ECONOMIC RESEARCH PROJECT

Reprint Series Nos. 2 and 3 January, 1963.

CAPITAL THEORY AND DEVELOPMENT PLANNING

and

THE ROBINSONIAN MODEL OF ACCUMULATION

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C O N T E N T S.

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Reprinted from the Review of Economic Studies
Vol. XXIX No.2, 1962

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CAPITAL THEORY AND DEVELOPMENTAL PLANNING*

This paper presents a normative model of long range developmental planning in a backward economy with an agricultural sector of peasant proprietors and a state operated industrial sector, treating the problem as a sequential decision process in the manner of modern capital theory of the Dorfman-Samuelson-Solow "turnpike" variety.[1] The first section of the paper gives a background discussion intended to assist the reader in gaining a perspective on the context of the model's application. The model itself is constructed in the second and third sections and the fourth section makes a brief attempt to relate the results of the model to the existing literature on the subject.

I

The pattern of planned industrialization exemplified by Soviet history is for the state to collectivize the peasantry to ensure sufficient deliveries of food to the urban industrial areas and, within industry itself, for a high proportion of available resources to be devoted to the capital-goods-producing department, leaving the production of consumer goods in comparative neglect.

This procedure of emphasizing capital goods production relatively early in the stage of industrialization had seemed to many Western observers to be putting the cart before the horse if compared with the "textiles first" path of development suggested by the experience of their own countries. The difference is no doubt partly due to the great changes in the character of the world trading system that have taken place since the nineteenth century when most of the Western economies "took off". It would have been, on both political and economic considerations, a suicidal policy for Soviet Russia to have expected to obtain the expanding capital requirements of her industrialization through imports from the capitalist west. Many underdeveloped countries today, faced with stagnant world demand for their primary exports or with tariff barriers for their light manufactures, seem to be finding the Soviet pattern of industrial development an increasingly attractive one for them to emulate.[2]

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* This paper is a revised version of a chapter of a doctoral thesis submitted to the Massachusetts Institute of Technology in May 1960. The author wishes to thank E.D. Domar, whose lectures on Soviet economics at M.I.T. stimulated the writing of the paper, and R.M. Solow, R.M. Sundrum and anonymous referees for valuable comments on previous drafts. He also wishes to thank the Ford Foundation for the award of a grant making his stay at M.I.T. possible.

1. See R.Dorfman, P.A. Samuelson and R.M.Solow, Linear programming and Economic Analysis, McGraw Hill, New York, 1958 , Chapter 12.
2. See S.J. Patel "Export Prospects and Economic Growth: India," Economic Journal, September, 1959 and a recent ECLA report on the possibilities of a Latin American Common Market.

Even if the view that international trade cannot be relied on to provide for the capital goods requirements of developing countries is accepted, it does not necessarily follow that these countries should expand current domestic production of capital goods to the fullest extent possible in the interest of growth. One reason for this is the fact that plant and equipment, once installed, become largely specific to the uses to which they have been committed. Thus if the objective of a particular long term plan is to maximize consumer goods output at the terminal date it is possible that the capital stock available for consumer goods production at that date would be smaller than it might have been as a result of excessive investment in the capital goods industries in the earlier years of the plan, so that too much of the capital stock of the country becomes specific to that sector.

It has however been shown, by P.C. Mahalanobis[1] and Evsey Domar[2], that the further away the terminal date set for the maximization of consumer goods output the greater should be the proportion of current investment allocated to the capital goods sector in spite of complete specificity of the capital stock in both the consumer and capital goods sectors. If the objective is the maximization of capital goods output at the terminal date it follows from the similar models of these authors that the entire current investment should be in the capital goods sector, leaving the flow of consumer goods output over time to depend purely on the existing capital stock in that sector. Thus, regardless of the desired composition of output at the horizon, the more emphasis should be given to capital goods the further away the horizon is set.

The theoretical model employed by these authors assumes that labour is not a scarce factor in the industrial sector. The justification for this assumption would no doubt be sought in the alleged existence of "disguised unemployment" in substantial quantities in the agricultural sectors of underdeveloped economies. This assumption alone, however, would not be sufficient. It would need to be supplemented with the further assumptions that the state both wishes and is able, by force if necessary, to transfer any required amount of labour from the rural sector, together with the means of subsistence that this labour would normally be consuming on the farms.

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1. P.C. Mahalanobis, "Some Observations on the Process of Growth of National Income," Sankhya, September, 1953.

