YANGON UNIVERSITY OF ECONOMICS

DEPARTMENT OF STATISTICS

APPLICATION OF QUEUING MODEL FOR BANKING SYSTEM: A CASE STUDY OF PUBLIC AND PRIVATE BANKS IN YANGON CITY

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M.Econ (Statistics)

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This thesis is submitted as a partial fulfillment towards

the Degree of Master of Economics (Statistics)

BY

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ABSTRACT

This study focuses on the waiting lines of public and private banks in Yangon City as a case study. This study contains the analysis of queuing system for single-channel and multi-channel for selected banks. This study is determined the waiting line characteristics such as the average arrival rate, the average service rate, the probability that there are no customers in the system, the average number of customers in the queue, the average number of customers in the system, the average time a customer spends in the queue, the average time a customer spends in the system and server utilization based on the single-channel and multi-channel queuing system and the total minimum expected cost of the selected banks. This study includes three service counters which are withdraw counter, deposit counter and remittance counter. Regarding the methodologies, the direct observation method is used to determine customers arrival and service distribution. Moreover, key informant interview is applied to calculate service cost, and the one-sample Kolmogorov-Smirnov test is used to test whether a sample comes from a population with a specified distribution. Based on findings, it is highlighted that most of the service counters must be extended to increase service rate and customers satisfaction and managers should be emphasized to reduce queuing time of the customers.

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LIST OF ABBREVIATIONS

FCFS	first come, first served
LCFS	last come, first served
SIRO	service in random order
GD	general service discipline
M/M/1	single-channel queuing model
M/M/s	multi-channel queuing model
KII	key informant interview
ECDF	empirical cumulative distribution function
MEB	Myanma Economic Bank
KBZ	KanBawZa Bank
CSO	Central Statistical Organization

CHAPTER I

INTRODUCTION

1.1 Rationale of the Study

In modern society, a competitive business environment is progressively turning into a service dominating one. Customer satisfaction and service operation capabilities represent as a provider of a successful organization. Lack of satisfying service facilities would cause the waiting line of customers to be formed. One of the techniques is making to increase the service capacity increasing the efficiency of the existing capacity to a higher level (Sheikh et al., 2013). If customers wait in line for hours, they might be dissatisfied with their waiting. Nowadays, bank service providers are more focusing on customer satisfaction for their services because it is important that only satisfied customers may loyal and remain in service and dissatisfied customers might not come back again. Highly satisfied customers will be very likely to give repeat business and spread an optimistic experience by word of mouth (advertising), resulting in increased profits and productions. Therefore, one must learn about the behavior of customers in the market to provide efficient service to the customer as well as to stabilize the loyalty of customers. Since waiting in line is one of the major factors to retain the customers' loyalty and to develop their business.

In 1903, the queuing theory (also called waiting line theory) was developed by A.K. Erlang, began the problem of congestion of telephone traffic. Waiting line or queuing theory has been applied to a wide variety of business situations. The trouble was that during occupied periods, phone administrators were not able handle the calls the minute they were made bringing about deferred calls. A.K. Erlang coordinated his first endeavors at discoveries the deferral for one administrator and later on, the outcomes were stretched out to discover the postponement for a few operators. After World War II, lines or holding up lines were reached out to other general issues.

Queuing theory is the mathematical study of queuing, or waiting in lines. Waiting lines are a regular event, influencing individuals looking for food supplies, purchasing gas, or making a bank store. The three basic components of a queuing process are arrivals, service facilities, and the actual waiting line. In the queuing problem, the number of arrivals per unit time can be estimated by the Poisson probability distribution. Arrivals are not always Poisson and should be examined to make certain that they are well approximated by Poisson before that distribution is applied. More often, service times are randomly distributed. In many cases, it can be assumed that service times are a negative exponential probability distribution. This is mathematical convenient assumption if arrival rates are Poisson distributed (Render et al., 2012). Management scientists have found that they are best described by the exponential probability distribution. Waiting line system was generally characterized by either (1) Poisson input (the number of calls), exponential holding (service) time, and multiple channels (servers), or (2) Poisson input, constant holding time and a single channel. The study of waiting lines, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques.

Waiting in lines seems to be a part of our daily life. Queuing can be increased the time of waiting and induce the customer to complain and dissatisfy. Checkout stands are the administration windows of banks, which reflect banks' pictures as well as partner with banks' administration quality and business productivity. Waiting time relies upon the quantity of clients (people or items) on line, the quantity of servers, and the measure of administration time for every client. The main goal of queuing management is to maximize the level of customer satisfaction with the service provided. Low level of service might not be expensive, at least in the short run, but may incur high costs of customer dissatisfaction, such as loss of future business. A high level of service would cost more to provide and would result in lower dissatisfaction costs.

In banking operations, the most common measure of customer satisfaction is the average waiting time, i.e., the time that customers wait before service. The delay in receiving service will lead to queuing and waiting cost to customers. The important goal of queuing is essentially to minimize the total cost, service cost plus waiting cost, of the system. Therefore, banks' managers are concerned about providing the optimal service configuration with optimal service cost and waiting cost. According to these situations, this study intends to examine the waiting line characteristics of customers for the public and the private banking system in Yangon City.

1.2 Objectives of the Study

The objectives of the study are as follows:

(i) To determine the waiting line characteristics based on the Single-Channel and Multi-Channel Queuing Model.

(ii) To find out the total minimum expected cost of the selected banks.

1.3 Method of Study

The required data were collected during the peak period for a multi-channel of the bank by using the personal interview. This study was based on the primary data. In this study, the single-channel queuing (M/M/1: FCFS) model and multi-channel queuing (M/M/s: FCFS) model were used. In addition, SPSS (Statistical Package for the Social Sciences), and TORA (Operation Research Software) were used in this study.

1.4 Scope and Limitations of the Study

Data for this study were collected from Myanma Economic Bank (Kamayut Branch and Insein Branch) and Kanbawza Bank (Branch 6). Arrival rate and service rate were checked whether they followed specified distribution by using a one-sample Kolmogorov-Smirnov test with 5% significant level. The data were gathered from the daily record of the queuing system over two weeks started from the last week of July to the first week of August in MEB (Kamayut) and last week of September in MEB (Insein) and KBZ (Kamayut). The customers who come to those banks between 09:00 AM to 03:00 PM were observed for the study.

1.5 Organization of the Study

This study is organized into five chapters. Chapter one is an introduction and it describes rationale, objectives, method of study, scope and limitations and organization of the study. Chapter two presents the literature review which includes application of queuing models in banking system and other organizations. Chapter three reviews the methodology. Chapter four describes application of queuing model in selected banks. Chapter five presents the conclusion with findings and suggestions.

CHAPTER II

THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 Definition and Concepts of Queuing Theory

Queuing theory is the mathematical study of the congestion and delays of waiting in line. It examines every component of waiting in line to be served, including the arrival process, service process, number of servers, number of system places, and the quantity of clients which may be individuals, information bundles, vehicles, and so forth. Real-life applications of queuing theory cover a wide range of applications, such as how to provide faster customer service, improve traffic flow, efficiently ship orders from a warehouse, and design of telecommunications systems, from data networks to call centers (Kenton, 2019).

Queuing theory is a part of operational research, and queuing theory is widely used to understand queuing system behavior to predict, control and optimize the firm's performance. It is a very important aspect of the successful management of the queuing system. Hence, the importance of queuing theory cannot be underestimated. One of the basic parts of queuing theory is the performance evaluation of a system (Kumar, Vij, & Kumar, 2015).

2.2 Theoretical Reviews on Queuing Models

2.2.1 Characteristics of a Queuing System

In this sub section, the queuing system was discussed. They are (1) the arrivals or inputs to the system (the calling population), (2) the queue or waiting line itself, and (3) the service facility. These three components have certain characteristics that must be examined before mathematical queuing models can be developed.

(1) Arrival Characteristics

The input source that generates arrivals or customers for the service system has three major characteristics. It is important to consider the size of the calling population, the pattern of arrivals at the queuing system, and the behavior of the arrivals.

Arrival characteristics are summarized in figure (1.1) as below:

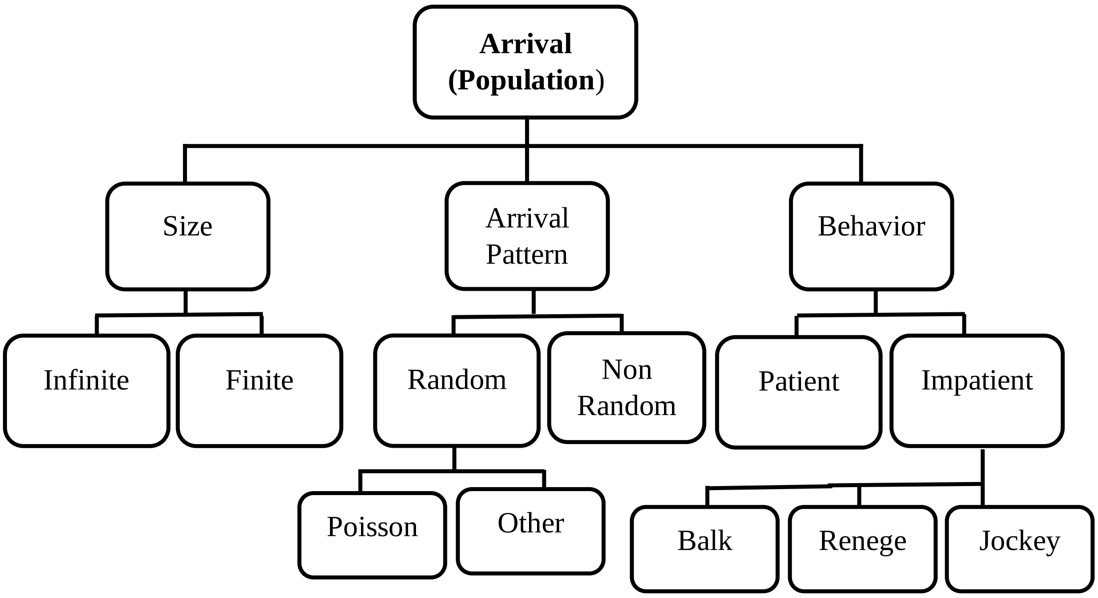

Figure 2.1: Arrival Characteristics

Size of the Calling Population: Population sizes are considered to be either finite or infinite. If there are only a few potential customers, the calling source of population is called finite. When the number of customers or arrivals on hand at any given moment is just a small portion of potential arrivals the calling population is considered infinite. Most queuing models assume such an infinite calling population. When this is not the case, modeling becomes much more complex.

Pattern of Arrivals at the System: Customers either arrive at a service facility according to some know schedule or else they arrive randomly. Arrivals may also be represented by the inter-arrival time, which is the period between two successive arrivals. The rate at which customers arrive to be served, i.e., the number of customers arriving per unit of time is called the arrival rate. Arrivals are considered random when they are independent of one another and their occurrence cannot be predicted exactly. Frequently in queuing problems, the number of arrivals per unit of time can be estimated by a probability distribution known as the Poisson distribution. The mean value of the arrival rate is represented by λ. It may be noted that the Poisson distribution with mean arrival rate λ is equivalent to the (negative) exponential distribution of inter-arrival times with mean inter-arrival time $1/\lambda$.

Poisson distribution can be established by using the following formula.

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \text{ (for } x = 0,1,2, \ldots)$$

where P(x) = probability of exactly x arrivals or occurrences

λ = average number of arrivals per unit of time (the mean arrival rate)

e = 2.718, the base of natural logarithm

x = number of arrivals per unit of time

The average arrival rate can be calculated as follow:

$$\text{average arrival rate} = \lambda = \frac{Total\ number\ of\ customers}{total\ interarrival\ time}$$

Behavior of the Arrivals: Most queuing models assume that an arriving customer is a patient customer. Patient customers are people or machines that wait in the queue until they are served and do not switch between lines. Balking refers to customers who refuse to join the waiting line because it is too long to suit their needs or interests. Reneging customers are those who enter the queue but then become impatient and leave without completing their transaction. When there are two or more parallel queues and the customers move from one queue to the other to reduce waiting time, they are said to be jockeying.

(2) Waiting Line Characteristics

The waiting line itself is the second component of a queuing system. Waiting lines characteristics is summarized in the figure (2.2) as below:

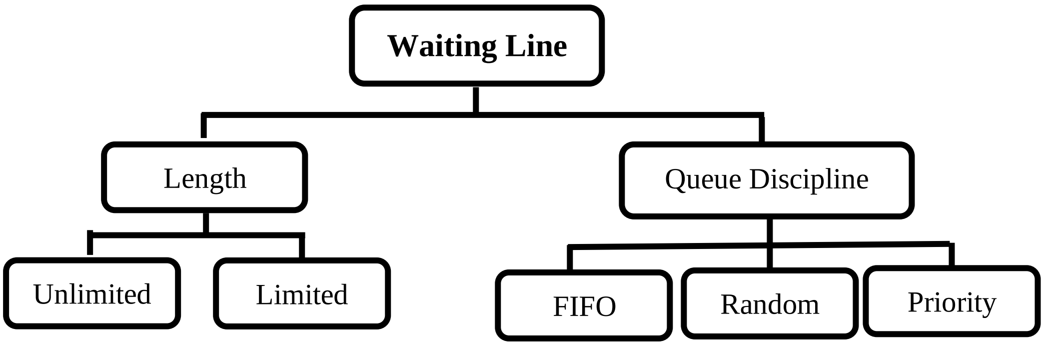

Figure 2.2: Waiting Line Characteristics

Length of Queue: The length of a line can be either limited or unlimited. A queue is limited when it cannot increase to an infinite length. It is unlimited when its size is unrestricted, as in the case of tollbooth serving arriving automobiles.

Queue Discipline: A second waiting line characteristic deals with queue discipline. This refers to the rule by which customers in the line are to receive service. The most common discipline is ‘first come, first served’, according to which the customers are served in the order of their arrival. The other discipline is ‘last come, first serve’, ‘service in random order (SIRO)’, and ‘priority’.

(3) Service Facility Characteristics

The third part of any queuing system is the service facility. It is important to examine two basic properties: (1) the configuration of the service system and (2) the pattern of service times.

Service facility characteristics are summarized in the following figure (1.3).

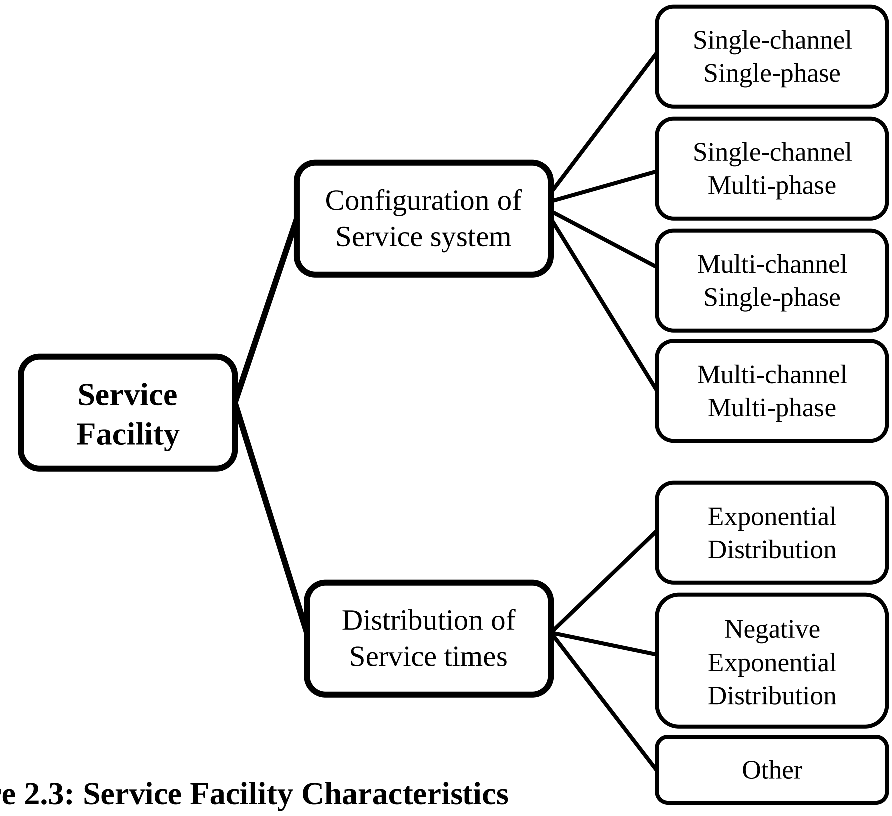

Figure 2.3: Service Facility Characteristics

Basic Queuing System Characteristics: Service systems are usually classified in terms of their number of servers and the number of service stops, that must be made. A *single-channel system* is typified by the drive-in bank that has only one open teller. On the other hand, if the bank had several tellers on duty then it would have a *multiple-channel system* at work.

