

DETECTION OF OUTLIERS IN WHEAT PRODUCTION OF MYANMAR (1950-2006)

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Abstract

Outliers are commonplace in statistical data analysis. Noting that the effects of outliers in a time series could be serious, the presence of the outliers has significant influence on the analysis of the series. Direct use of conventional statistical time series analysis may occasionally ignore the fact that the observed time series no longer covers the time period with the same condition. Consequently, it leads to use of the inadequate model and to the biased estimates of the parameters in time series analysis. Therefore, the consideration of outliers in a time series is a crucial aspect of time series analysis. This study reviews outliers in a time series, including definitions and types of outliers, ARIMA models for time series with outliers, as well as likelihood ratio test for detection of outliers in a time series. In addition, the detection and identification of outliers in wheat production series of Myanmar is also empirically investigated as an illustration in this study.

Key words: outlier, ARIMA models, likelihood ratio test.

1. Introduction

A time series is an ordered sequence of observations on a variable of interest collected usually in time, particularly in terms of some equally spaced time intervals. Time series exist in several fields such as agriculture, business, economics, engineering, geophysics, medical studies, meteorology, natural sciences and social sciences. An important feature of a time series is that, typically, adjacent observations are dependent or correlated. Because of dependence structure, statistical procedure and techniques that rely on independence assumptions are not applicable, and different statistical techniques are needed for a time series analysis at the presence of outliers.

Economic time series are sometimes more or less significantly influenced by certain external and special events or circumstances such as political or economic policy changes, strikes, outbreaks of war, monetary crises, implementation of a new rule and regulation, advertising promotions, and similar events. These events are referred to as intervention events and they usually bring outliers into the time series data.

Time series data with outlying observations needs to be analyzed using statistical outlier analysis. The effect of the change due to the unusual event and its position in time series should be analyzed in order to provide the most suitable and reliable forecasts for the future values. Thus, the investigation into the presence of outliers, identification of outliers, assessment of their effects and the remedial measures to accommodate the outliers become a crucial aspect of analyzing many economic time series and have gained much important momentum in recent years.

2. Types of Outliers

Outliers in a time series data set can rise for different reasons. There are two types of anomalies, namely gross errors and outliers. Gross errors are faulty observations, for example

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measurement, reading and typing errors. Identifying these is the least controversial aspect of outlier detection, since gross errors should naturally be identified and corrected whenever possible. If an observation treated as a potential outlier cannot be shown to be a gross error, it has to be considered as an outlier.

Outliers can take different forms in time series. The formal definition and classification of outliers in time series context were first proposed by Fox (1972). He proposed a classification of time series outliers as type I and type II, based on an autoregressive model. These two types have later been renamed as additive and innovational outliers, and are usually abbreviated as AO and IO, respectively. AO affects single observation, and there is no "carry-over" effect. IO affects the observations from the outlier position onwards, and it has "carry-over" effect as well as decays.

3. ARIMA Models for Time Series with Outliers

In the outlier literature (e.g. Tsay, 1986; Chen and Liu, 1993), a time series with outliers is modeled as ARIMA plus intervention. The basic reference to ARIMA model is Box and Jenkins (1976).

The parametric approach to modeling the time series in terms of linear difference equations has led to an important class of models, namely autoregressive integrated moving average model with order p , d and q , popularly known as ARIMA (p, d, q) (Box and Jenkins, 1976).

If Z_t is an observed time series, then the ARIMA (p, d, q) model is given by

$$\phi(B) (1-B)^d Z_t = \theta(B) a_t \quad (1)$$

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ are polynomials of degree p and q in B , $\phi_i, i = 1, 2, \dots, p$ and $\theta_j, j = 1, 2, \dots, q$ are the autoregressive and moving average parameters of the time series respectively and B is the backward shift operator, that is, $B^j Z_t = Z_{t-j}$. In the above model, a_t is the white noise or error series with mean zero, and variance σ_a^2 is referred to as the error variance.