2. E.D. Domar, <u>Essays on the Theory of Economic Growth</u>, Oxford University Press, New York, 1957, Chapter 9.

If either of these supplementary assumptions is not satisfied, as for example in Yugoslavia and India, labour could not be regarded as having zero real cost in the industrial sector in spite of "disguised unemployment" in agriculture. The level of industrial employment would depend on the quantity of manufactured consumer goods sold to the peasants in exchange for food. The relationship between capital and labour in the industrial sector would thus be competitive rather than complementary since more labour would require more consumer goods and therefore less capital goods could be produced. This exposes a conflict over the sectorial distribution of investment that is quite distinct from the one confronted by Mahalanobis and Domar and which consequently requires independent analysis.

In a recent book[1] Maurice Dobb has analyzed alternative models of planned development in which the determinant of the rate of growth is <u>either</u> the command over labour exercised by the level of consumer goods output in the planned industrial sector <u>or</u> the capacity of the capital goods producing department of that sector, as with Mahalanobis and Domar. What is lacking is an explicit and comprehensive attempt to reconcile these conflicting determinants within an integrated model in which both limitations on the rate of growth operate simultaneously.

The rest of this paper will be devoted to the formulation and application of a multiperiod, general-equilibrium model that is intended to accomplish this reconcilation as well as to answer some further questions arising out of the preceding discussion.

II

It is assumed that the economy, which carries on no foreign trade, is divided into two sectors "Industry" and "Agriculture". "Industry" is assumed to be a single decision unit, a "Ministry of Production" <u>a la</u> Barone, carrying out the directives of some supreme authority. It would be desirable to allow for decentralized implementation of these directives in the Lange-Lerner fashion of mimicry of perfect competition. Although this is not done explicitly in this paper, shadow prices will be derived that could serve the purpose of decentralizing decision.

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1. M.H. Dobb, <u>An Essay on Economic Growth and Planning</u>, Routledge and Kegan Paul, London, 1960. Also relevant is the well-known article by Arthur Lewis on "Economic Development with Unlimited Supplies of Labour," in <u>Manchester School</u>, May, 1954. A discussion of the Lewis paper in the present context would be interesting but is not attempted here for reasons of space.

Industry is assumed to produce only two commodities, Machines and Textiles. The inputs used in producing these outputs are the services of Labour and of the existing stock of the same Machines that the system itself produces. Since the economy is assumed not to engage in foreign trade, and it will later be assumed that production in Agriculture is purely by means of Labour and Land, there is no market for Machines, which therefore assume the status of an intermediate good in the Industrial sector. The services of Machines will henceforth be referred to as the factor of production Capital.[1]

Known production functions, relating the output of each good (Textiles and Machines) to inputs of Labour and Capital, are assumed to exist. Both production functions are taken to be homogeneous of the first degree in Labour and Capital, to ensure the viability of the Lange-Lerner type of decentralization that is assumed in the background. The technology for each good could thus be completely represented by a single "isoquant" which is assumed to be convex to the origin and to admit of continuous substitutability, at least over a wide range, between Labour and Capital.

Time is divided into "periods" during each of which the current output of new Machines is not available for use as an input. New Machines and Textiles are produced with Labour and the services of the stock of existing Machines. At the end of each period, the output of Machines produced during that period is added to the existing stock and is available for use as an input in the next period. For simplicity it is assumed that Machines are of constant efficiency and last foreover. Thus the stock of Machines for the second period is equal to the initial stock plus the output of Machines turned out during the first period.

During any period the currently available supplies of both Labou and Capital are assumed to be capable of being freely allocated between the production of either Textiles or Machines. The Domar-Maha lanobis feature of limited flexibility in allocating Capital could b introduced but since nothing of any consequence would follow in the present context this is not done for the sake of convenience.

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1. The flow of service units per Machine per period is assumed to be constant so that there is a simple proportional transformation connecting Machines and Capital.

Agriculture is assumed to produce a single homogeneous commodity, Corn. The inputs used in Corn production are Land, which is assumed to be homogeneous in quality and either in fixed supply or growing at some exogenously determined rate, and Labour, which is provided by peasant families, each of which cultivates its own small plot of Land. The Land in possession of each family is assumed to be so limited that if the family performs all the Labour services on it that it is willing to, the marginal productivity of Labour would be zero over a wide range. The implication of this is that if some members of the peasant family find non-agricultural occupations, the output of the farm would not decline until after a certain point. The remaining members of the family would, of course, have to work harder.

Remuneration to individual members of the peasant families is assumed to be on the basis of average productivity which is assumed to be able to meet the needs of subsistence. Since the marginal productivity of Labour is zero, a class of landless labourers could not exist in Agriculture.