A *single-phase system* is one in which the customer receives service from only one station and then exits the system. On the other hand, if the customer receives from one station, pay at a second and pick up the service another station, it becomes a *multiphase system*.

Service Times Distribution: Service pattern is like an arrival pattern in that they may be either constant or random. If service time is constant, it takes the same amount of time to take care of each customer. In many cases, service times are random, management scientists have found that they are best described by the exponential probability distribution. The number of customers served per unit of time is called the service rate. This rate assumes the service channel to be always busy, i.e., no idle time is allowed. The mean value of the service rate is represented by μ.

Exponential distribution can be established by using the following formula.

$$f(t) = \mu e^{-\mu t} \text{ for } t \geq 0$$

where t = service time (expressed in number of periods)

μ = average number of units that the service facility can handle in a specific period (the mean service rate)

e = 2.718, the base of the natural logarithm

The average service rate can be calculated as follow:

$$\text{average service rate} = \mu = \frac{Total\ number\ of\ customers}{total\ service\ time}$$

2.2.2 Operating Characteristics of a Queuing Model

Analysis of a queuing system involves a study of its different operating characteristics. Some of them are

1. Queue length (L_q) – the average number of customers in the queue waiting to get service. This excludes the customer(s) being served.
2. System length (L_s) – the average number of customers in the system including those waiting as well as those being served.
3. Waiting time in the queue (W_q) – the average time for which a customer has to wait in the queue to get service.
4. Total time in the system (W_s) – the average total time spent by a customer in the system from the moment he arrives till he leaves the system. It is taken to be the waiting time plus the service time.
5. Utilization factor (ρ) – it is the proportion of time a server spends with the customers. It is also called traffic intensity.

2.2.3 Assumptions of Queuing Model

Hiller and Hiller (2003) summarize the assumption generally made by queuing models of a basic queuing system. Each of these assumptions should not be taken for granted unless a model explicitly states otherwise.

1. Inter-arrival times are independent and identically distributed according to a specified probability distribution.

2. All arriving customers enter the queuing system and remain there until service has been completed.
3. The queuing system has a single infinite queue so that the queue will hold an unlimited number of customers.
4. The queue discipline is first-come, first-served.
5. The queuing system has a specified number of servers
6. Each customer is served individually by anyone of the servers.
7. Services times are independent and identically distributed according to a specified probability.

2.3 Kendall's Notation for Representing Queuing Models

D.G. Kendall (1953) introduced useful notation to summarize the characteristics of queuing models (Taha, 2017). The complete notation can be expressed as (a/b/c): (d/e/*f*)

where a = arrival (or interarrival) distribution,

b = departure (or service time) distribution,

c = number of parallel service channels in the system,

d = service discipline

e = maximum number of customers allowed in the system,

f = calling source or population

The following conventional codes are generally used to replace the symbols a, b and d: Symbols for a and b

M = Markovian (Poisson) arrival or departure distribution (or exponential interarrival or service time distribution),

E_k = Erlangian or gamma interarrival or service time distribution with parameter k,

GL= general independent arrival distribution,

G = general departure distribution,

D = deterministic interarrival or service times.

Symbols for d

FCFS = first come, first served,

LCFS = last come, first served,

SIRO = service in random order,

GD = general service discipline.

The symbols e and *f* represent a finite (N) or infinite (∞) number of customers in the system and calling source. For instance, (M, E_k, 1): (FCFS/N/∞) represents Poisson arrival (exponential interarrival), Erlangen departure, single server, 'first come, first served' discipline, maximum allowable customers N in the system and infinite population model.

2.4 Identifying Models Using Kendall Notation

2.4.1 Single-Channel Queuing Model with Poisson Arrivals and Exponential Service Times (M/M/1)

A single channel waiting line consists of only one service channel. The arriving units wait in a single waiting line. Once the first customer was served, the service provider calls the next customer in the waiting line. The formulas of the waiting line are applicable based on the following assumptions.

Assumptions of the Model

The single-channel, single-phase model is one of the most widely used and simplest queuing models. It involves the following seven conditions exist:

1. Arrivals are served on a FIFO basis.
2. Every arrival waits to be served regardless of the length of the line; that is, there is no balking or reneging.
3. Arrivals are independent of preceding arrivals, but the average number of arrivals (the arrival rate) does not change over time.
4. Arrivals are described by a Poisson probability distribution and come from an infinite or a very large population.
5. Service times also vary from one customer to the next and are independent of one another, but their average rate is known.
6. Service times occur according to the negative exponential probability distribution.
7. The average service rate is greater than the average arrival rate.

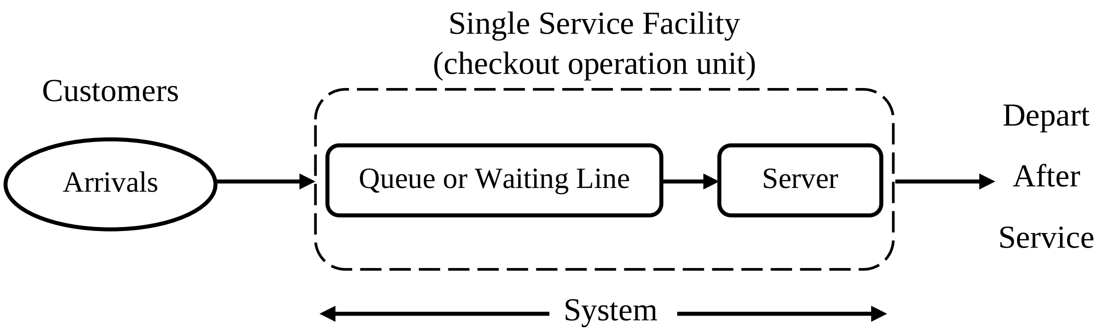

Figure 2.4: Single-channel Single-phase System

Equations for the Single-Channel Queuing Model

Let λ = mean number of arrivals per period, and

μ = mean number of served per period

When determining the arrival rate (λ) and the service rate (μ), the same period must be used. For example, if λ is the average number of arrivals per hour, then μ must indicate the average number of served per hour.

The queuing equations are as follow:

1. The average number of customers or units in the system, L_s, the number in line plus the number being served:

$$L_s = \frac{\lambda}{\mu - \lambda}$$

2. The average time a customer spends in the system, W_s, the time spent in line plus, the time spent being served:

$$W_s = \frac{1}{\mu - \lambda}$$

3. The average number of customers in the queue, L_q:

$$L_q = \frac{\lambda^2}{\mu\ (\mu - \lambda)}$$

4. The average time a customer spends waiting in the queue, Wq:

$$W_q = \frac{\lambda}{\mu\ (\mu - \lambda)}$$

5. The utilization factor for the system, ρ, the probability that the service facility is being used:

$$\rho = \frac{\lambda}{\mu}$$

6. The percent idle time, P_0, the probability that no one is in the system:

$$P_0 = 1 - \frac{\lambda}{\mu}$$

7. The probability that the number of customers in the system is greater than t, $P_{n > t}$:

$$P_{n > t} = \left(\frac{\lambda}{\mu}\right)^{t+1}$$

2.4.2 Multi-Channel Queueing Model with Poisson Arrivals and Exponential Service Times (M/M/s)

A multiple-channel queuing system in which two or more servers or channels are available to handle arriving customers. Assume that customers awaiting service from one single line and then proceed to the first available server. The formulas of multiple-channel waiting lines are applicable based on the following assumption.

Assumptions of the Model

Multi-channel, single-phase waiting lines are found in many banks today: A common line is formed, and the customer at the head of the line proceeds to the first free teller. The following assumptions are reasonable for multi-channel waiting line models.

1. The waiting line has two or more channels.
2. The pattern of arrivals follows as a Poisson probability distribution.
3. The service time for each channel follows an exponential probability distribution.
4. The mean service rate (μ) is the same for each channel.
5. The arrivals wait in a single waiting line and then move to the first open channel for served.
6. The queue discipline is first-come, first-served (FCFS).

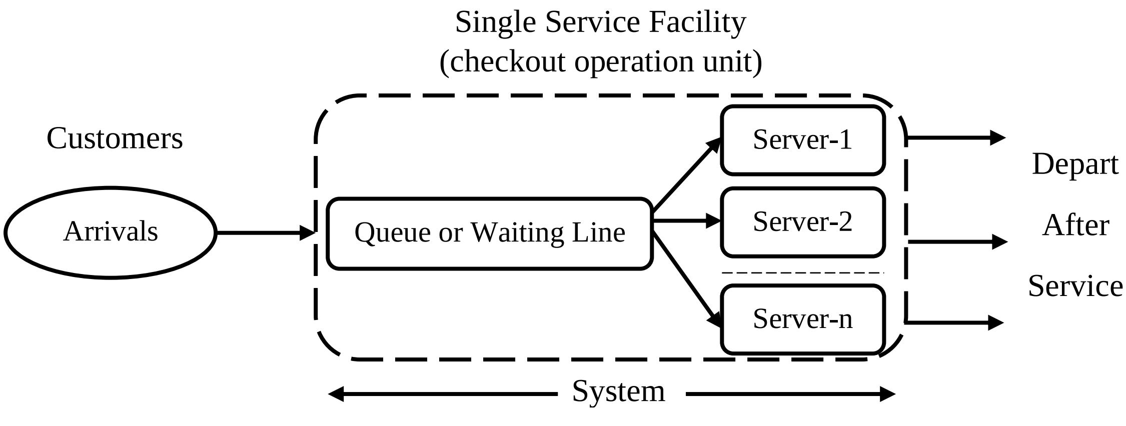

Figure 2.5: Multi-channel Single-phase System

Equations for the Multi-channel Queuing Model

Let s = number of channels open,

λ = average arrival rate, and

μ = average service rate at each channel

The queuing equations are as follow:

1. The probability that there are no customers or units in the system, P_0:

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s\frac{s\mu}{s\mu-\lambda}} \quad \text{for } s\mu > \lambda$$

2. The average number of customers or units in the system, L_S:

$$L_S = \frac{\lambda\mu\,(\lambda/\mu)^s}{(s-1)!(s\mu-\lambda)^2}\,P_0 + \frac{\lambda}{\mu}$$

3. The average time a unit spends in the waiting line or being serviced (namely, in the system), W_s:

$$W_s = \frac{\mu\,(\lambda/\mu)^s}{(s-1)!\,(s\mu-\lambda)^2}\,P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

4. The average number of customers or units in line waiting for service, L_q:

$$L_q = L - \frac{\lambda}{\mu}$$

5. The average time a customer or unit spends in the queue waiting for service, W_q:

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

6. Utilization rate, ρ:

$$\rho = \frac{\lambda}{s\mu}$$

These equations are more complex than the ones used in the single-channel model.

2.5 Introducing Costs into the Model

To solve a queuing problem, the service facility must be manipulated to obtain an optimum balance between service cost and waiting cost. The cost of waiting customers includes either the indirect cost of lost business or direct cost of idle equipment and persons. To determine how much business is lost, some type of experimentation and data collection is required. By increasing the investment in labor and service facilities, waiting time and the losses associated with it can be decreased.

To conduct an economic analysis of a waiting line model, a total cost model, which includes the cost of waiting and the cost of service, must be developed. These two costs, the waiting cost per period is usually more difficult to evaluate. The service cost per period is generally easier to determine. This cost would include the server`s wages, benefits, and any other direct cost associated with operating the service channel. To develop a total cost model for a waiting line system, the following notations defined as follow;

C_w = opportunity cost of waiting by the customers

W_s = average waiting time a customer spends in the system

C_s = service cost of each server

λ = average arrival rate per hour

s = number of servers

Total hourly waiting cost = (8 hours per day) $\lambda W_s C_w$

Total hourly service cost = (8 hours per day) sC_s

The total hourly cost of the system is the sum of the total hourly waiting cost and the total hourly service cost; that is,

Total hourly cost of the queuing system = Total hourly waiting cost + Total hourly service cost

2.6 Managerial Applications of Queuing Model

Queueing theory has been used for many real-life applications to the great advantage and a valuable tool for business decision-making. Queuing theory techniques can be applied to a wide variety of situations for scheduling such as:

(a) Planning, scheduling and sequencing of parts and components to assembly lines in a mass-production system.

(b) Scheduling of workstations and machines performing different operations in mass production

(c) Scheduling and dispatch of war material of special nature-based on operational needs

(d) Scheduling of service facilities in a repair and maintenance workshop.

(e) Scheduling of overhaul of used engines and other assemblies of aircraft, missile systems, transport fleet, etc.

(f) Scheduling of limited transport fleet to a large number of users

(g) Parts and components in assembly lines routing salespersons

(h) Scheduling of landing and take-off from airports with the heavy-duty of air traffic and air facilities

(i) Replacement of capital assets

(j) Minimization of congestion due to traffic delays at booths.

(k) The decision of replacement of plant, machinery, special maintenance tools and another equipment base on different criteria.

Queuing theory has generally been applied by factories, transport companies, telephone exchanges, computer centers, retail stores, cinema houses, restaurants, banks, insurance companies, traffic control authorities, hospitals, etc.

2.7 Benefits and Limitations of Queuing Theory

Gupta & Garg, (2012) conducted a study in the context of "A View of Queue Analysis with Customer Behavior, Balking and Reneging". The author proposed the benefits and limitations of queuing theory as follows.

2.7.1 Benefits of Queuing Theory

Special benefits which this technique enjoys in solving problems are

(1) Queueing theory attempts to solve problems based on a scientific understanding of the problems and optimally solving them so that facilities are fully utilized and waiting time is reduced to the minimum possible.

(2) Waiting time (or queueing) theory models can recommend the arrival of customers to be serviced, setting up workstations, the requirement of manpower, etc. based on probability theory.

2.7.2 Limitations of Queuing Theory

Queueing theory provides many organizations as a scientific method of understanding the queues and solving such problems, the theory has certain limitations which must be understood while using the technique, some of these are:

(1) Mathematical distribution, which assumes while solving queueing theory problems, is only a close approximation of the behavior of customers, the time between their arrival and service time required by each customer.

(2) Most real-life queuing problems are a complex situation and very difficult to use the queueing theory technique, even then uncertainty will remain.

(3) Many situations in industry and service are multi-channel queueing problems. When a customer has been attended to and the service provided, it may still have to get some other service from another service and may have to fall in the queue once again. Here the departure of one channel queue becomes the arrival of

another queue. In such situations, the problem becomes still more difficult to analyze.

(4) Queueing model may not be an ideal method to solve certain very difficult and complex problems and one may have to resort to other techniques like Discrete-Event simulation or Monte-Carlo simulation method.

2.8 Empirical Reviews on Queuing Models

2.8.1 Application of Queuing Models in Banking Systems

Sheikh, Singh, & Kashyap (2013) analyzed Multi-Server Queuing Models. In this paper, the Multi-Server Queuing Theory Model was used. By the investigation it was inferred that as the service level was expanded, the waiting time of the client was diminished, the administration proficiency was expanded and the consumer loyalty was expanded and just as it was discovered that at some particular quantities of service windows the complete system cost was limited. The authors also conducted "*Application of Queuing Theory for the Improvement of Bank Service*”. In that paper, the authors considered the M/M/Z/∞: FCFS model to convert into M/M/1/∞: FCFS to know which one was more efficient. In that study, multi-channel and single-channel queuing models were used. The results of that study were the time of customer queuing was reduced, the customer satisfaction was increased and it was proved that this optimal model of the queuing was feasible.

A few researchers used work-study techniques to improve the service quality of banking systems. Madadi et al., (2013) investigated and suggested the best configuration for a bank using a simulation model. In that study, Chi-square goodness of fit test was used to test the inter-arrival time of customers followed by a negative exponential distribution and WITNESS software was used to perform a simulation. That study attempted to investigate the best possible configuration for a bank in Malaysia through constructing computer-based simulation models. The creators suggested that the expansion of another counter, expelling the service table and institutionalizing the move of all counters was the most reasonable design as far as use rate, the normal waiting time of clients and the expense of usage.

Sahu & Sahu (2014) conducted on the title of “*Implementation of Single-Channel Queuing Model to Enhance Banking Services*”. In this study, the authors implemented a queuing management system in a bank single-channel queuing system.