It is assumed that the series $(1-B)^d Z_t$ is stationary, that is, the roots of $\phi(B) = 0$ lie outside the unit circle, and invertible, that is, the roots of $\theta(B) = 0$ lie outside the unit circle. When $d = 0$, Equation (1) represents a stationary process ARMA (p, q), given by

$$\phi(B) Z_t = \theta(B) a_t. \quad (2)$$

The ARMA (p, q) process Z_t can also be represented as a random shock model of the form

$$Z_t = \psi(B) a_t \quad (3)$$

where $\psi(B) = 1 + \psi_1 B + \psi_2 B^2 + \dots$ and ψ weights are calculated by equating the coefficients of B in the equation $\phi(B) \psi(B) = \theta(B)$. For the series to be stationary, it is assumed that $\psi(B)$ converges for $|B| \leq 1$; that is, the ψ weights have the condition $\sum_{j=0}^{\infty} |\psi_j| < \infty$. Similarly, Z_t can

also be represented as an inverted form of the model using the π weights as

$$\pi(B) Z_t = a_t \quad (4)$$

where $\pi(B) = 1 - \pi_1 B - \pi_2 B^2 - \dots$. The π weights are analogously obtained by equating coefficients of B in $\phi(B) = \theta(B) \pi(B)$. To satisfy the condition of invertibility it is assumed

that $\pi(B)$ converges on or within the unit circle. Alternatively, the π weights are assumed to satisfy the condition $\sum_{j=0}^{\infty} |\pi_j| < \infty$. (Box, Jenkins and Reinsel, 1994).

Following Box and Jenkins (1976), the analysis based on these models has been extensively studied in the literature and for details, Abraham and Ledolter (1983), Chatfield (1989), Kendal and Ord (1990), Wei (1990), Box et al. (1994), Mills (1994), Brockwell and Davis (1991, 1996) and Liu (2006) are referred to interested researchers and readers.

Box and Jenkins suggested that the principle of parsimony is important in model building; that is, the number of parameters p , d , and q of the fitted model must be minimum (Box et al., 1994). The inferential problems considered in the literature are usually identification of the order p , d , and q in the model, estimation of the time series parameters and error variance, diagnostic checking of the model, and forecasting of the future values, etc.

In this study, the analysis of stationary and invertible time series ARMA (p , q) with outliers are considered with the help of empirical analysis of time series data with outliers.

Let Y_t be the observed time series and Z_t be the underlying time series which is free of the impact of outliers. Assume that Z_t follows a general ARIMA model in Equation (1). Then the general outlier model for an observed time series Y_t is defined as

$$Y_t = f(t) + Z_t \quad (5)$$

where Z_t is a regular ARIMA model and outliers are incorporated into $f(t)$. The $f(t)$ can be denoted by different outliers types.

An additive outlier (AO) model, that is, $f(t) = \omega P_t^{(T)}$ at time T in ARMA (p , q) (Fox, 1972) is

$$Y_t = \omega P_t^{(T)} + Z_t \quad (6)$$

where Y_t is the observed series, Z_t is an unobserved outlier free series as in Equation (2), ω is the outlier parameter $-\infty < \omega < \infty$ and

$$P_t^{(T)} = 1, t = T, \\ = 0, t \neq T,$$

is the indicator variable representing the presence or absence of an outlier at time T . The presence of AOs, is clearly seen in a time sequence plot as AO does not have any carry-over effect.

An innovational outlier (IO) model at time T , that is, $f(t) = \omega \psi(B) P_t^{(T)}$, in ARMA (p , q) is specified by (Fox 1972; Abraham and Box, 1979)

$$Y_t = \omega \psi(B) P_t^{(T)} + Z_t \quad (7)$$

where, as before, Y_t is the observed series, Z_t is an unobserved outlier free series as in Equation (2), ω is the outlier parameter $-\infty < \omega < \infty$ and

$$P_t^{(T)} = 1, t = T,$$

$$= 0, t \neq T,$$

is the indicator variable which represents the presence or absence of an outlier at time T . The IO affects all observations Y_T, Y_{T+1}, \dots beyond time T and decays with ψ weights as it has carry-over effect.

It is not unusual to come across time series data with more than one outlier. The problem of handling multiple outliers in time series is more complicated, for the simple reason that the outliers could be of different types (Barnett and Lewis, 1994).