If it existed in isolation the population of the peasant economy would grow by the natural excess of births over deaths. What happens to average productivity of Labour of course depends on the supply of Land and the state of technical knowledge. It is assumed that either Land or technique or both are changing in such a way as either to leave average productivity of Labour constant or to increase it, but the increase in supply of Land or improvement in technique is insufficient to prevent the marginal productivity of Labour from always being zero. Thus the structure of the peasant economy is preserved in essential features in spite of the passage of time.

The peasant economy does not, however, exist in isolation. The presence of the Industrial sector, by creating a demand for Labour that can be met only by the members of the peasant families, alters the secular pattern of development for Agriculture.

The form of the supply function of Labour to Industry assumed is that it is horizontal within any period at a level equal to the average Corn productivity of Labour in the preceding period plus a constant margin. Thus, although horizontal in any particular period, the level of the supply curve can vary from period to period as a result of the changing relationships between Agriculture and Industry.

The fact that the Industrial sector produces no Corn implies that there must be a demand for Textiles from the peasant economy, since otherwise Industry could not make its demand for Labour effective. In this connection two possibilities arise. One is that the real wage in Industry is paid in terms of Textiles and that the price-ratio of Textiles to Corn is determined competitively by the exchange of Textiles in the hands of Labour employed in Industry for Corn produced by the population remaining in Agriculture. Average productivity in Agriculture, converted into terms of Textiles at the price-ratio thus established, would then determine the real wage in terms of Textiles for the next period. Given Industry's demang for Labour the wage-bill in the form of Textiles is determined and a new price-ratio is established by trade between workers and peasants, the process continuing in this fashion.

The other possibility is that the real wage is paid in Corn, which is obtained by the "Ministry of Production" engaging directly in trading the Textiles output for Corn from Agriculture with which to meet the wage-bill. The significant difference between these two alternatives is that for the same quantity of Corn and Textiles produced, the price-ratio will be more favourable to Textiles in the second case as it is traded monopolistically by the state instead of competitively by the workers employed in Industry. It is hence-forth assumed that the real wage is paid in terms of Corn.

Industry's demand for Labour has now to be explicitly introduced. This is best done with reference to Figure 1. TT represents the transformation curve in Industry between the production of Machines and Textiles. Production takes place with a given initial stock of Machines and an initial level of employment of Labour, wages having been paid out of an initial fund of Corn.

OO is the offer curve for Textiles in exchange for Corn. It is identical with the Marshallian offer curve that is a familiar tool in the theory of international trade. The slope of a vector from the origin to OO represents a price-ratio and the co-ordinates of the point at which it intersects OO indicates the quantities of Corn and Textiles that will be exchanged at that price-ratio. Since it is assumed that the wage-bill in terms of Corn is paid at the start of the current period, as in the "advances to labour" of the Physiocrats and Ricardo, OO represents the offer not only of the peasants in Agriculture but of the workers in Industry as well.

The "Ministry of Production" being in the position of a pure monopolist, can select the point on OO that is to its greatest advantage. Actually in making its decision, it will be with reference to some "conjectured" offer curve rather than to the actual one OO. Discrepancies will arise due to errors in anticipating exactly what the actual offer curve is. In practice this would be a very important problem but since it is only the logic of the process that is the concern here, it is assumed that the conjectured and actual offer curves always coincide.

Any decision as to the quantity of Textiles produced, i.e., the choice of a point on OO, at the same time determines a point on TT

