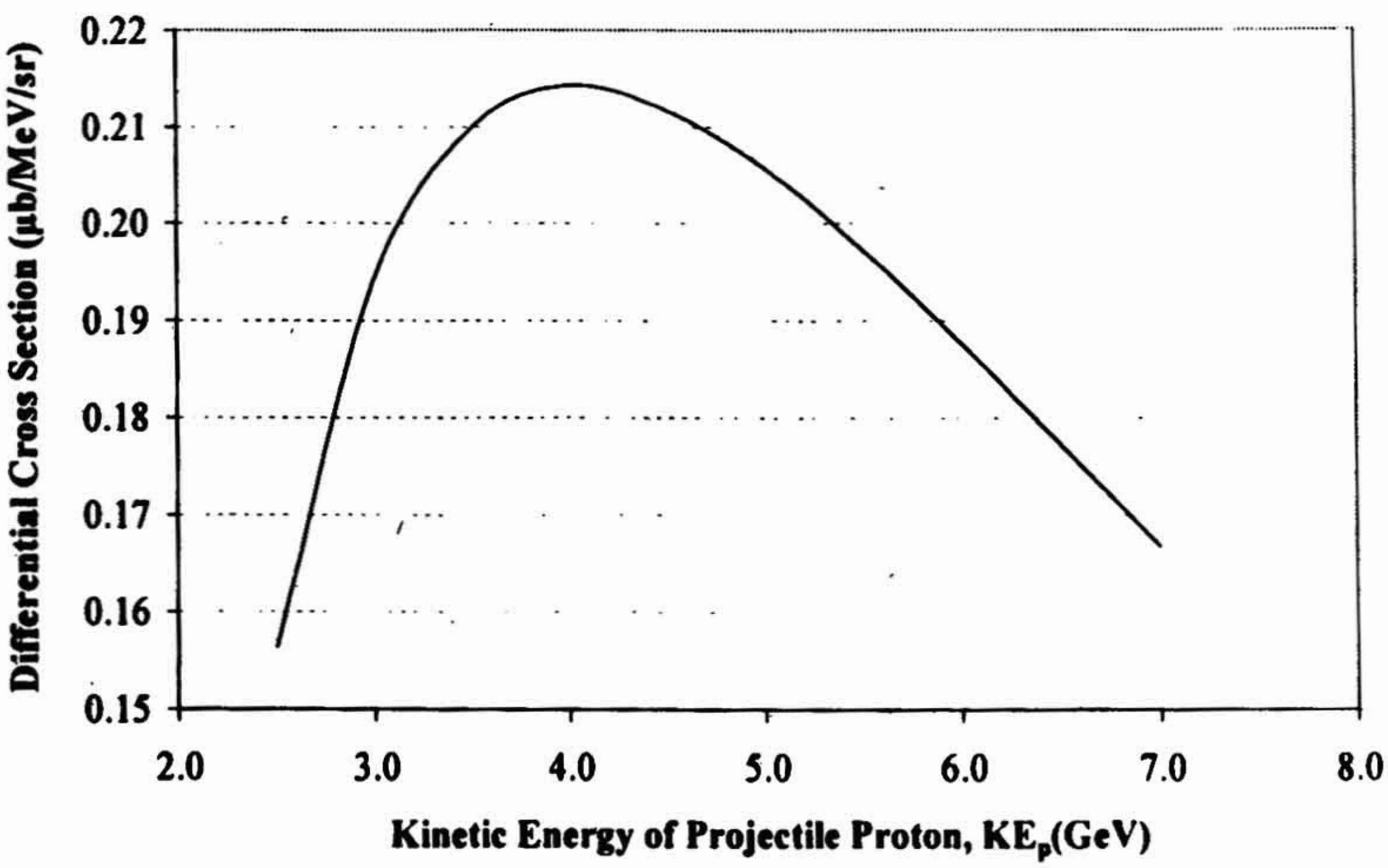

Figure (1) The curve of differential cross section for various kinetic energies of incident proton.

Table (2). The lists of various differential cross sections for various kinetic energies and for various boson masses.