More generally, an observed time series Y_t might be affected by outliers of different types at k points of time T_1, T_2, \dots, T_k and we have the following multiple outlier model of the general form

$$Y_t = \sum_{j=1}^k \omega_j V_j(B) P_t^{(T_j)} + Z_t \quad (8)$$

where k is the total number of outliers present in the series, $\omega_j, j = 1, 2, \dots, k$ are the corresponding outlier parameters which may not be distinct and

$$V_j(B) = 1 \quad \text{for an AO,}$$

$$= \psi(B) = \frac{\theta(B)}{\phi(B)} \quad \text{for an IO,}$$

when an outlier type presents at time point $T_j, j = 1, 2, \dots, k$.

Problems of interest associated with these types of outlier models are to be identified from the standpoints of the timing and the type of outliers and estimation of the magnitude ω_j of the outlier effect, so that the analysis of the time series will adjust for these outlier effects.

4. Likelihood Ratio Criterion for Detection of Outliers in a Time Series

In practice, the timing of an intervention event may or may not be known. Often in such cases, when the timing and causes of a series of interventions are known, an appropriate handling can be carried out using intervention analysis that was proposed by Box and Tiao (1975). In many situations, the timing of intervention is rarely known beforehand and it has significant influence on the analysis of time series. It leads to the general time series outlier analysis. The presence of outliers is often not known at the start of the time series data analysis; additional procedures for detection of outliers and assessment of their possible impacts are important in practice. Therefore, they need to be developed.

The well-known procedure for detection of outliers in a time series is the likelihood ratio test, which was first proposed by Fox (1972) followed by Chang and Tiao (1983), Tiao (1985), Tsay (1986), Chang, Tiao and Chen (1988) and Chan and Liu (1993).

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As stated above, Fox (1972) classified time series outliers as type I (additive outlier) and type II (innovational outlier) based on an autoregressive model. The basic idea (in an autoregressive

model) is to add a dummy variable for every observation in turn, maximize these likelihoods, and see whether the maximum of the likelihood ratio statistics thus achieved is significant. Fox also suggested the use of more practical simplifications of likelihood ratio test, such as standardized estimated errors in the observations being tested. These were developed by, among others, Muirhead (1986), Chang, Tiao and Chen (1988) and Tsay (1986), and are also used as a part of a complete outlier modeling strategy.

The models for additive outliers (AO) and innovational outliers (IO) are as described in Equations (6) and (7) respectively. These two models can be written in terms of the innovation sequence a_t 's as follows:

$$\text{AO: } Y_t = \frac{\theta(B)}{\phi(B)} a_t + \omega P_t^{(T)} \quad (9)$$

$$\text{IO: } Y_t = \frac{\theta(B)}{\phi(B)} (a_t + \omega P_t^{(T)}) \quad (10)$$

Thus, the AO case may be called a gross error model, since only the level of the t^{th} observation is affected. On the other hand, an IO represents an extraordinary shock at time point T influencing Y_T, Y_{T+1}, \dots , through the dynamic system described by $\frac{\theta(B)}{\phi(B)}$.

Let $e_t = \pi(B) Y_t$ for $t = 1, 2, \dots, n$ where $\pi(B) = \frac{\phi(B)}{\theta(B)}$. We can write Equations (9) and (10), respectively as

$$\text{AO: } e_t = \omega \pi(B) P_t^{(T)} + a_t \quad (11)$$

$$\text{IO: } e_t = \omega P_t^{(T)} + a_t \quad (12)$$

In other words, the information about an IO is contained in the residual e_T at that particular point T , whereas that of an AO is scattered over a string of residuals e_T, e_{T+1}, \dots

For n available observations, the AO model (11) can be written as

$$\begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_{T-1} \\ e_T \\ e_{T+1} \\ e_{T+2} \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix} = \omega \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \\ -\pi_1 \\ -\pi_2 \\ \cdot \\ \cdot \\ \cdot \\ -\pi_{n-T} \end{bmatrix} + \begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_{T-1} \\ a_T \\ a_{T+1} \\ a_{T+2} \\ \cdot \\ \cdot \\ \cdot \\ a_n \end{bmatrix} \quad (13)$$