The model illustrated in that bank was the single-channel queuing model with Poisson Arrival and Exponential Service Times (M/M/1). In that study, little's theorem and M/M/1 queuing model were derived. Little's theorem is easy to state, natural, broadly material, and depends just on powerless suspicions about the properties of the waiting line. That paper presumed that M/M/1 model gave the ideal outcome and had a high pace of usage proportion, working proficiency and less level of the inert worker. The normal number of clients in the queue and system, the normal time spent by any client in the system and queue and the traffic force of [M/M/1] queueing model were useful and better.

Queueing theory is the conventional investigation of waiting in line and a whole control inside the field of operations management. According to Okhuese (2018), problematic queuing systems can lead to the customer's perceptions of excessive, unfair, or unexplained waiting time. The author conducted "*Application of Queuing Theory: Analysis of Services of Commercial Banks (A case study of branches of some commercial banks in Abuja, Nigeria)*". The aim of this research paper was trying to estimate the waiting time and length of the queue(s). The single-queue multi-server model, single-queue single-server and multiple-queue multi-server models were used. Simulation was used to obtain a sample performance result and estimated solutions for multiple queuing models. The empirical analysis of the queuing system of the three commercial banks in that research as they may not be very efficient in terms of resource utilization. Queues form and customers wait even though counter-tellers or ATMs might be idle much of the time. It was a direct consequence of the variability of the arrival and service processes.

2.8.2 Application of Queuing Models in Sales Systems

V, Sikdar, & Medhi (2017) investigated a study of "*Modelling of Queueing System at Sales Checkout and Analysis of Consumer Behaviour*" with a Supermarket in Bengaluru as a case study. This examination was brought out through a structured questionnaire was administered to 100 customers at a select Grocery store in Bengaluru city. Exact information of appearance and administration times were recorded through observation methods to break down the presentation proportions of the multi-channel multi-queue system at the checkout counters. Monte Carlo Simulation technique and statistical test of Chi-square and correlation had been utilized. The average time customers wait in the queue and system was about 2 minutes and 5.5 minutes. The utilization at server one was 93.4% which indicated that it was busy most of the time. It had two servers and the utilization at server two was 62%. This analysis suggested that the cost associated with one of the servers could be reduced.

Jhala & Bhathawala (2017) examined efficient queue management in XYZ supermarket as a case study. The aimed of that paper was trying to estimate the waiting time and length of the queue(s). The researchers used direct observation methods, personal interviews, and questionnaires to collect the required data. Single-channel and multi-channel queuing theory were used. The authors concluded that single-queue multi-server was better in comparison to multi-queue multi-server. The waiting time of customers waiting in the queue was reduced almost 3 times to the previous one.

Queuing systems exist everywhere. Each restaurant might want to abstain from losing their ideal clients because of a long wait in the queue. Molla (2017) analyzed the restaurant queuing model with Shuruchi restaurant as a case study in Dhaka city. The aimed of that paper was to show the single-channel queuing model M/M/1. Little's theorem and the single-channel waiting line model were used. That research paper had concluded that it could help to do the betterment of the restaurant. Developing a simulation model, it allowed adding more complexity.

Dharmawirya & Adi (2012) explored the queuing model with the restaurant as a case study. From the empirical analysis, that study found that the arrival rate and service rates were 2.22 and 2.24 customers per minute. The probability of buffer overflow is the likelihood that clients will flee in light of the fact that they are eager to wait in the queue. That review inferred that the arrival rate would be lesser and the service rate would be more prominent on the off chance that it was on weekdays since the normal number of clients was less when contrasted with those on ends of the week.

Gumus, Bubou, & Oladeinde (2017) examined "*Application of Queuing Theory to a Fast Food Outfit: A Study of Blue Meadows Restaurant*". Multi-channel queuing model and chi-square goodness of fit test were used. The results obtained from the study showed that the arrival pattern followed a Poisson distribution and the service pattern followed an exponential distribution. That study concluded that to improve operations within the waiting line, the service rate should be improved. The result of the research work might serve as a reference to analyze the current system and improved the next system.

2.8.3 Application of Queuing Models in Health Care Systems

Waiting in lines seems to be a component of human daily life. Queues form when the demand for a service exceeds its supply. In hospitals, patients can wait a certain period (minutes, hours, days or months) to receive healthcare service. For many patients, waiting in lines or queue is an annoying or negative experience. Queuing theory is a case of the utilization in healthcare. It basically manages patients move through the system, in the event that the patient stream is great, at that patient queuing is minimized, on the off chance that it is terrible, at that point the system may endure the loss of business and patients may endure extensive queuing delays.

Mehandiratta (2011) and Singh (2017) applied queuing theory in health care. In the health sector, the queuing theory was mainly used in the Emergency Department (ED) wait line and staffing studies, analysis of queues in outpatient and ambulatory care settings and for disaster management. The authors attempted to analyze the queuing theory and instance of the use of queuing theory in health care organizations around the world and benefits acquired from the same. The authors concluded that with the increasing cost pressure, changing reimbursement mechanisms and affiliations, pressure for quality control, and awareness and demands of the patients.

According to Singh (2017), the goal of queuing analysis and its application in healthcare organizations is to “minimize cost” to the organization – both tangible and intangible. The cost that was considered: capacity costs, waiting costs, the cost of waiting space, the cost to the society and the effects of loss of business to healthcare organization if patients refused to wait and decided to go elsewhere.

CHAPTER III

METHODOLOGY

In this chapter, one-sample Kolmogorov-Simonov statistical test, application of Poisson probability distribution on customer arrival times, application of exponential probability distribution on customer service times, single-channel Poisson arrivals with exponential service, infinite population model, and multi-channel queuing model were mentioned.

3.1 One-Sample Kolmogorov-Smirnov Statistical Test

To test whether a sample comes from a population with a specified distribution, the Kolmogorov Smirnov (K-S) test is used. The K-S test is based on the empirical cumulative distribution function (ECDF), it measures the difference between the empirical cumulative distribution function and the hypothesized cumulative distribution function. The K-S test is distribution-free since its critical values do not depend on the specific distribution being tested.

In the K-S test, the values of the observed cumulative distribution of the random variable in a given situation are compared with the corresponding values of the theoretical cumulative distribution of the random variable and their absolute differences are calculated. Then, the maximum of these absolute differences is treated as the calculated statistic of the K-S test, namely D_{cal} which is given by the following formula:

$$D_{cal} = \max|OF_i - EF_i|$$

where OF_i is the observed cumulative probability for the i^{th} value of the random variable X, EF_i is the expected cumulative probability for the i^{th} value of the random variable X, from the theoretical distribution.

The Kolmogorov Smirnov (K-S) test is defined as follows:

H_0: The data follows a specified distribution

H_1: The data does not follow a specified distribution.

Reject the null hypothesis if the calculated value of D is greater than the theoretical value of D at a given significance level otherwise accept the null hypothesis.

3.2 Application of Poisson Probability Distribution on Customer Arrival Times

The Poisson distribution counts the number of discrete events in a fixed period; it is closely connected to the exponential distribution. Waiting line models assume that customers arrive according to a Poisson probability distribution. The Poisson probability distribution specifies the probability that a certain number of customers will arrive in a given period. Defining the arrival process for a queue involves determining the probability distribution for the number of arrivals in a given period. In many queuing situations, the arrivals occur in a random; each arrival is independent of other arrivals, and we cannot predict when an arrival will occur. In this situation, the Poisson probability distribution is used to describe the arrival pattern. The Poisson distribution is discrete; the random variable can only take non-negative integer values.

3.3 Application of Exponential Probability Distribution on Customer Service Times

The exponential distribution can take any (nonnegative) real value. The exponential distribution describes the service times as the probability that particular service time will be less than or equal to a given amount of time. The service time is the time that the customer spends at the service facility once the service has started. Service time normally varies according to individual situations.

3.4 Single-Channel Poisson Arrivals with Exponential Service, Infinite Population Model (M/M/1): (FCFS/∞/∞)

The following mathematical notation (symbols) were used in queuing models:

Let n = number of customers in the system (waiting line + service facility) at time t

λ = mean arrival rate (number of arrivals per unit of time)

μ = mean service rate per busy server (number of customers served per unit of time)

$\lambda\, dt$ = probability that an arrival enters the system between t and $t + dt$ time interval i.e., within time interval dt

$1-\lambda\, dt$ = probability that no arrival enters the system within interval dt plus higher-order terms in dt

$\mu\, dt$ = probability of no service completion between t and $t + dt$ time interval i.e., within time interval dt

1-μ dt = probability of no service rendered during the interval dt plus higher-order order terms in dt

P_n = steady-state probability of exactly n customers in the system

P_n (t) = transient state probability of exactly n customers in the system at time t, assuming the system started its operation at time zero

P_{n+1} (t) = transient state probability of having n + 1 customers in the system at time t

P_{n-1} (t) = transient state probability of having n – 1 customer in the system at time t

P_n (t + dt) = Probability of having n customers in the system at time t + dt

L_q = expected number of customers in the queue

L_s = expected number of customers in the system (waiting + being served)

W_q = expected waiting time per customer in the queue

W_s = expected time a customer spends in the system (in waiting + being served)

L_n = expected number of customers waiting in line excluding those times when the line is empty

W_n = expected time a customer waits in line if he has to wait at all

The probability of n units (customers) in the system at time t + dt can be determined by summing up probabilities of all the ways this event could occur. The event can occur in four mutually exclusive and exhaustive ways:

Event	No. of units at time t	No. of arrivals in time dt	No. of services in time dt	No. of units at time t + dt
1	n	0	0	n
2	n + 1	0	1	n
3	n -1	1	0	n
4	n	1	1	n

3.4.1 Steady state probability of exactly n units (customers) in the system

Compute the probability of occurrence of each of the events, the probability of a service or arrival is $\mu\, dt$ or $\lambda\, dt$ and $(dt)^2 \to 0$,

$$P_0\,(t+dt) = P_0(t)\,(1-\lambda dt) + P_1(t)(\mu dt) \qquad (1)$$

$$P_1\,(t+dt) = P_0(t)(\lambda dt) + P_1(t)(1-\mu dt)(1-\lambda dt) + P_2(t)(\mu dt) \qquad (2)$$

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$P_n(t+dt) = P_{n-1}(t)(\lambda dt) + P_n(t)(1-\mu dt)(1-\lambda dt) + P_{n+1}(t)(\mu dt)$ (3)

from eq (1);

$P_0(t+dt) = P_0(t) - P_0(t)(\lambda dt) + P_1(t)(\mu dt)$

$P_0(t+dt) - P_0(t) = P_1(t)(\mu dt) - P_0(t)(\lambda dt)$

$\frac{P_0(t+dt)-P_0(t)}{dt} = P_1(t)\mu - P_0(t)\lambda$

taking the limit when dt → 0,

$\lim_{dt\to 0}\frac{d}{dt}P_0(t) = P_1\mu - P_0\lambda = 0$

$P_1\mu = P_0\lambda$

$\therefore P_1 = \left(\frac{\lambda}{\mu}\right)P_0$

from eq (2);

$P_1\,(t+dt) = P_0(t)(\lambda dt) + P_1(t)(1-\mu dt)(1-\lambda dt) + P_2(t)(\mu dt)$

$= P_0(t)\lambda dt + P_1(t) - P_1(t)\lambda dt - P_1(t)\mu dt + P_1(t)\lambda\mu dt^2 + P_2(t)\mu dt$

$P_1(t+dt) - P_1(t) = P_0(t)\lambda dt - P_1(t)\lambda dt - P_1(t)\mu dt + P_1(t)\lambda\mu dt^2 + P_2(t)\mu dt$

$\frac{P_1(t+dt)-P_1(t)}{dt} = P_0(t)\lambda - P_1(t)\lambda - P_1(t)\mu + P_1(t)\lambda\mu dt^2 + P_2(t)\mu$

taking the limit when dt → 0 and the probability of a service or arrival is μ dt or λ dt and $(dt)^2 \to 0$,

$\lim_{dt\to 0}\frac{d}{dt}P_1(t) = \lambda P_0 - \lambda P_1 - \mu P_1 + \mu P_2 = 0$

$\lambda P_0 - (\lambda+\mu)P_1 + \mu P_2 = 0$

$\mu P_2 = (\lambda+\mu)P_1 - \lambda P_0$

$\mu P_2 = (\lambda+\mu)\left(\frac{\lambda}{\mu}\right)P_0 - \lambda P_0 \qquad (\because P_1 = \left(\frac{\lambda}{\mu}\right)P_0)$

$\mu P_2 = \left(\frac{\lambda^2+\mu\lambda-\mu\lambda}{\mu}\right)P_0$

$\mu P_2 = \left(\frac{\lambda^2}{\mu}\right)P_0$

$P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0$

$$\vdots$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$$

3.4.2 Probability of no units (customers) in the system, using normalization condition

The probability that no units are in the system, can be obtained as follow:

$$\sum_{n=0}^{\infty} P_n = 1$$

$$P_0 + P_1 + P_2 + \cdots = 1$$

$$P_0 + \left(\frac{\lambda}{\mu}\right) P_0 + \left(\frac{\lambda}{\mu}\right)^2 P_0 + \cdots = 1$$

$$P_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \cdots\right] = 1$$

$$P_0\ [1 + \rho + \rho^2 + \cdots] = 1 \quad \left(\because \rho = \frac{\lambda}{\mu}\right)$$

$$P_0 \left(\frac{1}{1-\rho}\right) = 1$$

$$\therefore P_0 = 1 - \rho$$

3.4.3 The average number of units (customers) in the system, L_s

The average number of units in the system (waiting + being served) is obtained by using the definition of expected value:

$$\text{E (L)} = \sum_{n=0}^{\infty} nP_n$$

$$= \sum_{n=0}^{\infty} n\rho^n(1-\rho)$$

$$= \rho(1-\rho) + 2\rho^2(1-\rho) + 3\rho^3(1-\rho) + \cdots$$

$$= \rho(1-\rho)[1 + 2\rho + 3\rho^2 + \cdots]$$

$$= \rho(1-\rho) \times \frac{1}{(1-\rho)^2}$$

$$= \frac{\rho}{1-\rho}$$

$$= \left(\frac{\lambda}{\mu}\right)\left(\frac{\mu}{\mu-\lambda}\right)$$

$$L_s = \frac{\lambda}{\mu-\lambda}$$

3.4.4 The average number of units (customers) in the queue, L_q

The average number of units in the queue, expected number of units in the system minus the expected number in service, can be obtained as follow:

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$= \frac{\lambda}{\mu-\lambda} - \frac{\lambda}{\mu}$$

$$= \frac{\mu\lambda-\lambda(\mu-\lambda)}{\mu(\mu-\lambda)}$$

$$= \frac{\mu\lambda-\lambda\mu+\lambda^2}{\mu(\mu-\lambda)}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

3.4.5 Expected waiting time per unit (customer) in the system, W_s

Expected waiting time per unit in the system, can be obtained as follow:

$$W_s = \frac{L_s}{\lambda} = \frac{\frac{\lambda}{\mu-\lambda}}{\lambda} = \frac{\lambda}{\lambda(\mu-\lambda)} = \frac{1}{\mu-\lambda}$$

3.4.6 Expected waiting time per unit (customer) in the queue, W_q

Expected waiting time per unit in the queue, expected time in system minus time in service, is shown below:

$$W_q = W_s - \frac{1}{\mu}$$

$$= \frac{1}{\mu-\lambda} - \frac{1}{\mu}$$

$$= \frac{\mu-\mu+\lambda}{\mu(\mu-\lambda)}$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

3.5 Multi-Channel Queueing Model (M/M/s): (FCFS/∞/∞)

Multi-channel queuing theory model treats the condition in which there are several service stations in parallel and each customer in the waiting line can be served by more than one station. Each service facility is prepared to deliver the same type of service. The new arrival selects one station without any external pressure.

Let n = number of units (customers) in the system

P_n = probability of n customers in the system

s = number of parallel service channels (s > 1)

λ = arrival rate of customers

μ = service rate of individual channel

When n < s, there is no queue because all arrivals are being serviced, and the rate of servicing will be nμ as only n channels are busy, each at the rate of μ. When n = s, all channels will be working and when n > s, there will be (n-s) customers in the queue and rate of service will be sμ as all the s channels are busy.