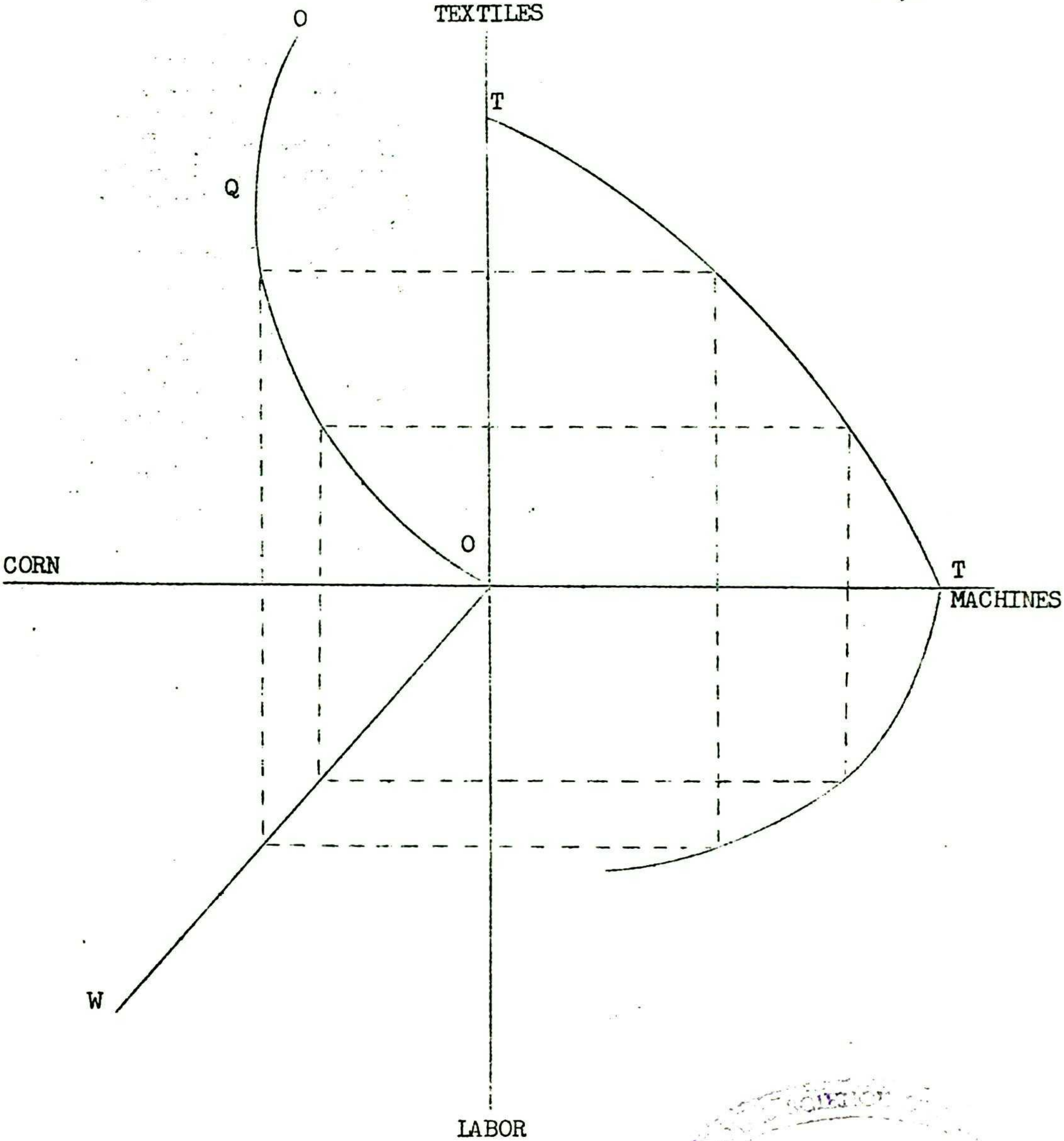

FIGURE I

and hence the output of new Machines. As the output of Textiles rises from zero, the Corn proceeds from their exchange increase also, but the output of new Machines declines. At the point Q on OO the Corn proceeds reach a maximum and decline afterward. It follows that Textile outputs above those corresponding to Q on OO will never be chosen since they imply both less Corn and fewer new Machines. Up to the point Q, however, a choice has to be made between the relative desirability of Corn and Machines since more of one can only be obtained at the expense of less of the other.

Machines are desired solely for their use in production. Corn could be desired for consumption by the state authorities but, since this use is not likely to be an important one, it is simplest to ignore it altogether, so that the sole reason the state has for acquiring Corn is to meet the wage-bill for the subsequent period.[1] Corn is thus a factor of production at one remove. This is displayed in the southwest quadrant of Figure 1. OW represents the wage-rate for the current period determined by Labour's average productivity in Agriculture Obviously, the more Corn the state can acquire. the greater the number of workers it can employ in the next period. The maximum employment possible in Industry is determined by the point Q on OO. The curve in the south-east quadrant shows the possible combinations of Labour and new Machines that can be chosen for the next period.[2]

It now becomes perfectly obvious that nothing can be decided as to the current allocation of resources between Textiles and Machines until some criterion is introduced that will take account of the future, since alternative choices of the current Machine-Textiles mix will give alternative factor endowment patterns to subsequent periods.

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1. The simplifying assumption that all stocks of Corn held by Industry at the start of a period are expended on Labour in that period itself is made in order to keep the problem manageable. It is also assumed that Textiles are not hoarded from one period to another but are exchanged for Corn in the same period as they are produced.