Let $\hat{\omega}_A$ be the least square estimator of ω for the AO model. Because $\{a_t\}$ is white noise, from the least squares theory, we have

$$\begin{aligned} \text{AO: } \hat{\omega}_A &= \frac{e_T - \sum_{j=1}^{n-T} \pi_j e_{T+j}}{\sum_{j=0}^{n-T} \pi_j^2} \\ &= \tau^2 \pi(F) e_T \end{aligned} \quad (14)$$

where $\tau^2 = \left(\sum_{j=0}^{n-T} \pi_j^2 \right)^{-1}$ and $\pi(F) = (1 - \pi_1 F - \pi_2 F^2 - \dots - \pi_{n-T} F^{n-T})$, F is the forward shift operator such that $Fe_t = e_{t+1}$. The variance of the estimator $\hat{\omega}_A$ is

$$\text{var}(\hat{\omega}_A) = \tau^2 \sigma_a^2 \quad (15)$$

Similarly, letting $\hat{\omega}_I$ be the least squares estimator of ω for the IO model, we have

$$\text{IO: } \hat{\omega}_I = e_T \quad (16)$$

and

$$\text{var}(\hat{\omega}_I) = \sigma_a^2. \quad (17)$$

Thus, the best estimate of the effect of an IO at time T is the residual e_T , whereas the best estimate of the effect of an AO is a linear combination of e_T, e_{T+1}, \dots and e_n with weight depending on the structure of the time series process. Since $\tau^2 \leq 1$, it is easily seen that $\text{var}(\hat{\omega}_A) \leq \text{var}(\hat{\omega}_I) = \sigma_a^2$ and in some cases, $\text{var}(\hat{\omega}_A)$ can be much smaller than σ_a^2 .

Let H_0 denote the null hypothesis that $\omega = 0$ in Equations (9) and (10), H_1 denote the situation $\omega \neq 0$ in Equation (9) for AO and H_2 denote the situation $\omega \neq 0$ in Equation (10) for IO. The likelihood ratio test statistics for AO and IO are given by

$$\begin{aligned} H_0 \text{ vs } H_1 : \quad \lambda_{1,T} &= \frac{\hat{\omega}_A}{\tau \sigma_a} \\ H_0 \text{ vs } H_2 : \quad \lambda_{2,T} &= \frac{\hat{\omega}_I}{\sigma_a} \end{aligned} \quad (18)$$

Under the null hypothesis H_0 , the statistics $\lambda_{1,T}$ and $\lambda_{2,T}$ both have the standard normal distribution.

The likelihood ratio method further leads to the criteria

$$\text{AO} : \quad \max |\lambda_{1,T}|$$

$$\text{IO} \quad : \quad \max_{t=1, \dots, n} |\lambda_{2,T}|$$

for testing the possibility of an AO or IO, respectively, at an unknown position in the series Y_1, \dots, Y_n .

A simple rule was mentioned by Fox (1972) as a possible way to distinguish between AO and IO. At any suspected point T , the possible outlier is classified as an AO if $|\lambda_{1,T}| > |\lambda_{2,T}|$ and it is classified as an IO if $|\lambda_{1,T}| \leq |\lambda_{2,T}|$.

In practice, the ARMA parameters and σ_a^2 are usually unknown. Estimates of these parameters, together with that of ω under either the AO or the IO case, can be obtained by maximizing the likelihood function of $(\phi_1, \dots, \phi_p, \theta_1, \dots, \theta_q, \omega, \sigma_a^2)$ in the same fashion as that described by Box and Jenkins (1976). Based on these estimates, the likelihood ratios can be computed accordingly for testing the hypotheses, one against another, in Equation (18).

The iterative procedure for the detection of outliers in a time series at unknown positions is as follows:

Step 1

Model the series $\{Y_t\}$ by assuming that there is no outlier. From the estimated model, compute the residuals, that is,

$$\hat{e}_t = \hat{\pi}(B)Y_t = \frac{\hat{\phi}(B)}{\hat{\theta}(B)} Y_t$$

where $\hat{\phi}(B) = (1 - \hat{\phi}_1 B - \dots - \hat{\phi}_p B^p)$ and $\hat{\theta}(B) = (1 - \hat{\theta}_1 B - \dots - \hat{\theta}_q B^q)$. Let

$$\hat{\sigma}_a^2 = \frac{1}{n} \sum_{t=1}^n \hat{e}_t^2$$

be the initial estimate of σ_a^2 .