Service rate = $\frac{\lambda}{s\mu} < 1$

$\lambda_n = \lambda$ = constant arrival

$\mu_n = n\mu$, n < s

$= s\mu$, n ≥ s

When n lies between 1 and s-1, all customers arriving will be immediately served and n channels out of s will be busy. First of all, find $P_n(t + dt)$. This event can occur in three exclusive and exhaustive ways:

Event	No. of units at time t	No. of arrivals in time dt	No. of services in time dt	No. of units at time t + dt
1	n	0	0	n
2	n - 1	1	0	n
3	n +1	0	1	n

3.5.1 Probability of n units (customers) in the system

To determine the properties of multi-channel system, it is necessary to find an expression for the probability of n customers in the system at time t as follow:

$$P_0\ (t + dt) = P_0(t)\ (1 - \lambda dt) + P_1(t)(\mu dt) \qquad (1)$$

$$P_n(t+dt) = P_{n-1}(t)(\lambda dt) + P_n(t)(1-n\mu dt)(1-\lambda dt) + (n+1)P_{n+1}(t)(\mu dt) \quad (2)$$

From eq (1);

$$P_0(t+dt) = P_0(t) - P_0(t)(\lambda dt) + P_1(t)(\mu dt)$$

$$P_0(t+dt) - P_0(t) = P_1(t)(\mu dt) - P_0(t)(\lambda dt)$$

$$\frac{P_0(t+dt)-P_0(t)}{dt} = P_1(t)\mu - P_0(t)\lambda$$

taking the limit when dt → 0,

$$\lim_{dt\to 0} \frac{d}{dt} P_0(t) = P_1\mu - P_0\lambda = 0$$

$$P_1\mu = P_0\lambda$$

$$\therefore P_1 = \left(\frac{\lambda}{\mu}\right) P_0$$

From eq (2);

$$P_n(t+dt) = P_{n-1}(t)\lambda dt + (P_n(t) - P_n(t)\lambda dt)(1-n\mu dt) + (n+1)P_{n+1}(t)\mu dt$$

$$= P_{n-1}(t)\lambda dt + P_n(t) - P_n(t)n\mu dt - P_n(t)\lambda dt + P_n(t)n\lambda\mu(dt)^2 + (n+1)\, P_{(n+1)}(t)\mu dt$$

$$P_n(t+dt) - P_n(t) = P_{n-1}(t)\lambda dt - P_n(t)n\mu dt - P_n(t)\lambda dt + P_n(t)n\lambda\mu(dt)^2 + (n+1)\, P_{n+1}(t)\mu dt$$

$$\frac{P_n(t+dt)-P_n(t)}{dt} = P_{n-1}(t)\lambda - P_n(t)n\mu - P_n(t)\lambda + P_n(t)n\lambda\mu + (n+1)P_{n+1}(t)\mu$$

taking the limit when dt → 0,

$$\lim_{dt\to 0} P_n(t) = P_{n-1}\lambda - P_n n\mu - P_n\lambda + (n+1)P_{n+1}\mu = 0$$

$$\lambda P_{n-1} - (\lambda + n\mu)P_n + (n+1)P_{n+1}\mu = 0 \quad (3)$$

If n < s; $\mu_n = n\mu$

when n = 1; substituting in eq (3)

$$\lambda P_0 - (\lambda+\mu)P_1 + 2\mu P_2 = 0$$

$$\lambda P_0 - (\lambda+\mu)\left(\frac{\lambda}{\mu}\right)P_0 + 2\mu P_2 = 0$$

$$\lambda P_0 - \frac{\lambda^2}{\mu}P_0 - \lambda P_0 + 2\mu P_2 = 0$$

$$2\mu P_2 = \frac{\lambda^2}{\mu}$$

$$P_2 = \frac{1}{2!}\left(\frac{\lambda}{\mu}\right)^2 P_0$$

$$P_3 = \frac{1}{3!}\left(\frac{\lambda}{\mu}\right)^3 P_0$$

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$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0;\ 1 \le n \le s-1 \qquad (4)$$

If n ≥ s; $\mu_n = s\mu$

when n = s-1; substituting in eq (3)

$\lambda P_{s-2} - (\lambda + (s-1)\mu)P_{s-1} + (s-1+1)P_{s-1+1}\mu = 0$

$\lambda P_{s-2} - (\lambda + (s-1)\mu)P_{s-1} + sP_s\mu = 0$

$sP_s\mu = (\lambda + (s-1)\mu)P_{s-1} - \lambda P_{s-2}$

$$P_s = \frac{1}{s\mu}[\lambda + (s-1)\mu]P_{s-1} - \frac{\lambda}{s\mu}P_{s-2} \qquad (5)$$

from eq (4);

$$P_{s-1} = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s-1} P_0$$

$$P_{s-2} = \frac{1}{(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0$$

Substituting above equation in eq (5);

$$P_s = \frac{1}{s\mu}[\lambda + (s-1)\mu]\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s-1} P_0 - \frac{\lambda}{s\mu}\frac{1}{(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0$$

$$= \frac{\lambda}{s\mu}\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s-1} P_0 + \frac{(s-1)\mu}{s\mu}\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s-1} P_0 - \frac{\lambda}{s\mu}\frac{1}{(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0$$

$$= \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s} P_0 + \frac{(s-1)\mu}{s\mu}\frac{\lambda}{\mu}\frac{1}{(s-1)(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0 - \frac{\lambda}{s\mu}\frac{1}{(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0$$

$$= \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s} P_0 + \frac{\lambda}{s\mu}\frac{1}{(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0 - \frac{\lambda}{s\mu}\frac{1}{(s-2)!}\left(\frac{\lambda}{\mu}\right)^{s-2} P_0$$

$$P_s = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s} P_0$$

when n = s+1; substituting in eq (3)

$\lambda P_s - [\lambda + (s-1)\mu]P_{s+1} + (s+2)P_{s+2}\mu = 0$

$$P_{s+1} = \frac{\lambda}{s\mu} P_s$$

$$= \frac{\lambda}{s\mu} \times \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^{s} P_0$$

$$P_{s+1} = \frac{1}{ss!}\left(\frac{\lambda}{\mu}\right)^{s+1} P_0$$

$$P_{s+2} = \frac{1}{s^2 s!}\left(\frac{\lambda}{\mu}\right)^{s+2} P_0$$

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$$P_n = \frac{1}{s^{n-s}s!}\left(\frac{\lambda}{\mu}\right)^{s} P_0$$

3.5.2 Probability of no unit (customer) in the system, P_0

The probability of no unit in the system, can be obtained as follow:

$$\sum_{n=0}^{\infty} P_n = 1$$

$$\sum_{n=0}^{s-1} P_n + \sum_{n=s}^{\infty} P_n = 1$$

$$\sum_{n=0}^{s-1} \frac{\rho^n P_0}{n!} + \sum_{n=s}^{\infty} \frac{\rho^n P_0}{s! s^{n-s}} = 1$$

$$P_0\left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \sum_{n=s}^{\infty} \frac{\rho^s}{s!}\frac{\rho^{n-s}}{s^{n-s}}\right] = 1 \ (\because \rho^n = \rho^s\, \rho^{n-s})$$

$$P_0[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\sum_{n=s}^{\infty} \frac{\rho^{(n-s)}}{s^{n-s}} = 1$$

$$P_0\left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\left\{\frac{\rho^{s-s}}{s^{s-s}} + \frac{\rho^{s+1-s}}{s^{s+1-s}} + \frac{\rho^{s+2-s}}{s^{s+2-s}} + \cdots\right\}\right] = 1$$

$$P_0\left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\left\{1 + \frac{\rho}{s} + \frac{\rho^2}{s^2} + \cdots\right\}\right] = 1$$

$$P_0\left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\left\{\frac{1}{1-\frac{\rho}{s}}\right\}\right] = 1$$

$$P_0\left[\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\left\{\frac{s}{s-\rho}\right\}\right] = 1$$

$$\therefore P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\left(\frac{s}{s-\rho}\right)}$$

3.5.3 Expected number of units (customers) waiting in the queue, L_q

Expected number of units in the queue, can be obtained as follow:

$$L_q = \sum_{n=0}^{\infty} (n-s) P_n$$

let j = n-s; n = j + s

$$L_q = \sum_{j=0}^{\infty} j P_{j+s}$$

$$= \sum_{j=0}^{\infty} j\, \frac{\rho^{j+s}}{s! s^j}\, P_0$$

$$= \frac{P_0}{s!} \sum_{j=0}^{\infty} \frac{\rho^{j+s}}{s^j}$$

$$= \frac{P_0}{s!} \left[\frac{\rho^{s+1}}{s} + \frac{\rho^{s+2}}{s^2} + \frac{\rho^{s+3}}{s^3} + \cdots \right]$$

$$= \frac{P_0}{s!} \frac{\rho^{s+1}}{s} \left[1 + \frac{\rho}{s} + \frac{\rho^2}{s^2} + \cdots \right]$$

$$= \frac{P_0}{s!} \frac{\rho^{s+1}}{s} \left[\sum_{j=0}^{\infty} \frac{d\left(\frac{\rho}{s}\right)^j}{d\left(\frac{\rho}{s}\right)} \right]$$

$$= \frac{P_0}{s!} \frac{\rho^{s+1}}{s} \frac{d}{d\left(\frac{\rho}{s}\right)} \sum_{j=0}^{\infty} \left(\frac{\rho}{s}\right)^j$$

$$= \frac{P_0 \rho^{s+1}}{s!s} \left[\frac{d}{d\left(\frac{\rho}{s}\right)} \frac{\frac{\rho}{s}}{1-\frac{\rho}{s}} \right]$$

$$= \frac{P_0 \rho^{s+1}}{s!s} \frac{1-\frac{\rho}{s}+\frac{\rho}{s}}{\left(1-\frac{\rho}{s}\right)^2}$$

$$= \frac{P_0 \rho^{s+1}}{s!s} \frac{s^2}{(s-\rho)^2}$$

$$= P_0 \frac{s\rho^{s+1}}{s(s-1)!(s-\rho)^2}$$

$$L_q = \frac{\rho^{s+1}}{(s-1)!(s-\rho)^2} P_0$$

3.5.4 Expected number of units (customers) in the system, L_s

Expected number of units in the system, can be obtained as follow:

$$L_s = L_q + \rho$$

$$= \frac{\rho^{s+1}}{(s-1)!(s-\rho)^2} P_0 + \rho$$

3.5.5 Average time a unit (customer) spends in the system, W_s

Average time a customer spends in the system, can be obtained as follow:

$$W_s = \frac{L_s}{\lambda} = \frac{\rho^{s+1}}{\lambda(s-1)!(s-\rho)^2} P_0 + \frac{\rho}{\lambda}$$

$$= \frac{\rho^{s+1}}{\lambda(s-1)!(s-\rho)^2} P_0 + \frac{1}{\mu}$$

3.5.6 Average waiting time of a unit (customer) in the queue, W_q

Average waiting time of a customer in the queue, can be obtained as follow:

$$W_q = \frac{L_q}{\lambda} = \frac{\rho^{s+1}}{\lambda(s-1)!(s-\rho)^2} P_0$$

CHAPTER IV

APPLICATION OF QUEUING MODEL IN SELECTED BANKS

In this section, the waiting line characteristics of two Myanma Economic banks and one KanBawZa bank in Yangon City were studied. To offset the negative effects of waiting time, there will be several solutions to be implemented such as shortening waiting lines, informing customers about the length of their waiting time and providing customers with different kinds of entertainment.

4.1 Types of Service Provided by Myanma Economic Bank (MEB)

MEB's E-Remittance service started in June 2015 at Naypyidaw and Pyinmana branch. Internal Remittance services such as Bank draft, Telegraphic transfer, and Fax rendered by MEB. Also, it has been serving external fund transfer services between two foreign countries, Malaysia and Thailand, since Oct 2012 and Apr 2017. It was intended for migrant Myanmar workers to send back smoothly for their families. In August 2012, MEB provided E-pension System for Pensioners. MEB accepts saving deposits in all of its bank branches and everyone can open saving accounts at any MEB branches. Depositors are allowed to withdraw from their saving deposit accounts only once a week but there is no limit on the amount for withdrawal. There are five types of saving deposits such as Single Account, Minor Account, Joint (A) or Joint (B) Account, Security Deposit Account, and Public Account. Interest rates are 8% per annum on saving deposits. MEB is selling five kinds of Saving Certificates with different face values.

MEB offers to customers five types of Fixed Deposits with different maturities, three months, six months, nine months and one year. Current Accounts can be opened by Individuals, Business Persons, Companies, Social and Community Groups. To businesses engaged in production, Trade, Transportation, Construction and Service Industries, it provides three types of loans. They are Short-term loans (up to one year), Mid-term loans (three to five years), and Long-term loans (above five years up to ten years). To promote Small and Medium-Sized Enterprises in Myanmar, MEB is implementing the Two-Step Loan project. Other types of services provided by MEB are Payment Order, Automated Teller Machine System (ATM), Online Phone Billing System, Mobile Payment System, FOREX services, and Border Trade Services between sixteen townships. MEB is implementing one of the plans as Centralized

Online Real-time Exchange (CORE) Banking Solution to support most of the current and future needs of the MEB and its customers (“MEB Services | Ministry of Finance & Planning,” n.d.).

4.2 Characteristics of Staff in MEB (Kamayut)

In this section, the characteristics of staff in MEB (Kamayut) were presented by gender, educational attainment, and their position.

4.2.1 Gender

There were 74 staff in MEB (Kamayut). The gender distribution of the staffs of the bank was presented in Table (4.1).

Table 4.1: Gender Distribution

Gender	No. of Staff	Percentage
Male	10	13.51
Female	64	86.49
Total	**74**	**100.00**

Source: MEB (Kamayut)

According to Table (4.1), the majority of staff 64 (86.49%) were female and the remaining 10 (13.51%) of the staffs were male. It has been found that the number of female staff was higher than that of male staff.

4.2.2 Educational Attainment

In Table (4.2), the educational attainment of the staffs is presented.

Table 4.2: Educational Attainment

Education	No. of Staff	Percentage
High School	6	8.11
Graduate	67	90.54
Post Graduate	1	1.35
Total	**74**	**100.00**

Source: MEB (Kamayut)

According to Table (4.2), it could be seen that 90.54% of the staffs were graduates, 8.11% were high school and one of the staffs was postgraduate. It has been found that most of the staffs are graduates.

4.2.3 Position of Staffs

The position of staff was shown in Table (4.3).

Table 4.3: Position of Staffs

Position	No. of Staff	Percentage
Manager	1	1.35
Assistant Manager	4	5.41
Supervisor	5	6.76
Assistant Supervisor	12	16.22
Senior Clerk	31	41.89
Junior Clerk	15	20.27
Peon	6	8.11
Total	**74**	**100.00**

Source: MEB (Kamayut)

According to Table (4.3), 41.89% of the staffs were senior clerks who were the highest percentage and 1.35% of the staffs were manager and the lowest. Therefore, senior clerk was the most of the staff in MEB (Kamayut).

4.3 Single-Channel Waiting Line Model for MEB (Kamayut)

In this section, the waiting line characteristics of the single-channel waiting line model were calculated. Before calculation, the arrival rate and service rate were checked whether they follow Poisson probability distribution and exponential probability distribution by using the Kolmogorov-Smirnov test with 5% significance level.

There were three types of service counters that are operating for bank service. They were withdraw counter, deposit counter, and remittance counter. Each counter serves by a single channel. According to the observation, the arrival rate and service rate for three counters were computed and shown in Table (4.4). Data could be seen in Appendix (A - 4).

Table 4.4: Poisson and Exponential Probability Distribution Test for Average Daily Arrival and Service Rate in MEB (Kamayut)

Test / Counter	Poisson Distribution Test			Exponential Distribution Test		
	D_{max}	P-value	Poisson	D_{max}	P-value	Exponential
Withdraw	1.020	0.249	**Yes**	1.273	0.078	**Yes**
Deposit	0.996	0.275	**Yes**	1.348	0.053	**Yes**
Remittance	1.517	0.020	**No**	1.108	0.171	**Yes**

Source: Survey Data

According to Table (4.4), the Poisson probability distribution test for withdraw counter and deposit counter were not statistically significant and the remittance counter was statistically significant at 5% level. Therefore, arrivals of the withdraw counter and deposit counter followed the Poisson distribution. But arrivals of the remittance counter did not follow the Poisson distribution. And then, the Exponential probability distribution test for three counters were not statistically significant at 5% level. But, the service rate of three counters followed an exponential distribution.

4.3.1 Waiting Line Characteristics of Withdraw Counter in MEB (Kamayut)

Waiting line characteristics formula for a single-channel waiting line model can be applied only if the service rate (μ) is greater than the arrival rate (λ).

Table (4.5) showed that the single-channel waiting line characteristics for withdraw counter by using the average arrival rate was 22.39 customers per hour and the average service rate was 28.72 customers per hour. The average arrival rate and average service rate could be seen in Appendix (A - 4).