2. The convexity of this curve follows from that of the transformation and offer curves.

This criterion is assumed to be provided in the form of a function showing combinations of Machines and Textiles output between which the state is indifferent, and the decision rule for the "Ministry of Production" is taken to be the maximization of this function over some specified time-horizon.[1] Thus a set of curves could be drawn in Machines-Textiles space and the problem of Industry conceived as pushing the transformation curve out as far as possible so as to be tangential to the highest "social indifference curve". For simplicity these curves are assumed to be either convex or straight lines. The latter case would correspond to maximizing national income in Industry at some socially determined fixed price-ratio between Machines and Textiles.

The locus of possible factor endowments for the next period is readily derived from the curve in the southeast quadrant of Figure 1. The existing stock of Machines can be added to the new Machines produced and the total amount of Capital thereby obtained. The FF curve in Figure 2 indicates the pattern of factor endowments possible for the next period.

From the FF curve it is possible to derive the production possibilities or transformation curve between Machines and Textiles for the next period. This curve would differ from the one most familiar to economists since factor endowments are variable instead of being fixed as in the usual derivation from the Edgeworth-Bowley box diagram. The method of construction in the present case, as depicted in Figure 2, is as follows. Take the isoquant for any specified level of the output of one commodity. Slide this isoquant along the FF curve in such a way that it is always tangential to it. The point of origin of the isoquant diagram will then trace out a locus within the FF curve that is labelled F*F* . This locus shows the quantity of each factor available after various possible factor combinations to produce the specified output have been subtracted from the total factor availabilities. The isoquant map for the other commodity can then be super-imposed on Figure 2 and the point on F*F* that is tangential to the highest isoquant determines the factor endowment that maximizes the output of one commodity given the output of the other. Once the point on F*F* is obtained, the origin of the isoquant diagram for the commodity the output of which

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1. This procedure is of course highly arbitrary since the valuations of the two goods in the terminal period will depend on their uses in further time periods. This is perhaps the most fundamental problem confronting the theory of capital, with the exception of considerations relating to uncertainty. Frank Ramsey's well-known solution to the problem was to take the terminal point as determined by either utility satiation or technical saturation.