Step 2

Calculate $\hat{\lambda}_{1,t}$ and $\hat{\lambda}_{2,t}$ for $t = 1, 2, \dots, n$, using the estimated model. Define

$$\hat{\lambda}_T = \max\{|\hat{\lambda}_{1,t}|, |\hat{\lambda}_{2,t}|\}$$

for $t = 1, 2, \dots, n$, where T denotes the time when the maximum occurs. If $\hat{\lambda}_T = |\hat{\lambda}_{1,T}| > C$, where C is a predetermined positive constant, then there is the possibility of an AO at time T with its effect estimated by $\hat{\omega}_A$ in Equation (14). The effect of AO can be removed by defining new residuals

$$\tilde{e}_t = \hat{e}_t - \hat{\omega}_A \hat{\pi}(B)P_t^{(T)} \quad \text{for } t \geq T.$$

If $\hat{\lambda}_T = |\hat{\lambda}_{2,T}| > C$, then there is the possibility of an IO at time T. The impact of IO is estimated by $\hat{\omega}_1$ in Equation (16). Then, the effect can be eliminated by defining a new residual

$$\tilde{\epsilon}_T = \hat{\epsilon}_T - \hat{\omega}_1 = 0 \quad \text{at time T.}$$

In practice, Chang et. al. (1988) recommended using $C = 3$ for high sensitivity, $C = 3.5$ for median sensitivity and $C = 4$ for low sensitivity in the outlier detecting procedure when the length of the series is less than 200. In either of preceding cases, a new estimate $\tilde{\sigma}_a^2$ is computed from modified residuals.

Step 3

If an IO or an AO is identified in Step 2, recompute $\hat{\lambda}_{1,T}$ and $\hat{\lambda}_{2,T}$ based on the same initial estimates of time series parameters, but using the modified residuals $\tilde{\epsilon}_t$'s and the estimate $\tilde{\sigma}_a^2$, and repeat Step 2.

Step 4

Continue to repeat Steps 2 and 3 until no further outliers can be identified.

Step 5

Suppose that Step 4 terminated and k outliers have been tentatively identified at times T_1, T_2, \dots, T_k . Treat these time points as known and estimate the outlier parameters simultaneously using general outlier model of the form

$$Y_t = \sum_{j=1}^k \omega_j V_j(B) P_t^{(T_j)} + \frac{\theta(B)}{\phi(B)} a_t \quad (19)$$

which is equivalent to Equation (8) where $V_j(B) = 1$ for an AO and $V_j(B) = \frac{\theta(B)}{\phi(B)}$ for an IO at time T_j .

Treating Equation (19) as the suggested model, we start the outlier detection stage again. If no other outliers are found, we stop. Otherwise, the estimation stage is repeated, with the newly identified outliers incorporated into the model (19), until no more outliers can be found, and all of the outlier effects have been simultaneously estimated with the time series parameters. Thus, we have the following fitted outlier model:

$$Y_t = \sum_{j=1}^k \hat{\omega}_j V_j(B) P_t^{(T_j)} + \frac{\hat{\theta}(B)}{\hat{\phi}(B)} a_t \quad (20)$$

where $\hat{\omega}_j$, $\hat{\phi}(B) = (1 - \hat{\phi}(B) - \dots - \hat{\phi}_p B^p)$ and $\hat{\theta}(B) = (1 - \hat{\theta}_1 B - \dots - \hat{\theta}_q B^q)$ are obtained in the final iteration.

5. Detection of Outliers in Wheat Production Series

The likelihood ratio test is used for the detection of outliers in the "Wheat Production Series (in thousand metric ton) of Myanmar from 1950-51 to 2005-06". The data is plotted in Figure 1.