Table 4.5: Single-Channel Waiting Line Characteristics for Withdraw Counter in MEB (Kamayut) ($\lambda = 22.39, \mu = 28.72$)

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.22
L_q	The average number of customers in the queue	2.758 customers
L_s	The average number of customers in the system	3.537 customers
W_q	The average time a customer spends in the queue	7.39 minutes
W_s	The average time a customer spends in the system	9.478 minutes
ρ	Service Utilization	0.78 < 1

In the withdraw counter, the probability that a customer has to wait for service was 0.78 and the probability that no customers at the withdraw counter was 0.22. The average queue length was 2.758 (3) customers and the average number of customers in the system of withdraw counter was 3.537 (4) customers. The average time a customer spends in the queue and the system was 7.39 minutes and 9.478 minutes respectively.

4.3.2 Waiting Line Characteristics of Deposit Counter in MEB (Kamayut)

Table (4.6) showed the single-channel waiting line characteristics for the deposit counter by using the average arrival rate was 21.59 customers per hour and the average service rate was 21.96 customers per hour. The average arrival rate and average service rate could be seen in Appendix (A - 4).

Table 4.6: Single-Channel Waiting Line Characteristics for Deposit Counter in MEB (Kamayut) ($\lambda = 21.59, \mu = 21.96$)

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.017
L_q	The average number of customers in the queue	57.37 customers
L_s	The average number of customers in the system	58.35 customers
W_q	The average time a customer spends in the queue	2.39 hours
W_s	The average time a customer spends in the system	2.42 hours
ρ	Service Utilization	0.98 < 1

In the deposit counter, the probability that a customer has to wait for service was 0.98 and the probability that no customers at the deposit counter was 0.017. The average queue length was 57 customers and the average number of customers in the system of deposit counter was 58 customers. The average time a customer spends in the queue and the system was 2.39 hours and 2.42 hours respectively.

4.3.3 Waiting Line Characteristics of Remittance Counter in MEB (Kamayut)

Table (4.7) showed the single-channel waiting line characteristics for the remittance counter by using the average arrival rate was 11.24 customers per hour and the average service rate was 16.22 customers per hour. The average arrival rate and average service rate could be seen in Appendix (A - 4).

Table 4.7: Single-Channel Waiting Line Characteristics for Remittance Counter in MEB (Kamayut) ($\lambda = 11.24, \mu = 16.22$)

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.307
L_q	The average number of customers in the queue	1.56 customers
L_s	The average number of customers in the system	2.25 customers
W_q	The average time a customer spends in the queue	8.35 minutes
W_s	The average time a customer spends in the system	12.05 minutes
ρ	Service Utilization	0.693 < 1

In the remittance counter, the probability that a customer has to wait for service was 0.69 and the probability that no customers at the deposit counter was 0.307. The average queue length was 1.56 customers and the average number of customers in the system of remittance counter was 2.25 customers. The average time a customer spends in the queue and the system was 8.35 minutes and 12.05 minutes.

4.3.4 Economic Analysis of Waiting lines for MEB (Kamayut)

To assess and decide the ideal number of servers in the system, the following costs must be considered in settling on these decisions. It should be noted that the hourly service cost of each server is the marginal cost per hour to run a server. It is calculated as the hourly salary of each staff at the service counter. To determine the hourly income of the customers who are getting service from the service channel, the waiting cost of a customer is calculated by using the per capita income. This income is converted into hourly income to be consistent with the queuing system (Nahar, Islam, & Islam, 2016).

At the MEB (Kamayut), the salary of each staff at the service counter was 190,000 Kyats per month. Based on the 22 (working) days, the hourly salary of each staff was nearly 1080 Kyats. And then, the hourly service cost (C_s) of each server was 1080 Kyats. According to CSO (2018), the per capita annual income of a person in Myanmar was about 1,694,219 Kyats and the monthly income of a person was about 141,185 Kyats. Therefore, the hourly income of a person in Myanmar was about 588 Kyats. Since hourly waiting cost (C_w) of a customer was about 588 Kyats per hour.

In Table (4.8), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost for the withdraw counter. In this Table, the results showed that the minimum total expected cost for withdraw counter was

2701 Kyats and the optimal server was 2.

Table 4.8: Determining Optimal Server Number at Minimum Total Expected Cost for Withdraw Counter

No. of Server (S)	Total hourly service cost, $E(SC) = S.C_s$	Arrival rate (λ per hour)	The expected waiting time in the system (W_s per hour)	Total hourly waiting cost, $E(WC) = \lambda W_s C_w$	Total hourly Expected cost, $E(TC) = E(SC) + E(WC)$
1	1080 Kyats	22.39	0.1579	2079 Kyats	3159 Kyats
2	**2160 Kyats**	**22.39**	**0.0411**	**541 Kyats**	**2701 Kyats**
3	3240 Kyats	22.39	0.0356	469 Kyats	3709 Kyats
4	4320 Kyats	22.39	0.0349	460 Kyats	4780 Kyats
5	5400 Kyats	22.39	0.0348	458 Kyats	5858 Kyats

Source: MEB (Kamayut) and CSO (2018)

Figure (4.1) showed the average waiting time in each server. Therefore, server 1 was the highest in waiting time and server 5 was the lowest in waiting time in the system. By comparing the steady-state operating characteristics of the two-channel system to the original single-channel system calculated in Table (4.5), the average time a customer spends in the system was reduced from W_S= 0.1579 hours to W_S = 0.0411 hours, the probability that there were no customers in the system is increased from 22% to 44%, the probability that a customer has to wait for service was reduced from 78% to 39%. Therefore, it was seen that the two-channel system will be significantly improved the operating characteristics of the waiting line.

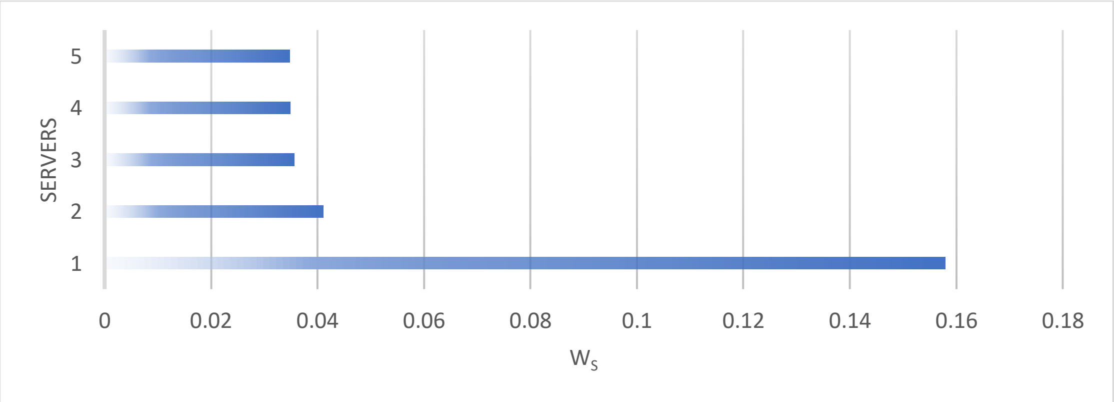

Fig 4.1: Average Waiting Time in the System Vs Number of Servers for Withdraw Counter in MEB (Kamayut)

Source: Table (4.8)

Figure (4.2) showed that the optimal server at the withdraw counter was achieved when the number of servers was 2 with a minimum total cost of Kyats 2701 per hour as against the present server level of 1 at the service counter with a high total cost of Kyats 3159 per hour.

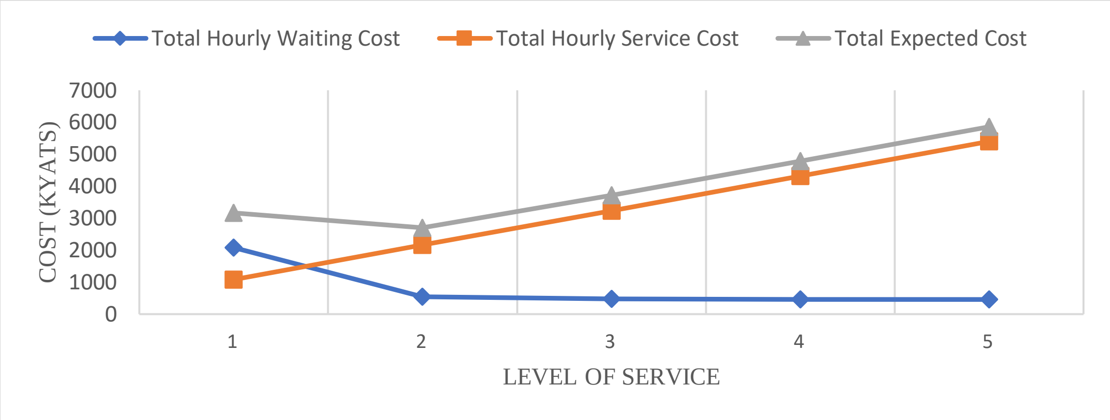

Fig 4.2: Chart for Determining Optimal Server Number at Minimum Total Cost for Withdraw Counter in MEB (Kamayut)

Source: Table (4.8)

In Table (4.9), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost for the deposit counter. In this Table, the results showed that the minimum total expected cost for deposit counter was 2923 Kyats and the optimal server was 2.

Table 4.9: Determining optimal server number at minimum total expected cost for Deposit counter

No. of Server (S)	Total hourly service cost, $E(SC) = S.C_s$	Arrival rate (λ per hour)	Expected waiting time in the system (W_s per hour)	Total hourly waiting cost, $E(WC) = \lambda W_s C_w$	Total hourly Expected cost, $E(TC) = E(SC + E(WC)$
1	1080 Kyats	21.59	2.7027	34311 Kyats	35391 Kyats
2	**2160 Kyats**	**21.59**	**0.0601**	**763 Kyats**	**2923 Kyats**
3	3240 Kyats	21.59	0.0475	603 Kyats	4023 Kyats
4	4320 Kyats	21.59	0.0458	581 Kyats	4901 Kyats
5	5400 Kyats	21.59	0.0456	579 Kyats	5979 Kyats

Source: MEB (Kamayut) and CSO (2018)

Figure (4.3) showed the average waiting time in each server. Therefore, server 1 was the highest in waiting time and server 5 was the lowest in waiting time in the system. By comparing the steady-state operating characteristics of the two-channel system to the original single-channel system calculated in Table (4.6), the average time a customer spends in the system was reduced from W_s= 2.073 hours to W_s = 0.060 hours, the probability that there are no customers in the system was increased from 1.69% to 34%, the probability that a customer has to wait for service was reduced from 98% to 49%. Therefore, it was seen that the two-channel system will be significantly improved the operating characteristics of the waiting line.

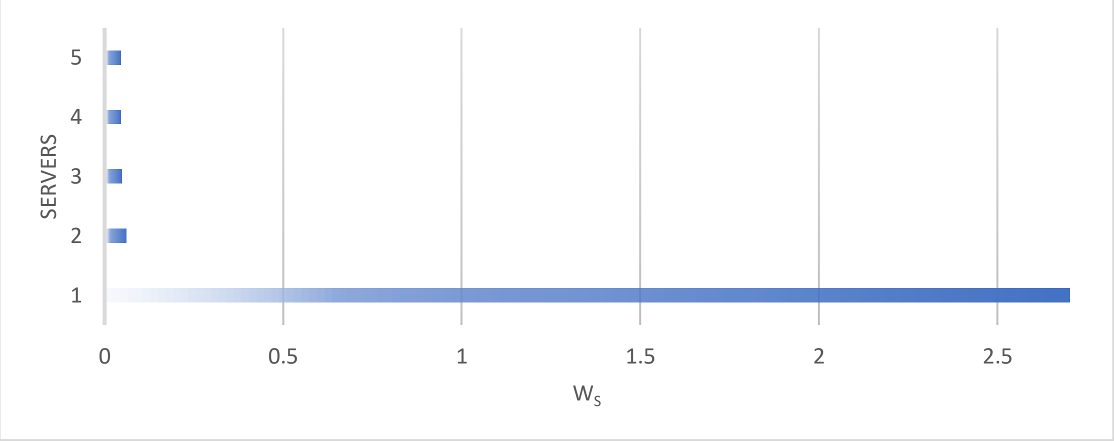

Fig 4.3: Average Waiting Time in the System Vs Number of Servers for Deposit Counter in MEB (Kamayut)

Source: Table (4.9)

Figure (4.4) showed that optimal server number at the deposit counter was achieved when the number of servers was 2 with a minimum total cost of Kyats 2923 per hour as against the present server level of 1 at the service counter with a high total cost of Kyats 35391 per hour.

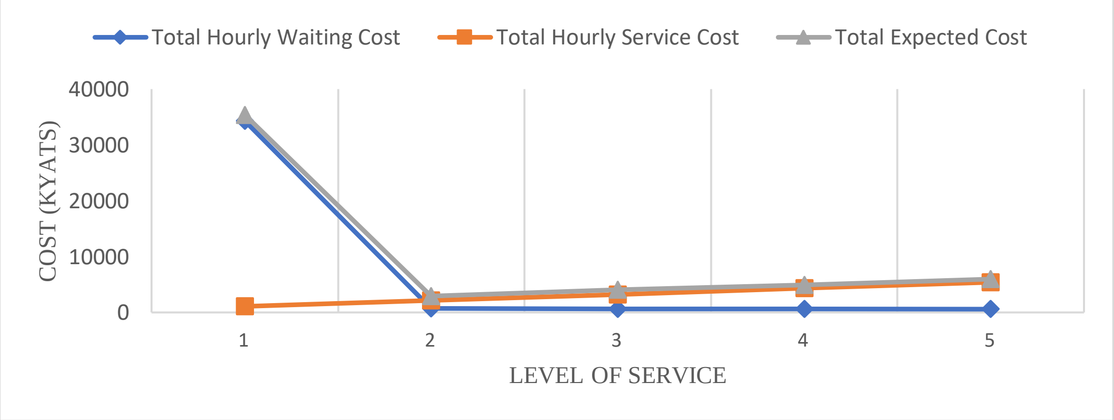

Fig 4.4: Chart for Determining Optimal Server Number at Minimum Total Cost for Deposit Counter in MEB (Kamayut)

Source: Table (4.9)

In Table (4.10), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost for the remittance counter. In this Table, the results showed that the minimum total expected cost for the remittance counter was 2655 Kyats and the optimal server was 2.

Table 4.10: Determining Optimal Server Number at Minimum Total Expected Cost for Remittance Counter

No. of Server (S)	**Total hourly service cost, $E(SC) = S.C_s$**	**Arrival rate (λ per hour)**	**Expected waiting time in system (W_s per hour)**	**Total hourly waiting cost, $E(WC) = \lambda W_s C_w$**	**Total hourly Expected cost, $E(TC) = E(SC + E(WC)$**
1	1080 Kyats	11.84	0.2283	1589 Kyats	2669 Kyats
2	**2160 Kyats**	**11.84**	**0.0711**	**495 Kyats**	**2655 Kyats**
3	3240 Kyats	11.84	0.0628	437 Kyats	3677 Kyats
4	4320 Kyats	11.84	0.0618	430 Kyats	4750 Kyats
5	5400 Kyats	11.84	0.0617	430 Kyats	5830 Kyats

Source: MEB (Kamayut) and CSO (2018)

Figure (4.5) showed the average waiting time in each server. Therefore, server 1 was the highest in waiting time and server 5 was the lowest in waiting time in the system. By comparing the steady-state operating characteristics of the two-channel system to the original single-channel system calculated in Table (4.7), the average time a customer spends in the system was reduced from W_S= 0.20 hours to W_S = 0.07 hours, the probability that there are no customers in the system was increased from 30.70% to 48.54%, the probability that a customer has to wait for service was reduced from 69.30% to 34.65%. Therefore, it was seen that the two-channel system will be significantly improved the operating characteristics of the waiting line.