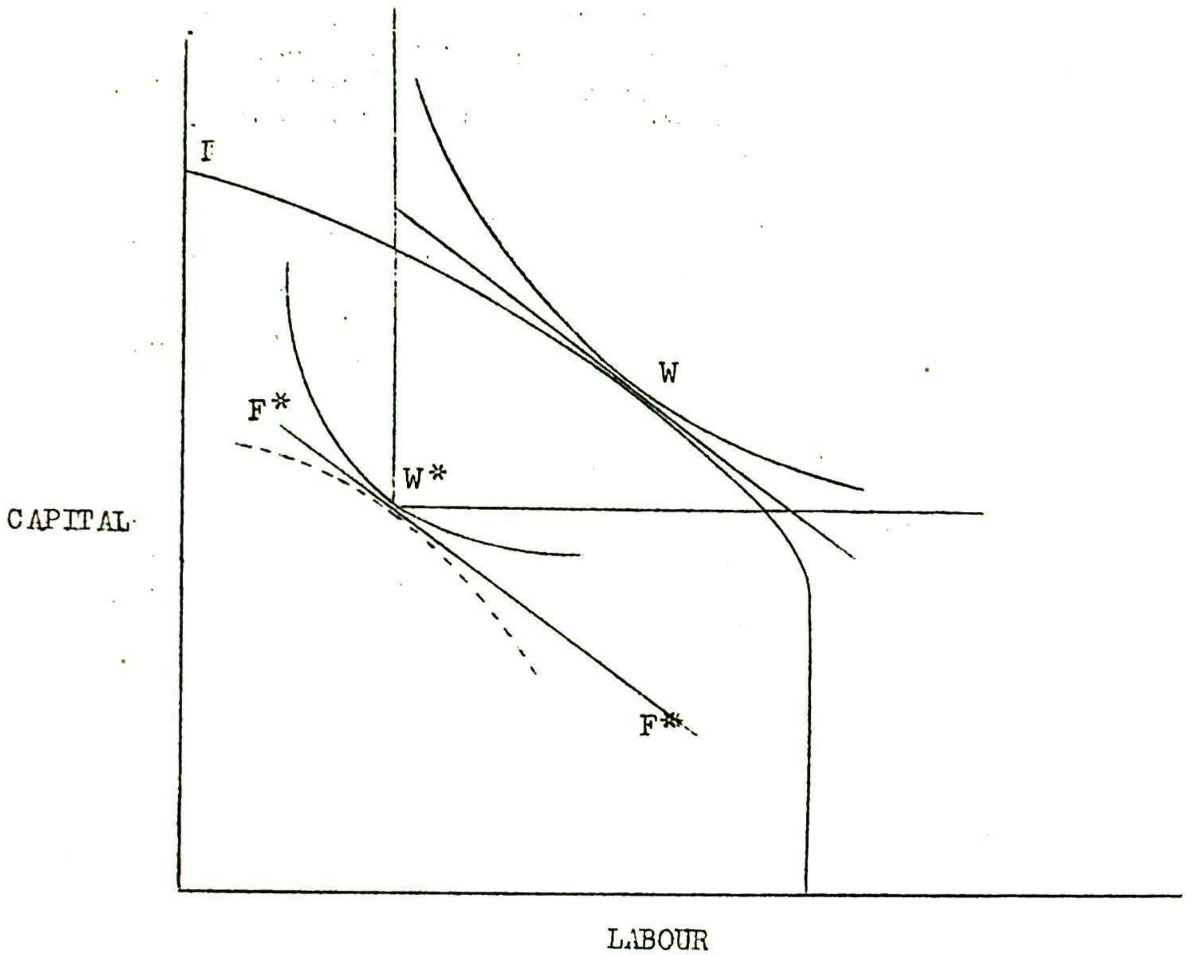

FIGURE II

is specified can be set so as to coincide with it, and the poing of tangency between the isoquant for the specified output and FF gives the optimal total factor endowment.

An Edgeworth-Bowley box with the dimensions selected in this fashion can then be set up and a production-possibilities curve derived from it. Thus for each specified level of one output there will be a unique production-possibilities curve and the envelope of all these curves constitutes the production frontier for society in the next period, as indicated by T'T' in Figure 3.

Every point on T'T' corresponds to a point on TT that generates the factor endowments which make it attainable. Every point on TT, however, does not correspond to a point on T'T'. This is shown by the following argument. Consider the FF curve in Figure 2. Suppose that the output of either good was set at zero. Then the maximum amount of the other good that could be produced would be determined by the tangency of FF with an isoquant for that commodity In this case FF and F*F* coincide. Let us suppose further that one of the goods is capital-intensive and the other labour-intensive (which one it is which does not matter) in the Lerner-Samuelson sense.[1] The factor endowment that maximizes the output of the capital-intensive good will therefore have more Capital and less Labour than that which maximizes the output of the labour-intensive good. The optimal factor endowment for any mix of both the goods will thus lie between these limits. Corresponding to these two limiting points on FF there will be two points on TT indicated by X and Y. It is therefore only the points on TT lying on the XY segment that generate all the points on T'T'. Points to the left of X and to the right of Y will never be chosen, since the factor endowments that they determine for the next period will be suboptimal for any output-mix in that period.

III

It has thus been shown that given the initial endowments of Labour and Capital in Industry, the offer curve of workers and peasants and the supply curve of Labour, it is possible to describe the production frontier for Industry in the next period, assuming no change in technology. If any point on T'T' is specified as a target for Industry, a point on TT can be found that will provide just enough of both factors to make the target point attainable.

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1. See A.P. Lerner, "Factor Prices and International Trade," Economica, Vol XIX, February, 1952 and P.A. Samuelson, "International Trade and Factor Price Equalization Once Again," Economic Journal, Vol. LVIV, June 1949.