Table (2.a). exchange particle φ meson

KE_p (GeV)	E_{Λ^*p} (MeV)	$d\sigma/dE_1/d^2\Omega_1$ (μb/MeV/sr)
2.5	-64.73	0.1928
3.0	-64.88	0.2393
3.5	-65.17	0.2586
4.0	-64.04	0.2641
4.5	-65.20	0.2611
5.0	-64.89	0.2536
5.5	-64.61	0.2433
6.0	-64.29	0.2314
6.5	-63.89	0.2186
7.0	-65.07	0.2056

Table. (2.b) exchange particle ρ meson

KE_p (GeV)	E_{Λ^*p} (MeV)	$d\sigma/dE_1/d^2\Omega_1$ (μb/MeV/sr)
2.5	-64.73	0.1564
3.0	-64.88	0.1941
3.5	-65.17	0.2097
4.0	-64.04	0.2142
4.5	-65.20	0.2117
5.0	-64.89	0.2057
5.5	-64.61	0.1973
6.0	-64.29	0.1877
6.5	-63.89	0.1773
7.0	-65.06	0.1667

Table (2.c). exchange particle K meson

KE_p (GeV)	E_{Λ^*p} (MeV)	$d\sigma/dE_1/d^2\Omega_1$ (μb/MeV/sr)
2.5	-64.73	0.1407
3.0	-64.88	0.1747
3.5	-65.17	0.1887
4.0	-64.04	0.1928
4.5	-65.20	0.1906
5.0	-64.89	0.1851
5.5	-64.61	0.1776
6.0	-64.29	0.1689
6.5	-63.89	0.1596
7.0	-65.06	0.1501

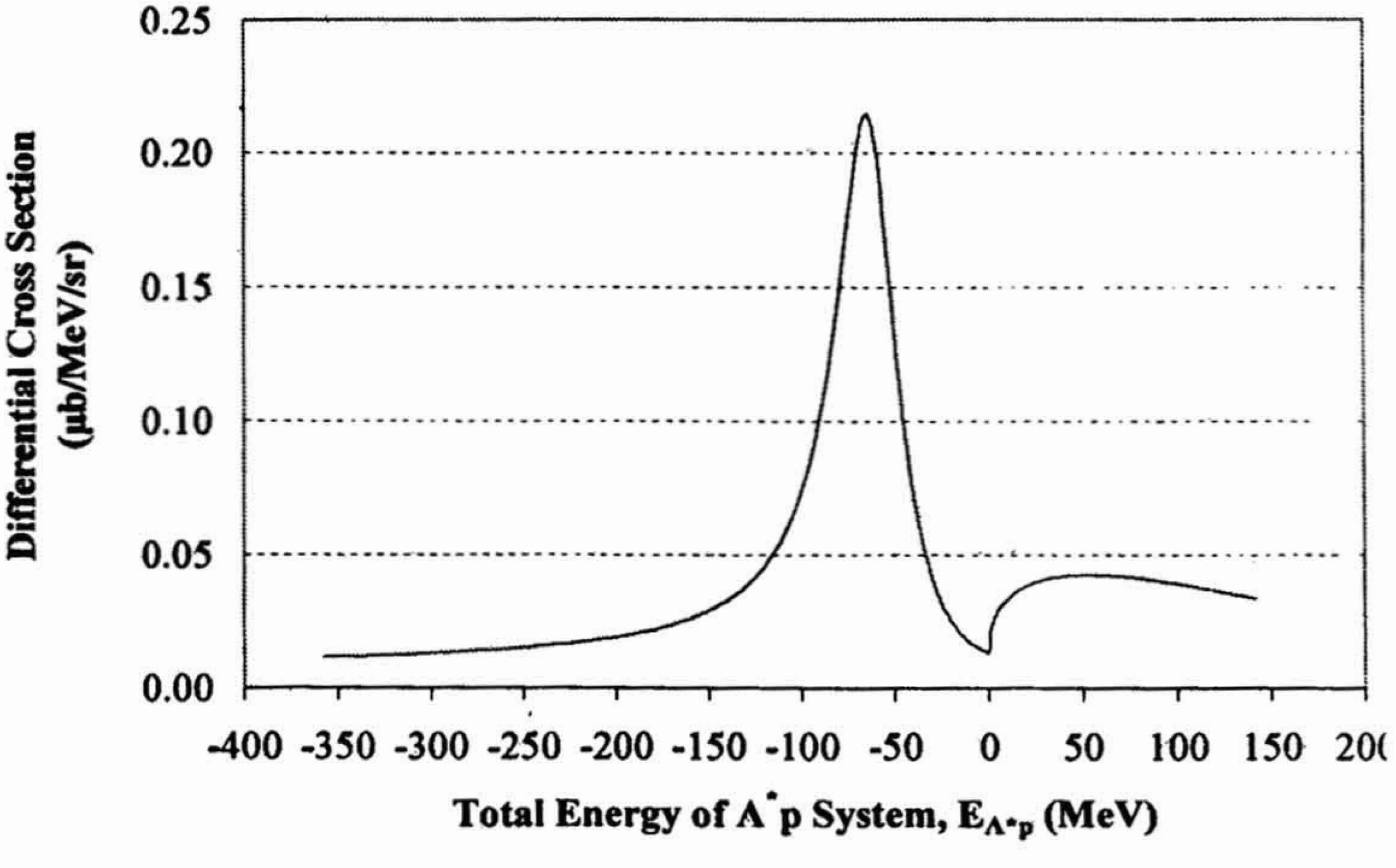

Figure (2) The curve of differential cross sections with respect to the relative energies of Λ^*p at KE_p = 4.0 GeV.