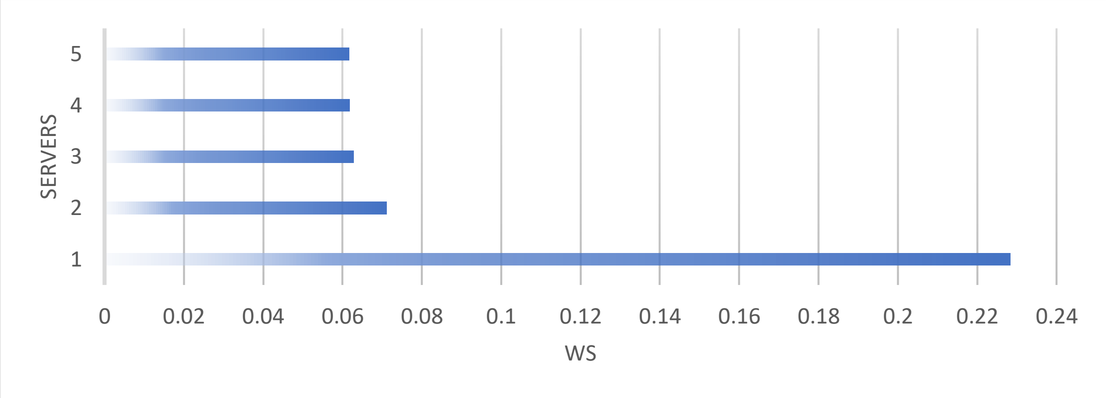

Fig 4.5: Average Waiting Time in the System Vs Number of Servers for Remittance Counter in MEB (Kamayut)

Source: Table (4.10)

Figure (4.6) showed that the optimal server number at the remittance counter was achieved when the number of servers was 2 with a minimum total cost of Kyats 2655 per hour as against the present server level of 1 at the service counter with a high total cost of Kyats 2669 per hour.

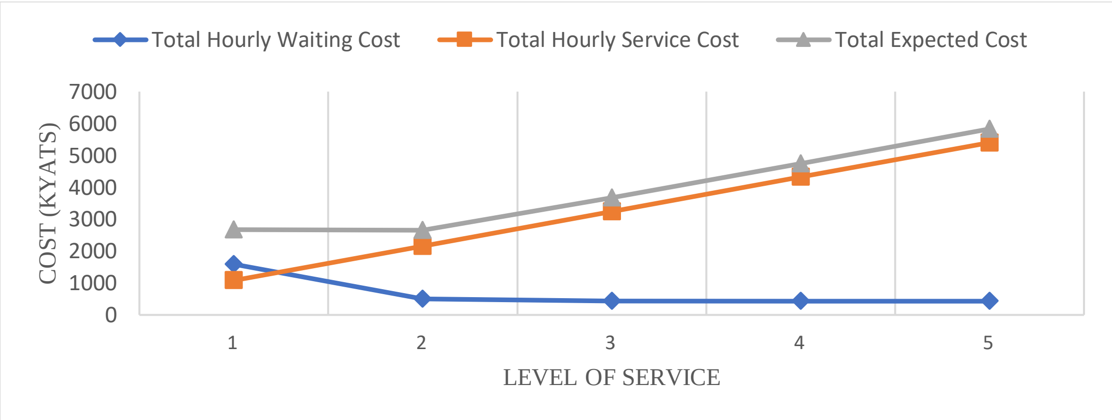

Fig 4.6: Chart for Determining Optimal Server Number at Minimum Total Cost for Remittance Counter in MEB (Kamayut)

Source: Table (4.10)

4.4 Characteristics of Staffs in MEB (Insein)

In this section, the characteristics of staff in MEB (Insein) were presented by gender, educational attainment, and their position.

4.4.1 Gender

There were 102 staff in MEB (Insein). The gender distribution of the staffs of the bank was presented in Table (4.11).

Table 4.11: Gender Distribution

Gender	No. of Staff	Percentage
Male	25	24.51
Female	77	75.49
Total	**102**	**100.00**

Source: MEB (Insein)

According to Table (4.11), the majority of staff 77 (75.49%) were female and the remaining 25 (24.51%) of the staffs were male. It has been found that the number of female staff was higher than the number of male staff.

4.4.2 Educational Attainment

In Table (4.12), the educational attainment of the staffs was presented.

Table 4.12: Educational Attainment

Education	No. of Staff	Percentage
High School	3	2.94
Graduate	97	95.09
Post Graduate	2	1.96
Total	**102**	**100.00**

Source: MEB (Insein)

It can be seen that 95.09% of the staffs were graduates, 2.94% were high school and 1.96% were postgraduate. It has been found that most of the staffs were graduates.

4.4.3 Position of Staffs

The position of staff is shown in Table (4.13).

Table 4.13: Position of Staffs

Position	No. of Staff	Percentage
Manager	1	0.98
Assistant Manager	6	5.88
Supervisor	5	4.90
Assistant Supervisor	15	14.71
Senior Clerk	45	44.12
Junior Clerk	23	22.55
Peon	7	6.86
Total	**102**	**100.00**

Source: MEB (Insein)

According to Table (4.13), 44.12% of staffs were senior clerk who are the highest in percentage and 0.98% of staff are manager and the lowest. Therefore, senior clerk is the most of the staff in MEB (Insein).

4.5 Single-Channel and Multi-Channel Waiting Line Model for MEB (Insein)

In this section, the waiting line characteristics of the single-channel waiting line model and multi-channel waiting line model were studied. Hence, the arrival rate and service rate were checked whether they follow Poisson probability distribution and exponential probability distribution by using the Kolmogorov-Smirnov test with 5% significance level.

There were three types of service counters that were operating for bank service. They were withdrawn counter, deposit counter, and remittance counter. Withdraw counter and deposit counter serve by a single channel and remittance counter served by a multi-channel. According to the observation, the arrival rate and service rate for three counters were computed and shown in Table (4.14). Data could be seen in Appendix (B-4).

Table 4.14: Poisson and Exponential Probability Distribution Test for Average Daily Arrival and Service Rate in MEB (Insein)

Test / Counter	Poisson Distribution Test			Exponential Distribution Test		
	D_{max}	P-value	Poisson	D_{max}	P-value	Exponential
Withdraw	1.008	0.261	**Yes**	1.226	0.099	**Yes**
Deposit	0.629	0.824	**Yes**	1.300	0.068	**Yes**
Remittance	0.437	0.991	**Yes**	0.965	0.310	**Yes**

Source: Survey Data

According to Table (4.14), average daily arrival and service rate for three counters were not statistically significant at 5% level. Therefore, average daily arrival and service rate for three counters followed Poisson and exponential probability distribution.

4.5.1 Waiting Line Characteristics of Withdraw Counter in MEB (Insein)

Table (4.15) showed that the single-channel waiting line characteristics for withdraw counter by using the average arrival rate was 16.22 customers per hour and the average service rate was 55.22 customers per hour. The average arrival rate and average service rate could be seen in Appendix (B - 4).

Table 4.15: Single-Channel Waiting Line Characteristics for Withdraw Counter in MEB (Insein) ($\boldsymbol{\lambda = 16.22, \mu = 55.22}$)

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.71
L_q	The average number of customers in the queue	0.12 customers
L_s	The average number of customers in the system	0.42 customers
W_q	The average time a customer spends in the queue	0.45 minutes
W_s	The average time a customer spends in the system	1.54 minutes
ρ	Service Utilization	0.29 < 1

In the case of the considered withdraw counter, the probability that a customer has to wait for service was 0.29 and the probability that no customers at the withdraw counter was 0.71. The average queue length was 0.12 customers and the average number of customers in the system of withdraw counter was 0.42. The average time a customer spends in the queue and the system was 0.45 minutes and 1.54 minutes.

4.5.2 Waiting Line Characteristics of Deposit Counter in MEB (Insein)

Table (4.16) showed that the single-channel waiting line characteristics for the deposit counter by using the average arrival rate was 7.86 customers per hour and the average service rate was 11.26 customers per hour. The average arrival rate and average service rate could be seen in Appendix (B - 4).

Table 4.16: Single-Channel Waiting Line Characteristics for Deposit Counter in MEB (Insein) ($\boldsymbol{\lambda = 7.86, \mu = 11.26}$)

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.302
L_q	The average number of customers in the queue	1.61 customers
L_s	The average number of customers in the system	2.31 customers
W_q	The average time a customer spends in the queue	12.32 minutes
W_s	The average time a customer spends in the system	17.65 minutes
ρ	Service Utilization	0.698 < 1

In the deposit counter, the probability that a customer has to wait for service was 0.698 and the probability that no customers at the deposit counter was 0.302. The average queue length was 1.61 customers and the average number of customers in the system of deposit counter was 2.31 customers. The average time a customer spends in the queue and the system was 12.32 minutes and 17.65 minutes.

4.5.3 Waiting Line Characteristics of Remittance Counter in MEB (Insein)

Table (4.17) was shown that the multi-channel waiting line characteristics for the remittance counter by using the average arrival rate was 30.86 customers per hour and the average service rate was 17.18 customers per hour. The average arrival rate and average service rate could be seen in Appendix (B - 4).

Table 4.17: Multi-Channel Waiting Line Characteristics for Remittance Counter in MEB (Insein) ($\lambda = 30.86, \mu = 17.18$)

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.054
L_q	The average number of customers in the queue	7.49 customers
L_s	The average number of customers in the system	9.29 customers
W_q	The average time a customer spends in the queue	14.57 minutes
W_s	The average time a customer spends in the system	18.06 minutes
ρ	Service Utilization	0.898 < 1

In remittance counter, the probability that a customer has to wait for service was 0.898 and the probability that no customers at the deposit counter was 0.054. The average queue length was 7.49 customers and the average number of customers in the system of remittance counter was 9.29 customers. The average time a customer spends in the queue and the system was 14.57 minutes and 18.06 minutes.

4.5.4 Economic Analysis of Waiting lines for MEB (Insein)

The salary of each staff at the service counters of MEB (Insein) was the same as MEB (Kamayut). Therefore, the hourly service cost (C_s) of each server was 1080 Kyats and the hourly waiting cost (C_w) of a customer was about 588 Kyats.

In Table (4.18), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost for the withdraw counter. In this Table, the results showed that the minimum total expected cost for withdraw counter was 1036.54 Kyats and server 1 was optimal.

Table 4.18: Determining Optimal Server Number at Minimum Total Expected Cost for Withdraw Counter

No. of Server (S)	Total hourly service cost, $E(SC) = S.C_s$	Arrival rate (λ per hour)	Expected waiting time in system (W_s per hour)	Total hourly waiting cost, $E(WC) = \lambda W_s C_w$	Total hourly Expected cost, $E(TC) = E(SC) + E(WC)$
1	**1080 Kyats**	**16.22**	**0.02564**	**245 Kyats**	**1325 Kyats**
2	2160 Kyats	16.22	0.01851	177 Kyats	2337 Kyats
3	3240 Kyats	16.22	0.01813	173 Kyats	3413 Kyats
4	4320 Kyats	16.22	0.01811	173 Kyats	4493 Kyats
5	5400 Kyats	16.22	0.01811	173 Kyats	5573 Kyats

Source: MEB (Insein) and CSO (2018)

Figure (4.7) showed the average waiting time in each server. Therefore, server 1 was the highest in waiting time and server 5 was the lowest in waiting time in the system. But server 1 was still optimal for withdraw counter.

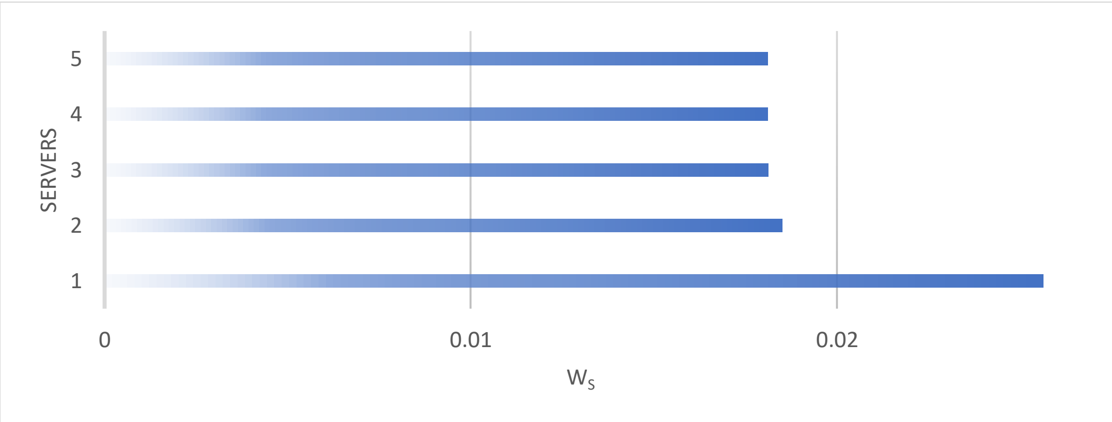

Fig 4.7: Average Waiting Time in the System Vs Number of Servers for Withdraw Counter in MEB (Insein)

Source: Table (4.18)

Figure (4.8) shows that server 1 was still optimal because it has the lowest minimum total expected cost with Kyats 1325 than others.

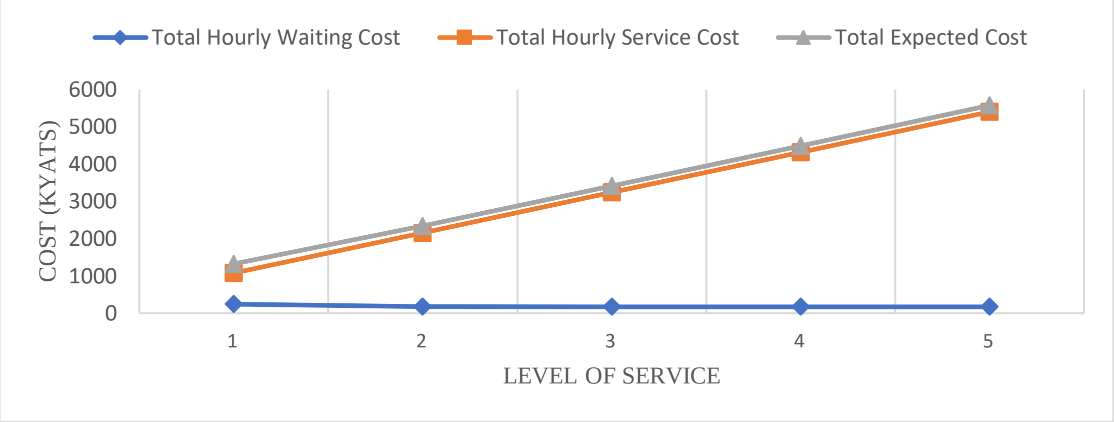

Fig 4.8: Chart for Determining Optimal Server Number at Minimum Total Cost for Withdraw Counter in MEB (Insein)

Source: Table (4.18)

In Table (4.19), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost for the deposit counter. In this Table, the results show that the minimum total expected cost for deposit counter was 2439 Kyats and server 1 was optimal.

Table 4.19: Determining Optimal Server Number at Minimum Total Expected Cost for Deposit Counter

No. of Server (S)	**Total hourly service cost, $E(SC) = S.C_s$**	**Arrival rate (λ per hour)**	**Expected waiting time in system (W_s per hour)**	**Total hourly waiting cost, $E(WC) = \lambda W_s C_w$**	**Total hourly Expected cost, $E(TC) = E(SC+WC)$**
1	**1080 Kyats**	**7.86**	**0.29412**	**1359 Kyats**	**2439 Kyats**
2	2160 Kyats	7.86	0.10113	467 Kyats	2627 Kyats
3	3240 Kyats	7.86	0.09022	417 Kyats	3657 Kyats
4	4320 Kyats	7.86	0.08897	411 Kyats	4731 Kyats
5	5400 Kyats	7.86	0.08883	411 Kyats	5811 Kyats

Source: MEB (Insein) and CSO (2018)

Figure (4.9) shows the average waiting time in each server. Therefore, server 1 was the highest in waiting time and server 5 was the lowest in waiting time in the system. By comparing the steady-state operating characteristics of the two-channel system to the original single-channel system calculated in Table (4.16), the average time a customer spends in the system was reduced from W_s= 0.29 hours to W_s = 0.10 hours, the probability that there are no customers in the system was increased from

30.20% to 48.26%, the probability that a customer has to wait for service was reduced from 69.81% to 34.90%. Therefore, it was seen that the two-channel system will be significantly improved the operating characteristics of the waiting line.