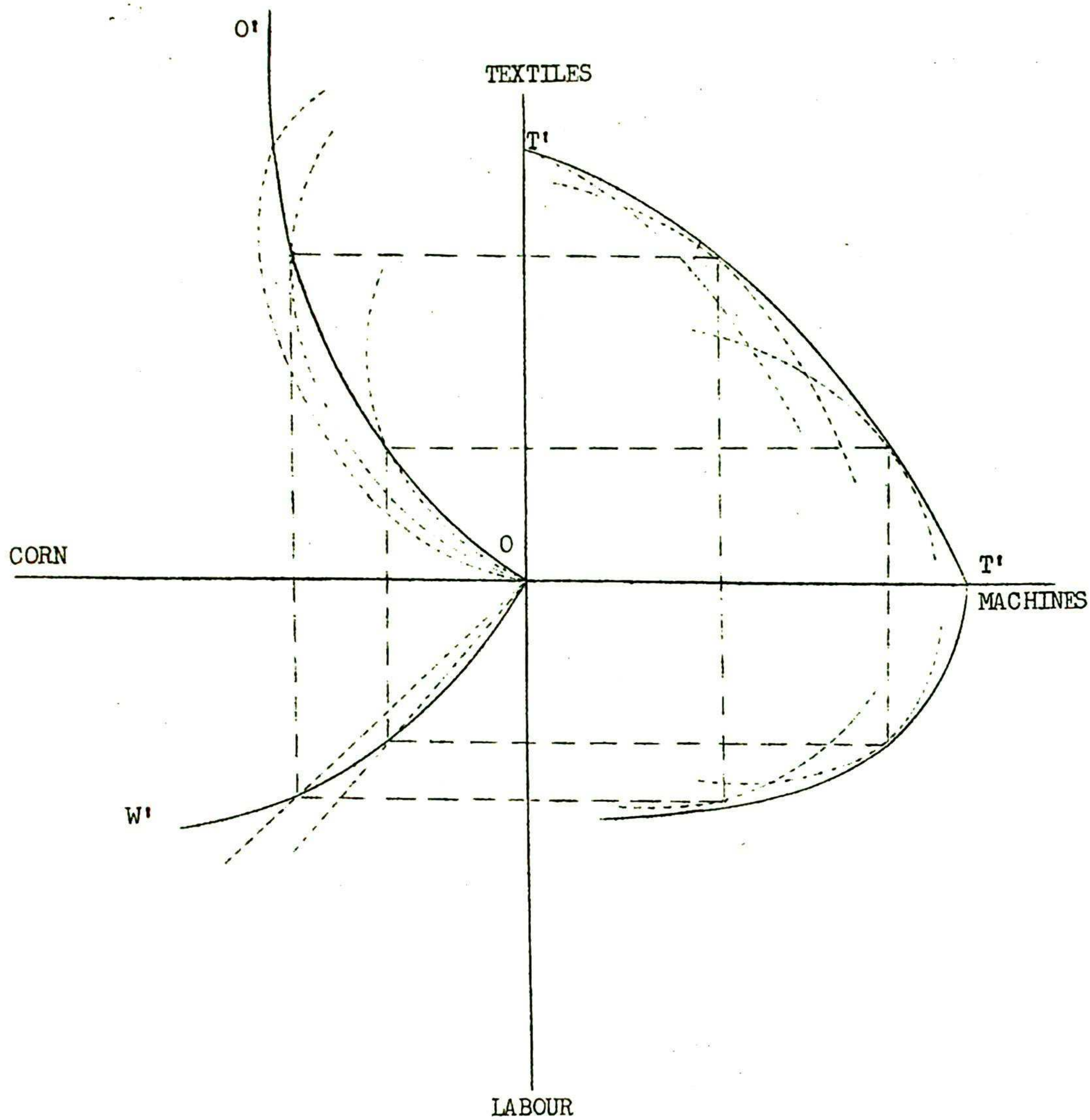

FIGURE III

Suppose, however, that a target were set for Industry not in terms of the next period's output-mix but any finite number of periods away. Will it then still be possible to determine a point on TT such that the target is attained ? At the outset, there does not seem to be any great difficulty in doing this. It has already been shown that given TT, OO, and OW it is possible to obtain T'T', the production frontier for the next period. Given corresponding offer curves and supply curves, the production possibilities curves of the additional periods could be derived in the same way as T'T' was. If the objective is maximization of the given preference function over the specified number of periods, the point of tangency between the production-possibilities curve for the last period and the highest attainable social indiference curve would determine the outputs of the target period and its optimal factor endowment. In order to make the target point attainable, a unique point on the production-possibilities curve of the preceding period will have to be chosen; and this in turn will involve a unique point on all previous curves until the optimal point on TT is reached.

The process of generating successive production-possibilities curves, however, encounters what appears to be a serious obstacle as soon as it is attempted to proceed from T'T' to the curves for later periods. This is because the offer curve and labour supply curve, which together with T'T' determine the locus of possible factor endowments for the next period, cannot be given independently of the choice of outputs for the initial period.

In Figure 1, as the output of Textiles is substituted for that of Machines, the Corn proceeds increase up to the level of Textiles output corresponding to the point Q on OO. This means that employment in Industry in the next period will rise as we proceed along TT in the direction of greater Textiles output. The more people there are in Industry, the less there will be in Agriculture. On the one hand this makes for increasing per capita real income in Agriculture, since there are fewer individuals among whom any given output is to be shared. On the other hand, total output might fall. It shall be assumed, however, that the "disguised unemployment" and the increase in population is so large that the maximal withdrawal of Labour into Industry still leaves the marginal productivity of Labour zero in Agriculture.