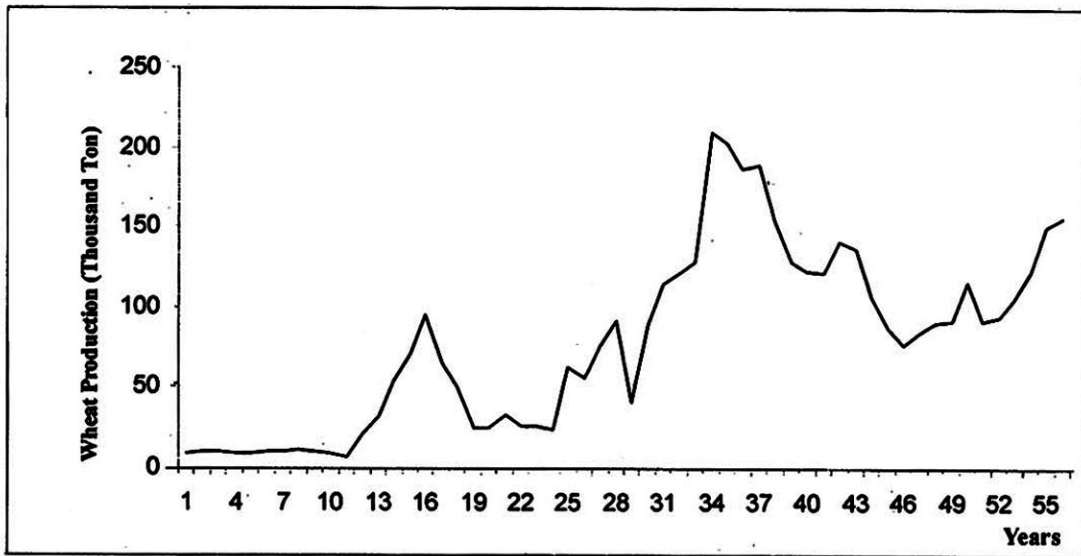


Figure 1: Plot of Wheat Production Series (1950-1951 to 2005-2006)

In Figure 1, the unusual peaks at $t = 16, 29,$ and 34 look like the possible outliers in this series. But the question is whether all are outliers or not and what are the types of outliers to which these outliers belong. It may be difficult to get the correct answer by visualization from the time series sequence plot as in Figure 1 and the detection of outliers is an important issue in such case.

Firstly, the identification of the time series model is needed for the observed data series. Using SPSS software, the plots of autocorrelation function (*acf*) and partial autocorrelation function (*pacf*) of the observed series are obtained which show a tail off pattern of *acf* and a cut off after lag 1 for *pacf*, respectively. Hence, the model suggested for this data is AR(1) specified by

$$(1 - \phi B)Z_t = \theta_0 + a_t \tag{21}$$

where θ_0 is the overall constant in the model. Based on the tentative model AR(1) without outlier, fitted model is obtained as

$$(1 - 0.938 B)Z_t = 79.832 + a_t \tag{22}$$

with $\hat{\sigma}_a^2 = 530.151$.

By likelihood ratio test under the AR (1) model, an AO at $t = 29$ and an IO at $t = 34$ are identified. Thus, the fitted AR(1) model with an AO $t = 29$ and an IO at $t = 34$ is given by

$$Z_t = \theta_0 + \frac{1}{1 - \phi B} [\omega_1 P_t^{(34)} + a_t] + \omega_2 P_t^{(29)} \tag{23}$$

The estimates of parameters for model in Equation (23) on the basis of the wheat production series are presented in the following table.

**Table (1)
Estimated Parameters of AR(1) with an AO and an IO Outliers
for Wheat Production Series**

Parameter	Estimate	S. E.
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θ_0	64.839	26.308
ϕ	0.936	0.035
ω_1	86.220	15.628
ω_2	-49.744	11.223
σ_a^2	287.981	-

Hence, the simultaneous estimation of the parameters of the model is given by

$$Z_t = 64.839 + \frac{1}{1-0.936B} [86.220P_t^{(34)} + a_t] - 49.744\omega_2P_t^{(29)} \quad (24)$$

(26.308)
(0.035)
(15.628)
(11.223)

with $\hat{\sigma}_a^2 = 287.981$, where the values in parentheses below the parameter estimates are the associated standard errors.

From the above table, it is also noticed that the estimate of error variance σ_a^2 from model (24) is smaller in comparison with the estimate obtained on ignoring the outliers from model (22). The reduction percentage in error variance by model (24) for wheat production series is given in the following table.

Table (2)
The Reduction Percentage in Error Variance for Wheat Production Series

Model	Estimated Variance	Reduction Percent
AR(1) with 2 outliers (24)	287.981	45.67%
AR(1) with no outlier (22)	530.151	-

From Table (2), it is noticed that the error variance for model (24) is smaller than that for model (22), and the reduction percentage in error variance by model (24) is 45.67% when the effects of an IO at $t = 34$ and an AO at $t = 29$ are taken into account.

For the diagnostic checking, the residuals series of the fitted model with two outliers is examined to check the residuals are white noise or not. Thus, the *acf* and *pacf* of the residual series for the fitted model with outliers are plotted in the following figure.