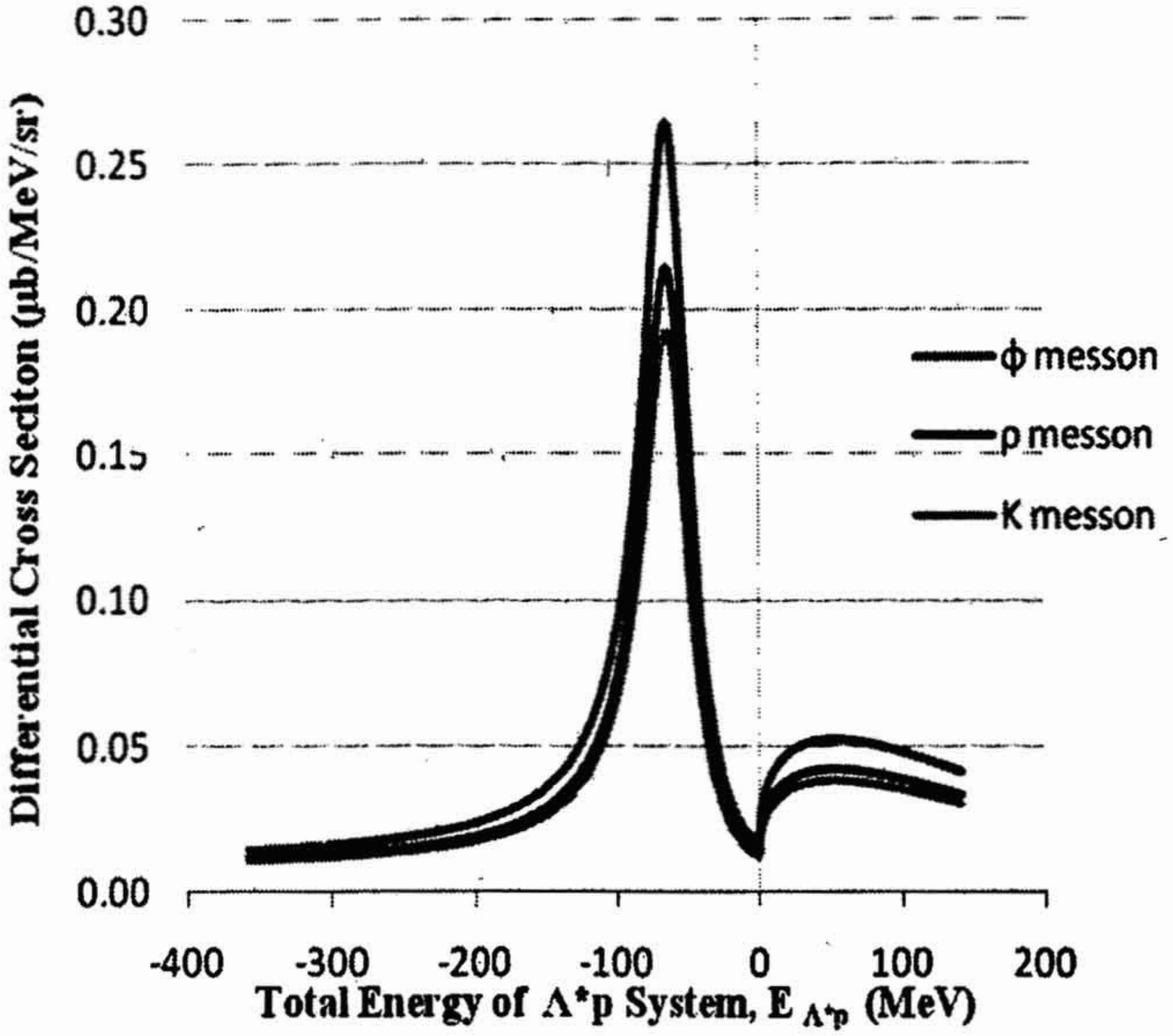

Figure (3) The curve of differential cross sections with respect to the relative energies of Λ^*p at KE_p = 4.0 GeV with various boson masses.