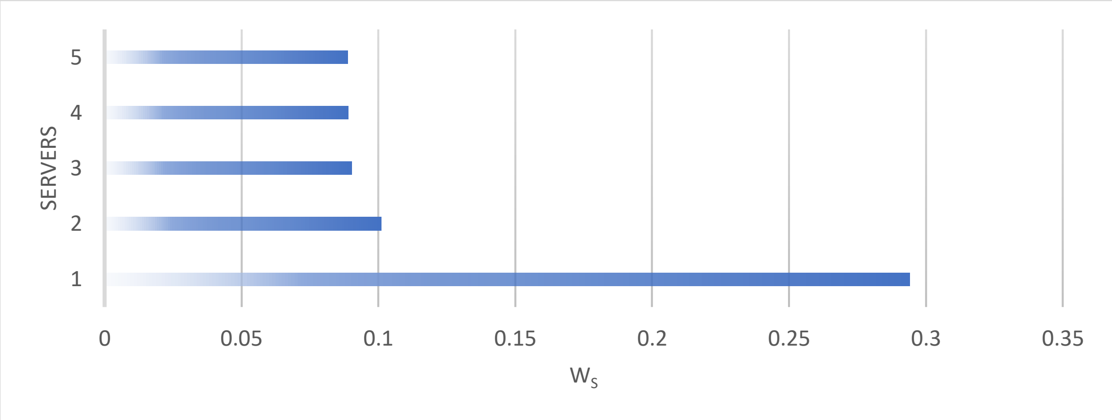

Fig 4.9: Average Waiting Time in the System Vs Number of Servers for Deposit Counter in MEB (Insein)

Source: Table (4.19)

Figure (4.10) shows that optimal server number at the Deposit counter was achieved when the number of servers was 1 with a minimum total cost of Kyats 2439 per hour.

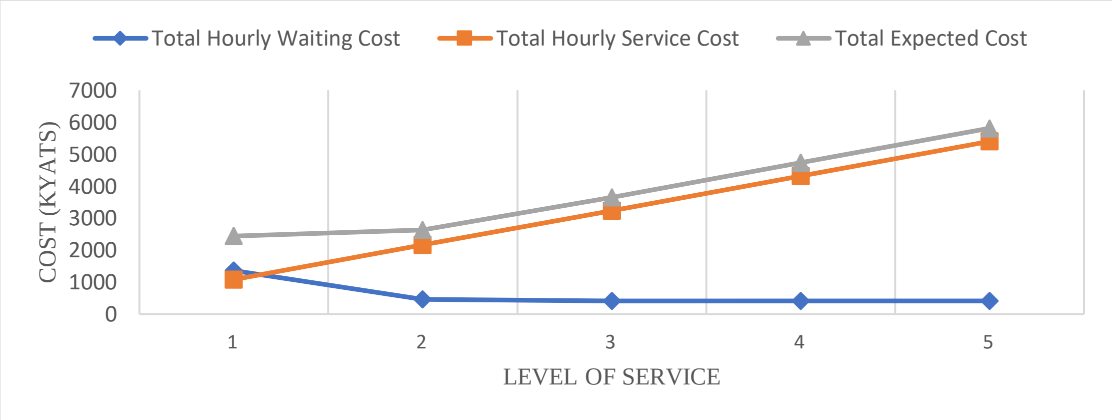

Fig 4.10: Chart for Determining Optimal Server Number at Minimum Total Cost for Deposit Counter in MEB (Insein)

Source: Table (4.19)

In Table (4.20), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost for the remittance counter. The remittance counter at MEB (Insein) serves by a multi-channel. In this Table, the results showed that the minimum total expected cost for the remittance counter was 4606 Kyats and the optimal server was 3.

Table 4.20: Determining Optimal Server Number at Minimum Total Expected Cost for Remittance Counter

No. of Server (S)	Total hourly service cost, $E(SC) = S.C_s$	Arrival rate (λ per hour)	Expected waiting time in system (W_s per hour)	Total hourly waiting cost, $E(WC) = \lambda W_s C_w$	Total hourly Expected cost, $E(TC) = E(SC + WC)$
2	2160 Kyats	30.86	0.30105	5463 Kyats	7623 Kyats
3	**3240 Kyats**	**30.86**	**0.07528**	**1366 Kyats**	**4606 Kyats**
4	4320 Kyats	30.86	0.06158	1117 Kyats	5437 Kyats
5	5400 Kyats	30.86	0.05894	1070 Kyats	6470 Kyats
6	6480 Kyats	30.86	0.05836	1059 Kyats	7539 Kyats

Source: MEB (Insein) and CSO (2018)

Figure (4.11) showed the average waiting time in each server. Therefore, server 2 was the highest in waiting time and server 6 was the lowest in waiting time in the system. By comparing the steady-state operating characteristics of the three-channel system to the original two-channel system calculated in Table (4.17), the average time a customer spends in the system was reduced from W_S= 0.30 hours to W_S = 0.08 hours, the probability that there are no customers in the system was increased from 5.37% to 14.67%, the probability that a customer has to wait for service was reduced from 89.81% to 59.88%. Therefore, it was seen that the three-channel system will be significantly improved the operating characteristics of the waiting line.

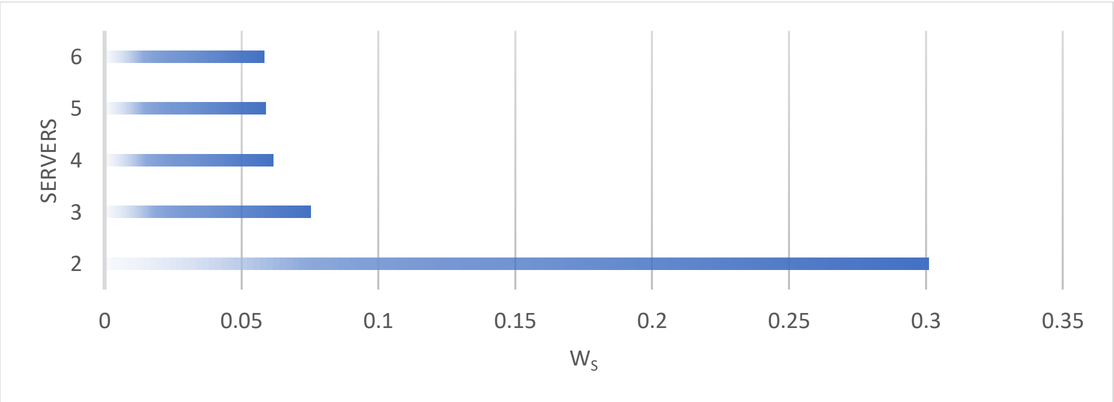

Fig 4.11: Average Waiting Time in the System Vs Number of Servers for Remittance Counter in MEB (Insein)

Source: Table (4.20)

Figure (4.12) showed that the optimal server number at the Remittance counter was achieved when the number of servers was 3 with a minimum total cost of Kyats 4606 per hour as against the present server level of 2 at the service counter with the high total cost of Kyats 7623 per hour.

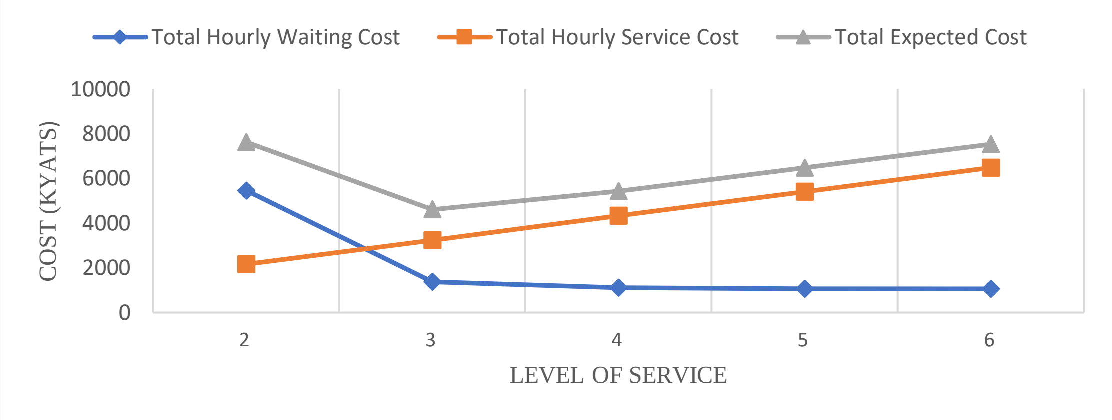

Fig 4.12: Chart for Determining Optimal Server Number at Minimum Total Cost for Remittance Counter in MEB (Insein)

Source: Table (4.20)

4.6 Types of Service Provided by KBZ Bank

KBZ bank offers several types of service. They are six types of accounts, seven types of cards, three types of loans, and two types of remittance. Six types of accounts are a current account, savings deposit account, fixed deposit account, call deposit account, children's saving account, and foreign currency account. Seven types of cards are explored debit card, explore MPU e-commerce, prepaid card, teen card, international card acceptance, KBZ UnionPay credit card, and KBZ visa credit card. Three types of loans are hire purchase, overdraft, and home loan. Two types of remittance are loan remittance and international remittance (“Collection Services”, n.d.). KBZ also provides other services such as safe deposit locker, gift cheques, KBZ quick pay, currency exchange, E-commerce and a bank certificate (“Other Services,” n.d.).

4.7 Characteristics of Staffs in KBZ (Kamayut)

In this section, the characteristics of staff in KBZ (Kamayut) were presented by gender, educational attainment, and their position.

4.7.1 Gender

There were 63 staff in KBZ (Kamayut). The gender distribution of the staffs of the bank was presented in Table (4.21).

Table 4.21: Gender Distribution

Gender	No. of Staff	Percentage
Male	24	38.10
Female	39	61.90
Total	**63**	**100.00**

Source: KBZ (Kamayut)

Table (4.21) showed the gender distribution of staff of the KBZ (Kamayut). Therefore, the majority of staff 39 (61.90%) were female and the remaining 24 (38.10%) of the staffs were male. It has been found that the number of female staff was higher than those of male staff.

4.7.2 Educational Attainment

In Table (4.22), the educational attainment of the staffs was presented.

Table 4.22: Educational Attainment

Education	No. of Staff	Percentage
High School	8	12.70
Graduate	55	87.30
Total	**63**	**100.00**

Source: KBZ (Kamayut)

As mentioned in Table (4.22), It was seen that 87.30% of the staffs were graduates and 12.70% were high school. Since most of the staffs were graduates.

4.7.3 Position of Staffs

The position of staffs was shown in Table (4.23).

Table 4.23: Position of Staffs

Position	No. of Staff	Percentage
Manager	4	6.35
Assistant Manager	1	1.59
Supervisor	2	3.17
Assistant Supervisor	4	6.35
Senior Assistant	13	20.63
Junior Assistant	31	49.21
Peon	8	12.70
Total	**63**	**100.00**

Source: KBZ (Kamayut)

As stated in Table (4.23), 49.21% of staff were junior assistants who were the highest in percentage and 1.59% of staff were assistant managers who were the lowest in percentage. It has been found that junior assistants were most of the staff in KBZ (Kamayut).

4.8 Multi-Channel Waiting Line Model for KBZ (Kamayut)

In this section, the multi-channel waiting line characteristics of KBZ (Kamayut) were studied. Therefore, arrival rate and service rate were checked whether they followed Poisson probability distribution and exponential probability distribution by using the Kolmogorov-Smirnov test with 5% significance level. The calculated values were presented in Table (4.24). Data can be seen in Appendix (C - 2).

KBZ (Kamayut) provided by multi-channel and single-channel for several types of service as mentioned above. Among them, KBZ (Kamayut) offered multi-channel for three types of services such as withdraw, deposit and remittance. According to the observation, the arrival rate and service rate for three types of service were studied as follows.

Table 4.24: Poisson and Exponential Probability Distribution Test for Average Daily Arrival and Service Rate in KBZ (Kamayut)

Test / Counter	Poisson Distribution Test			Exponential Distribution Test		
	D_{max}	P-value	Poisson	D_{max}	P-value	Exponential
Multi-Channel	0.563	0.909	**Yes**	1.319	0.062	**Yes**

Source: Survey Data

According to Table (4.24), the average daily arrival and service rate for multi-channel was not statistically significant at 5% level. Therefore, average daily arrival and service rate for multi-channel followed Poisson and exponential probability distribution.

4.8.1 Multi-Channel Waiting Line Characteristics for KBZ (Kamayut)

Table (4.25) presented that the multi-channel waiting line characteristics by using the average arrival rate were 29.14 customers per hour and the average service rate was 7.70 customers per hour. The average arrival rate and average service rate could be seen in Appendix (C - 2).

Table 4.25: Multi-Channel Waiting Line Characteristics for KBZ (Kamayut)
$(\lambda = 29.14, \mu = 7.7)$

	Waiting Line Characteristics	Calculated value
P_0	The probability that there are no customers in the system	0.0056
L_q	The average number of customers in the queue	15.50 customers
L_s	The average number of customers in the system	19.29 customers
W_q	The average time a customer spends in the queue	31.92 minutes
W_s	The average time a customer spends in the system	39.71 minutes
ρ	Service Utilization	0.946 < 1

As stated in the above Table, the multi-channel waiting line characteristics were the probability that a customer has to wait for service was 0.946 and the probability that no customers at the service counters was 0.0056. The average queue length was 15.5 customers and the average number of customers in the system of the service counter was 19.29. The average time a customer spends in the queue and the system were 31.92 minutes and 39.71 minutes.

4.8.2 Economic Analysis of Waiting lines for KBZ (Kamayut)

At the KBZ (Kamayut), there were two staff, senior assistant and junior assistant, at the service counters. One of the staff called the token number to service the customers while another one was servicing the customers. The salary of the senior assistant was 310,000 Kyats per month and those of junior assistant was 230,000 Kyats per month. Based on the 22 (working) days, the service cost of a service counter was 540,000 Kyats per month. The service cost of a service counter was 540,000 Kyats per month. The hourly service cost (C_s) of a service counter was nearly 3068 Kyats. According to CSO (2018), per capita income in Myanmar was about 1,694,219 Kyats and the monthly income of a person was about 141,185 Kyats. Therefore, the hourly income of a person in Myanmar was about 588 Kyats. Since hourly waiting cost (C_w) of a customer was about 588 Kyats per hour.

In Table (4.26), the hourly service costs and hourly waiting costs were identified as mentioned above to calculate the total cost. In this Table, the results showed that the minimum total expected cost was 18275 Kyats and the optimal server was 5.

Table 4.26: Determining Optimal Server Number at Minimum Total Expected Cost for Multi-Channel

No. of Server (S)	**Total hourly service cost,** $E(SC) = S.C_s$	**Arrival rate** $(\lambda\ per\ hour)$	**Expected waiting time in system** $(W_s\ per\ hour)$	**Total hourly waiting cost,** $E(WC) = \lambda W_s C_w$	**Total hourly Expected cost,** $E(TC) = E(SC) + E(WC)$
4	12272 Kyats	29.14	0.66189	10762 Kyats	23034 Kyats
5	**15340 Kyats**	**29.14**	**0.18051**	**2935 Kyats**	**18275 Kyats**
6	18408 Kyats	29.14	0.14364	2336 Kyats	20744 Kyats
7	21476 Kyats	29.14	0.13419	2182 Kyats	23658 Kyats
8	24544 Kyats	29.14	0.13125	2134 Kyats	26678 Kyats

Source: KBZ (Kamayut) and CSO (2018)

Figure (4.13) showed the average waiting time in multi-servers. Therefore, server 4 was the highest in waiting time and server 8 was the lowest in waiting time in the system. By comparing the steady-state operating characteristics of the five-channel system to the original four-channel system calculated in Table (4.25), the average time a customer spends in the system was reduced from W_s= 0.66 hours to W_s = 0.18 hours, the probability that there were no customers in the system was increased from 0.56% to 1.78%, the probability that a customer has to wait for service was reduced from 94.61% to 75.69%. Therefore, it was seen that the five-channel system will significantly improve the operating characteristics of the waiting line.

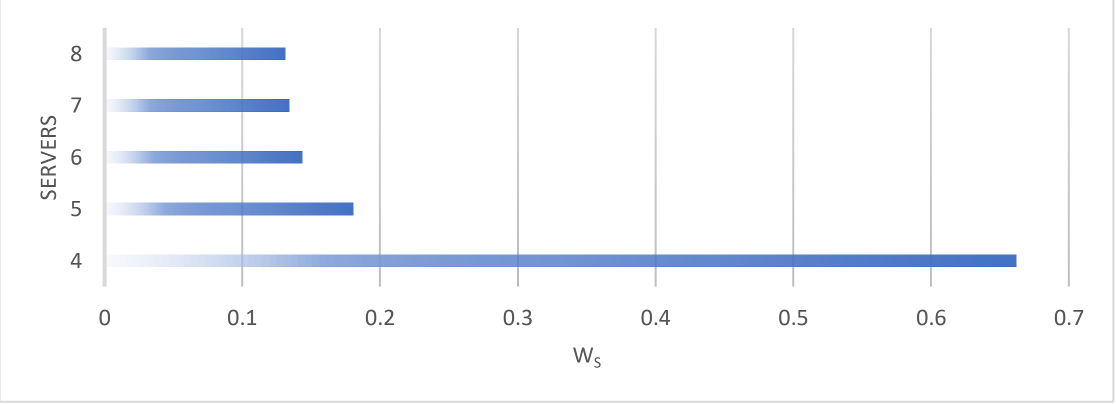

Fig 4.13: Average Waiting Time in the System Vs Number of Servers for Multi-Servers in KBZ (Kamayut)

Source: Table (4.26)

Figure (4.14) showed that the optimal server number was achieved when the number of servers was 5 with a minimum total cost of Kyats 18275 per hour as against the present server level of 4 at the service counter with a high total cost of Kyats 23034 per hour.