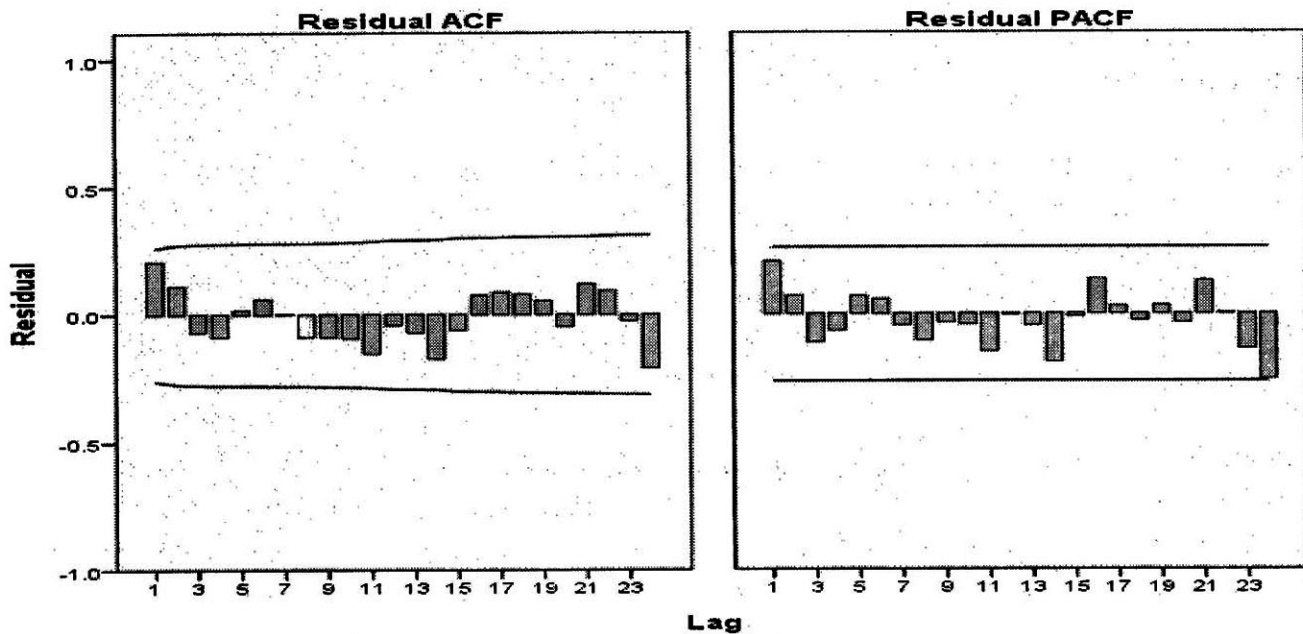


Figure 2 The *acf* and *pacf* of Residual Series for Model (24)

From Figure 2, the *acf* and *pacf* of the residuals series of the fitted model do not form any pattern, and they are statistically significant since the *acf* and *pacf* lie within two standard deviations for 5% level of significance.

Besides, Box-Ljung Q statistic of the fitted model is 12.407 which is not significant at 5% level of significance. This means that the fitted model is adequate for the wheat production series. Based on the results of these residual analyses, the tentative model with an AO at $t = 29$ (1978-79) and an IO at $t = 34$ (1983-84) is found to be adequate for the observed time series on wheat production of Myanmar.

First, an AO with negative sign at $t = 29$ is found when the sharp fall of wheat production occurred in 1978-79. It might be due to the fact that the number of irrigated area for wheat cultivation decreased relative to the previous years, and the utilization of fertilizer and amount of pure strain of seeds distributed also declined. Consequently, the wheat production declined to 41 thousand metric tons in 1978-79.

Second, an IO with positive sign at $t = 34$ is detected as wheat production increased to 210.2 thousand metric tons in 1983-84. The causes of such increase were increase in the supply of pure strain of seeds, the utilization of fertilizer and insecticides as wheat was considered as one of the most important crops. Besides, new pure strain seeds, which became suitable for the climate of Myanmar, could be produced after doing necessary agricultural research. Then, the State took necessary measures to supply the pure strain of seeds to the wheat cultivators. And, it rained enough for the crops in late monsoon days, then the volume of wheat production not only surpassed the production of previous years but also it exceeded the production target. Similarly, production of wheat in 1984-85 onwards continued rising but it dropped considerably in years to come.