Table (3) Differential cross sections with respect to various potential strengths

Λ^*p Potential Strength (MeV)	Range Parameter (fm)	Root Mean Square Distance (fm)	Differential Cross Section (μb/MeV/sr)
(-714.50,-59.00)	0.50	0.80	0.8036
(-413.30,-35.33)	0.75	0.93	0.4926
(-295.00,-42.00)	1.00	1.05	0.2142
(-155.00,-20.00)	2.00	1.46	0.0997
(-119.00,-25.00)	3.00	1.79	0.0427
(-93.08,-20.72)	5.00	2.33	0.0261

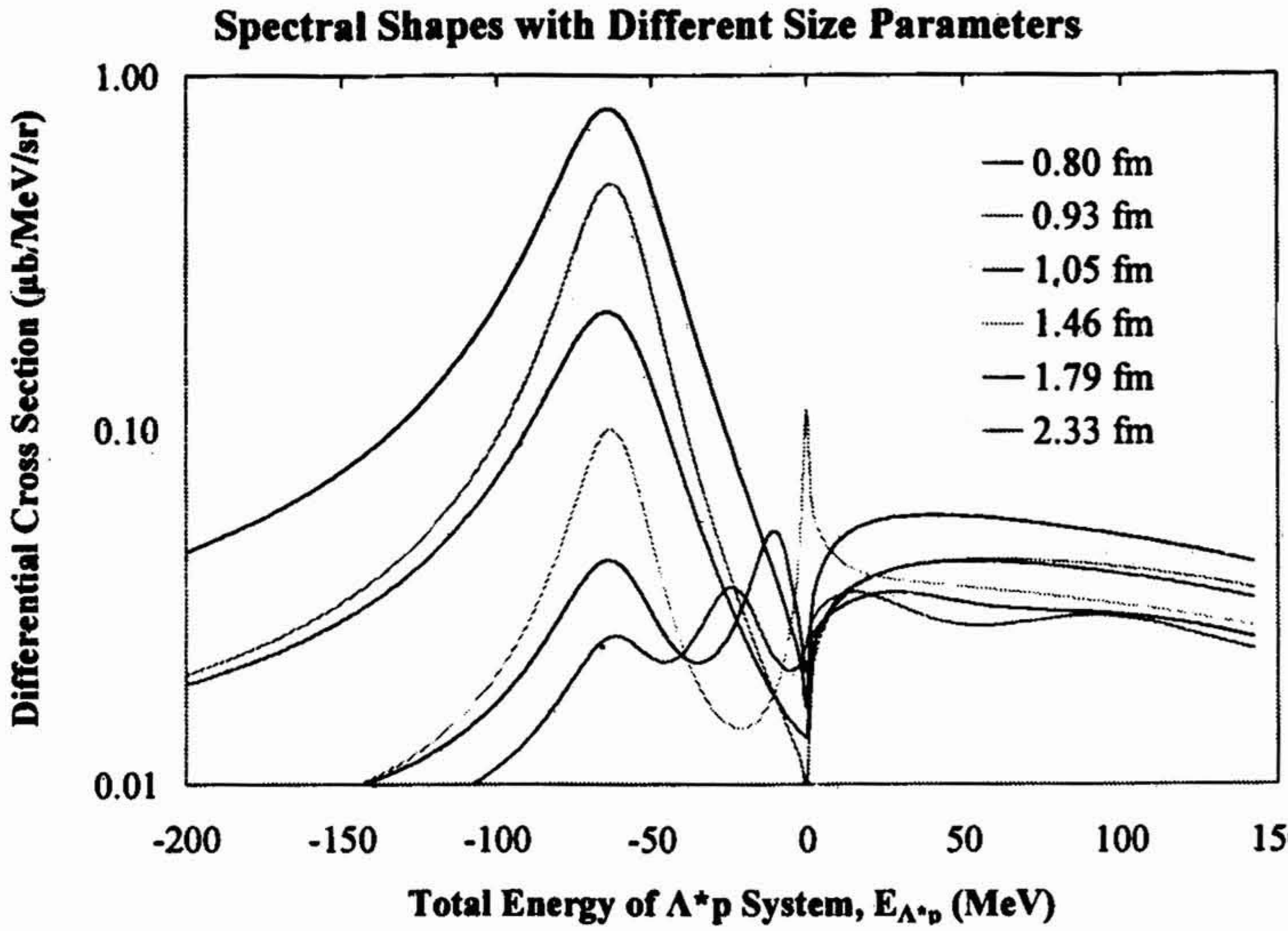

Figure (4) Spectral shapes of differential cross section with different size parameters.

References

Akaishi, Y., and Yamazaki, T. (2002). Nuclear $\overline{K}$ bound sates in light nuclei. Phys. Rev. **C** **65**(044005), 1-9.

Morimatsu, O., and Yazaki, K. (1985). The formation probabilities of Σ-Hypernuclei and the "Unstable Bound State". Nuclear Physics **A435**, 727-737.

Yamazaki, T., and Akaishi, Y. (2006). Enhanced formation of a dense $\overline{K}$ nuclear cluster $K^{-}pp$ in pp collisions - Λ*p doorway dominance, 0604049, 1-8. Retrieved April 20, 2006, from http://xxx.lanl.-gov/arXiv:-nuclthl.