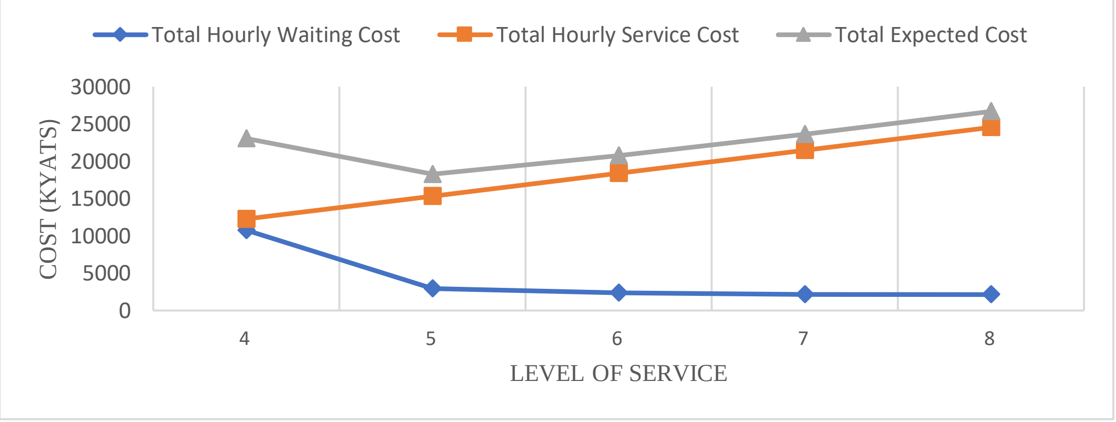

Fig 4.14: Chart for Determining Optimal Server Number at Minimum Total Cost for Multi-Servers in KBZ (Kamayut)

Source: Table (4.26)

CHAPTER V

CONCLUSION

Queuing theory is a powerful mathematical approach to the analysis of the waiting lines of business organizations. The queuing characteristics of the selected public and private banks were analyzed by using a single-channel and multi-channel queuing model and the waiting and service cost were calculated to determine the optimal service level of each selected bank. The optimal service counters were determined based on the minimum total expected cost. To get the required assumptions of the waiting line model, the observed data was tested by using a one-sample Kolmogorov-Smirnov test.

Based on the result of MEB (Kamayut), it was found that the arrival rate of observed data, except for the remittance counter, followed Poisson distribution and service rate followed an exponential distribution. According to data analysis, it has been found that the optimal server for each service counters was 2. Therefore, the manager should be considered to change the service counters from the single-channel system to the multi-channel system.

By analyzing the waiting line characteristics of the MEB (Insein), single-channel and multi-channel queuing models were used. Based on the result, it was found that the arrival rate of observed data followed Poisson distribution and service rate followed an exponential distribution. As stated in data analysis, the optimal server for withdraw and deposit counters was 1 and for remittance counter was 3. Hence, the manager should be considered to increase the remittance counter from 2 to 3 service counters.

As the results of KBZ (Kamayut), it was observed that the arrival rate varied from 26.2 to 33.22 customers per hour while the overall average arrival rate was 29.14 customers per hour. Similarly, the service rate varied from 6.84 to 8.45 customers per hour for each counter while the overall average service rate was 7.70 customers per hour for each counter. Based on the result, it was found that the arrival rate of observed data followed Poisson distribution and service rate followed an exponential distribution. The numbers of servers available at the KBZ (Kamayut) varied from 4 to 8 and they were one-stop service counter. During the data collection period, it was operated by 4 service counters. The time a customer spends in the queue could range from 1 minute to 15 minutes and those in the system could range from 3 minutes to 35 minutes. Overall the average time a customer spent in the queue was 31.92 minutes and those in the system was 39.71 minutes. As observed at the KBZ (Kamayut), long queues were more common before lunch-time and after 14:00. According to the calculated waiting lines characteristics, the probability that 94.6% of the service facility was being used. As mentioned in the above results, the manager should be considered to increase the service counter from 4 to 5 and reduce customers' waiting time and dissatisfaction. On the other hand, server 5 has the lowest total minimum expected cost.

References

Collection Services. (n.d.). Retrieved from KBZ Bank: https://www.kbzbank.com/en/cash-management/collection-services/

Dharmawirya, M., & Adi, E. (2012). Case Study for Restaurant Queuing Model. *International Conference on Management and Artificial Intelligence*, 52-54. doi:10.2139/ssrn.2014470

Gumus, S., Bubou, G. M., & Oladeinde, M. H. (2017). Application of queuing theory to a fast food outfit: a study of blue meadows restaurant. *Independent Journal of Management & Production, 8*(2), 441-455. doi:10.14807/ijmp.v8i2.576

Gupta, N., & Garg, R. (2012). A View of Queue Analysis with Customer Behaviour, Balking and Reneging. *University of Science & Technology, Faridabad, Haryana*, 906-909.

Hiller, F., & Hiller, M. (2003). *Introduction to management science: a modeling and case studies approach with spreadsheets* (Fifth edition ed.). New York, NY: McGraw-Hill.

Jhala, N., & Bhathawala, P. (2017). Analysis and Application of Queuing Theory in Supermarkets. *International Journal of Innovative Research in Science, Engineering and Technology, 6*(9), 6.

Kendall, D. G. (1953). *Stochastic Process Occuring in the Theory of Queues and their Analysis by the Method of the Imbedded Markov Chain* (Vol. 24). doi:10.1214/aoms/1177728975

Kenton, W. (2019). *Queuing Theory and the Business of Waiting in Line*. Retrieved 2 7, 2017, from Investopedia: https://www.investopedia.com/terms/q/queuing-theory.asp

Kumar, B., Vij, A., & Kumar, P. (2015, 12 20). Some Basic Concepts in Queuing Theory. *International Advanced Research Journal in Science, Engineering and Technology, 2*(12), 56-60. doi:10.17148/IARJSET.2015.21209

Madadi, N., Roudsari, A., Wong, K., & Galankashi, M. (2013). Modeling and Simulation of a Bank Queuing System. *2013 Fifth International Conference on Computational Intelligence, Modelling and Simulation (CIMSim)*, (pp. 209-215). Seoul, Korea (South). doi:10.1109/CIMSim.2013.41

MEB Services | Ministry of Finance & Planning. (n.d.). Retrieved from https://www.mopf.gov.mm/en/page/finance/

Mehandiratta, R. (2011). Applications of Queuing Theory in Health Care. *International Journal of Computing and Business Research, 2*(2), 11.

Molla, M. A.-A. (2017). Case Study for Shuruchi Restaurant Queuing Model. *IOSR Journal of Business and Management, 19*(02), 93-98. doi:10.9790/487X-1902039398

Myanmar Statistical Year Book. (2018). Centaral Statistical Organization. Retrieved from http://www.csostat.gov.mm

Nahar, K., Islam, Z., & Islam, M. (2016). Cost Effective Decisions on Service Rate at a Customer Care Centre with Multi-Server Queuing Model. *Journal of Modern Science and Technology, 4*(1), 144-153.

Okhuese, V. A. (2018). Application of Queuing Theory: Analysis of Services of Commercial Banks. *The Journal of Science Teachers Association of Nigeria, 48*(1), 19.

Other Services. (n.d.). Retrieved from KBZ Bank: https://www.kbzbank.com/en/other-services/

Render, B., Stair, R., & Hanna, M. (2012). *Quantitative analysis for management* (11th ed.). Pearson Prentice Hall.

Sahu, C., & Sahu, S. (2014). Implementation of Single Channel Queuing Model to Enhance Banking Services. *International Journal of Management, Information*, 8.

Sheikh, T., Singh, S. K., & Kashyap, A. K. (2013). Application of Queuing Theory for the Improvement of Bank Service. *International Journal of Advanced Computational Engineering and Networking,, 1*(4), 4.

Singh, V. (2017, 8 25). Use of Queuing Models in Health Care. 35.

Taha, H. (2017). *Operations research an introduction* (Tenth edition ed.). Pearson.

V, R., Sikdar, K., & Medhi, D. (2017, 6 30). Modelling of Queuing System at Sales Checkout and Analysis of Consumer Behaviour: An Empirical Study at a Supermarket in Bengaluru. *International Journal of Advanced Research (IJAR), 5*(6), 311-324. doi:10.21474/IJAR01/4415

Appendix A

Appendix A-1

For Withdraw Counter

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5		Day 6		Day 7		Day 8		Day 9		Day 10	
		Arrival	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**
1	09:00-10:00	5	3	15	12	1	-	13	-	64	51	15	2	15	7	20	11	16	2	18	3
2	10:00-11:00	22	15	21	19	21	14	25	25	26	21	15	25	22	24	19	21	18	23	25	20
3	11:00-12:0	15	14	23	21	11	12	12	17	23	36	9	11	17	20	25	19	15	19	23	23
4	12:00-13:00	9	14	21	22	17	15	6	10	8	12	15	6	17	15	16	26	20	22	15	13
5	13:00-14:00	8	11	13	17	14	18	11	11	6	5	17	18	17	17	19	17	15	15	11	27
6	14:00-15:00	6	8	8	10	8	13	4	8	-	2	4	13	6	11	7	12	2	5	9	15
Total		**65**	**65**	**101**	**101**	**72**	**72**	**71**	**71**	**127**	**127**	**75**	**75**	**94**	**94**	**106**	**106**	**86**	**86**	**101**	**101**

Source: Survey Data

Appendix A-2

For Deposit Counter

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5		Day 6		Day 7		Day 8		Day 9		Day 10	
		Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service
1	09:30-10:00	9	5	10	6	-	-	13	7	8	8	6	1	22	14	5	-	9	7	15	10
2	10:00-11:00	18	15	22	18	13	11	15	21	24	19	24	22	44	42	14	17	14	14	16	20
3	11:00-12:00	16	14	19	19	30	28	15	13	18	20	28	33	39	38	31	25	16	18	20	20
4	12:00-13:00	15	17	13	15	9	11	13	15	23	22	34	34	28	25	15	17	22	20	14	20
5	13:00-14:00	13	17	11	14	7	6	13	9	10	12	31	31	16	21	9	12	18	20	14	6
6	14:00-15:00	9	12	10	13	15	18	8	12	-	2	16	18	7	16	4	7	10	10	11	14
Total		**80**	**80**	**85**	**85**	**74**	**74**	**77**	**77**	**83**	**83**	**139**	**139**	**156**	**156**	**78**	**78**	**89**	**89**	**90**	**90**

Source: Survey Data

Appendix A-3

For Remittance Counter

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5		Day 6		Day 7		Day 8		Day 9		Day 10	
		Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service
1	09:30-10:00	5	3	5	3	-	-	2	2	5	1	-	-	2	2	-	-	1	-	-	-
2	10:00-11:00	7	6	9	10	1	1	3	1	9	12	6	4	2	2	9	8	4	3	5	3
3	11:00-12:00	15	12	15	12	-	-	7	6	5	5	4	6	3	3	18	17	6	6	7	6
4	12:00-13:00	10	15	18	19	-	-	-	3	11	7	6	5	1	1	11	12	2	4	8	8
5	13:00-14:00	9	8	10	12	-	-	3	1	8	13	4	5	-	-	8	6	6	6	5	7
6	14:00-15:00	4	6	8	9	3	3	4	6	-	-	2	2	-	-	7	10	3	3	-	1
Total		**50**	**50**	**65**	**65**	**4**	**4**	**19**	**19**	**38**	**38**	**22**	**22**	**8**	**8**	**53**	**53**	**22**	**22**	**25**	**25**

Source: Survey Data

Appendix A-4

Table: Daily Record of Average Arrival Rate and Service Rate

Day / Counter	Day 1		Day 2		Day 3		Day 4		Day 5		Day 6		Day 7		Day 8		Day 9		Day 10		Average	
	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
Withdraw	23.6	21.3	37	26.7	15.4	42.4	13.1	14.8	22.8	18.2	14.9	62.5	15.6	25.6	18.9	26.4	18.4	23.9	44.2	25.4	**22.39**	**28.72**
Deposit	20.9	21.7	25.5	30.7	11.1	12.8	13.5	18.6	15.4	17.5	28.8	27.8	30.1	25.6	16.7	12.2	17.3	20.9	36.5	31.8	**21.59**	**21.96**
Remittance	21.7	23.1	26	30	1	24	3.5	9.5	8.87	10.9	5.1	8.9	4.1	21.8	11.7	14.5	4.5	7.0	31.9	12.5	**11.84**	**16.22**

Source: Survey Data

Appendix B

Appendix B-1

For Withdraw Counter

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5	
		Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service
1	09:00-10:00	5	2	6	5	8	2	12	2	7	2
2	10:00-11:00	13	13	10	8	13	16	12	20	14	16
3	11:00-12:00	13	10	4	2	16	15	16	14	9	11
4	12:00-13:00	8	13	15	13	7	7	14	12	8	9
5	13:00-14:00	9	8	5	10	15	14	9	14	8	6
6	14:00-15:00	9	11	2	3	6	11	6	7	3	5
Total		**57**	**57**	**42**	**42**	**65**	**65**	**69**	**69**	**49**	**49**

Source: Survey Data

Appendix B-2

For Deposit Counter

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5	
		Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service
1	09:00-10:00	3	3	4	4	6	6	2	2	6	6
2	10:00-11:00	4	3	6	5	13	11	14	14	12	9
3	11:00-12:00	8	7	11	11	9	10	11	9	8	10
4	12:00-13:00	-	2	10	10	8	10	9	8	5	5
5	13:00-14:00	5	4	5	6	9	8	-	3	4	5
6	14:00-15:00	5	6	1	1	5	5	6	6	4	4
Total		**25**	**25**	**37**	**37**	**50**	**50**	**42**	**42**	**39**	**39**

Source: Survey Data

Appendix B-3

For Remittance Counter

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5	
		Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service	Arrival	Service
1	09:00-10:00	9	6	22	21	19	17	12	10	11	8
2	10:00-11:00	16	14	65	63	35	31	33	32	34	34
3	11:00-12:00	16	19	50	45	22	27	33	28	40	35
4	12:00-13:00	24	19	39	39	57	50	29	33	31	37
5	13:00-14:00	21	25	26	31	39	45	31	35	25	25
6	14:00-15:00	11	14	14	17	6	8	10	10	3	5
Total		**97**	**97**	**216**	**216**	**178**	**178**	**148**	**148**	**144**	**144**

Source: Survey Data

Appendix B-4

Table: Daily Record of Average Arrival Rate and Service Rate

Day / Counter	Day 1		Day 2		Day 3		Day 4		Day 5		Average	
	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
Withdraw	38.0	43.9	4.8	48.5	14.3	60	13.9	64.7	10.1	59	**16.22**	**55.22**
Deposit	4.8	11.8	8.2	9.8	10.4	10.6	8.0	10.0	7.9	14.1	**7.86**	**11.26**
Remittance	18.7	9.7	46.6	23.8	30.1	20.3	27.3	15.4	31.6	16.7	**30.86**	**17.18**

Source: Survey Data

Appendix C

Appendix C-1

For Multi-Channel

S/N	Time	Day 1		Day 2		Day 3		Day 4		Day 5	
		Arrival	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**	**Arrival**	**Service**
1	09:00-10:00	11	5	5	3	17	12	18	13	14	8
2	10:00-11:00	22	24	45	34	30	23	21	20	41	27
3	11:00-12:00	37	25	26	34	33	40	29	29	36	40
4	12:00-13:00	23	32	39	30	22	22	33	22	26	37
5	13:00-14:00	24	27	21	27	25	22	31	33	44	41
6	14:00-15:00	18	22	20	28	14	22	17	32	25	33
Total		**135**	**135**	**156**	**156**	**141**	**141**	**149**	**149**	**186**	**186**

Source: Survey Data

Appendix C-2

Table: Daily Record of Average Arrival Rate and Service Rate

Day / Counter	Day 1		Day 2		Day 3		Day 4		Day 5		Average	
	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
Multi-Channel	26.2	6.84	29.9	7.92	27.7	7.56	28.7	7.62	33.22	8.45	**29.14**	**7.70**

Source: Survey Data