6. Conclusion

Outliers do exist in economic time series data and can at least theoretically have harmful effects on their analysis. It is difficult to say that how frequently outliers occurred and which is

the best way to describe them (that is, additive or innovational outlier or others), how serious a threat they pose in practice, and how to handle them or indeed whether anything at all should be done about them.

In practice, the presence of outliers is assumed as indicated in the graph at the start of analysis; additional procedures for detection of outliers and assessment of their possible impacts are essential. We should stress the importance of adjusting outliers prior to and during the analysis of time series data. All outliers lead to the worst type of forecasting; consequently, it is necessary to detect these outliers. Even more emphasis should perhaps be placed on examining and explaining the possible causes of detected outliers in the observed data.

For detection of outliers, simple statistical tools such as time series plots, frequency distributions and simple t-tests can be used. These methods are simple and perhaps useful in some cases, but obviously not sufficient for the wide variety of situations encountered in empirical time series analysis. It is, therefore, necessary to consider more complicated but effective methods one of which is likelihood ratio test for the detection of outliers in a time series. It has become almost a standard method for detection and identification of outliers in time series. It is already featured in some computer software packages and easy to understand as well as seem to work reasonably well in most practical situations, especially when used iteratively.

The effect of the change in the economic time series due to special events should be analyzed by statistical outlier analysis in order to provide the timings and the causes of occurring these events which could be useful in future planning. In Myanmar, many economic time series were affected by events that are planned by decision and policy makers and caused by economic changes, weather conditions, out-of-stock situations, and similar events. This study attempts to detect the timing of the presence of outliers, identifies the type of outliers in wheat production series of Myanmar and also explains the possible causes of occurrence of outliers in the observed data series. In the detection of outliers, this study focuses only on the applications of likelihood ratio test. Then, ARIMA models with outliers are constructed and the fitted models could be useful in decision-making and planning purposes. For detection of outliers in wheat production series of Myanmar, AR(1) model with an AO outlier at $t = 29$ and an IO outlier at $t = 34$ is obtained.

In this study, emphasis has been put only on two types of outliers, namely, additive outlier (AO) and innovational outlier (IO) which can occur most often in practical time series. It is also needed to detect and investigate the effects of other types of outliers using suitable outlier detection procedures as well as causes of presence of these types of outliers in observed data series as a further research in this field of study. The detection of outliers can also be extended to seasonal time series models. The detection of outliers in multivariate time series should be investigated as a further study. It is also recommended that the outlier detection and model fitting to practically important sets of time series data should be carried out now and often in order to have better estimates as well as forecasts.

Appendix

Wheat Production of Myanmar (Thousand Metric Ton)

Years	Wheat Production	Years	Wheat Production
1950-51	9.5	1978-79	41

1951-52	10.6	1979-80	89.4
1952-53	10.4	1980-81	114.9
1953-54	9.6	1981-82	122
1954-55	8.8	1982-83	128
1955-56	10.1	1983-84	210.2
1956-57	10.7	1984-85	203
1957-58	10.9	1985-86	186.9
1958-59	10.7	1986-87	188.7
1959-60	9.7	1987-88	154.4
1960-61	7.3	1988-89	128.1
1961-62	20.7	1989-90	122.2
1962-63	31.9	1990-91	121.5
1963-64	53.4	1991-92	141.1
1964-65	70.6	1992-93	136.4
1965-66	94.8	1993-94	106.9
1966-67	65.7	1994-95	87.7
1967-68	50.3	1995-96	76.7
1968-69	25.4	1996-97	85.4
1969-70	25.4	1997-98	90.7
1970-71	32.9	1998-99	92
1971-72	26.5	1999-00	115.3
1972-73	26.3	2000-01	92.1
1973-74	24.3	2001-02	94.4
1974-75	62.6	2002-03	105.7
1975-76	55.7	2003-04	122.4
1976-77	75.2	2004-05	150
1977-78	92.2	2005-06	156.2

Source: Report to the People (1964-1965 to 1979-1980); Report to the Phyithu Hluttaw (1971-1972 to 1988-1989); Review of Financial, Economic and Social Conditions (1989-1990 to 1997-1998); Selected Monthly Economic Indicators (1955-2008)

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