Thus the average Corn productivity of Labour in the initial period, and hence the real wage to Industry in the next period, increases as Textiles are substituted for Machines along the "efficient" XY segment of TT. Since each point on T'T' corresponds to a unique point on this segment the real wage in the next period will be different for each point on T'T'. Whether it rises or falls as Textiles are substituted for Machines along T'T' depends upon the assumptions regarding the relative factor intensities of the two goods. If Textiles were the labour-intensive commodity the real wage would rise as Textiles were substituted for Machines along T'T'.

If the factor intensities were reversed, the opposite would happen. In either case there would be a family of straight wage-rate lines in the southwest quadrant of Figure 3 instead of the single line in Figure 1. Each one of these lines would correspond to one of the transformation curves enveloped by T'T'.

The position and shape of the next period's offer curve is also dependent on the decision for the current period. The more the current output-mix favours Textiles, the greater will be the quantity of Corn in the hands of peasants and workers in the next period. This is because the quantity of Corn that is in their hands is equal to next period's harvest, which, because of the "disguised unemployment" assumption, is independent of decisions in Industry, and the Industrial wage-bill for the next period, whichh is an inventory carry-over from the current period that increases as Textiles is substituted for Machines in the current output-mix of Industry.

Therefore, corresponding to each point on T'T' there will be an entire offer curve which shows the reciprocal supply and demand at various alternative price-ratios for a given real income of workers and peasants in terms of Corn. As Textiles are substituted for Machines along T'T' this real income either increases or decreases continually, depending upon the relative factor intensities of the two goods. On the assumption that Textiles are not inferior goods, the offer curves will either lie further and further out as Textiles are substituted for Machines or further and further in, depending upon whether real income is rising or falling during the process. The fact that the offer curve is shifting from point to point in this way seems to introduce an essential indeterminacy into the picture.

This, however, is by no means necessarily the case, as will be shown by the following argument. Suppose that the tastes of workers and peasants as a group could be adequately represented by a system of indifference curves that have the usual "well-behaved" properties, and that these tastes were known to the "Ministry of Production." Then, for any given real income in terms of Corn an offer curve could be derived showing the reciprocal supply and demand at each price-ratio. Since each level of Textile output implies a certain level of real income in terms of Corn, an offer curve could be drawn for each such level of Textile output. Thus a family of offer curves is generated in the northwest quadrant of Figure 3 instead of a single curve as in Figure 1. For each transformation curve enveloped by T'T' there is a corresponding unique offer curve and wage-rate line. For any level of Textiles output the real income, and hence the offer curve, is known so that the price-ratio that clears the market can be located on the offer curve. The locus of all such points for each level of Textile output could be drawn to form a "composite" offer curve OO' along which not only the price-ratio, but real income as well, would vary. Similarly, OW' is the "composite wage-rate line". Figure 3 is drawn on the assumption that Textiles are the labour-intensive good.

Thus no indeterminacy exists. For each point on T'T' the Corn-proceeds and the wage-rate are known so that a curve can be drawn in the southeast quadrant of Figure 3 showing the possible combinations of new Machines and Labour that could be used in the next period. This curve is the envelope of all the curves that could be drawn for each of the transformation curves enveloped by T'T'. Factor endowments, and hence production possibilities, for the next period, can now be derived. Passage to subsequent periods presents no additional problems since the analysis for the second period would still apply without modification.

The problem of what the current allocation of resources should be between Textiles and Machines can now be solved, given the preference function which is to be maximized over any specified number of periods. This is because the terminal position of Industry is arrived at by the tangency of the last period's transformation curve with the highest social indifference curve, and that implies a unique path to this position. The production point on the initial transformation curve thus determined generates a particular transformation curve for the next period, and, the production point on this and all subsequent curves being determined as well, the evolution to the optimal terminal position is completely described. In the process, the terms of trade between Textiles and Corn and the real wage of Labour, are determined for each period, together with the Capital and Labour in each sector of Industry. The failure to be at the "correct" point on any of the intervening periods' transformation curve would make the optimal terminal postion unattainable.

The problem that was posed has therefore been solved, but not, however, without paying the price of making certain strong restrictive assumptions. The major one is, of course, the fact that Industry is assumed to be able to predict with perfect accuracy the tastes of workers and peasants, total output in Agriculture, the total population, and the supply curve of Labour.

It would be wrong to criticize the model as "unrealistic" if it were considered simply as the framework for economic planning, since, in some way or the other, the authorities would have to estimate the information that is necessary abour the behaviour of the workers and peasants. Once the plans were adopted and put into effect, however, errors in forecasting are bound to appear. How the system would respond to these errors is not investigated at all here. It therefore cannot serve as a "behavioural" model of a planned economy. Only if all forecasts turn out to be a descriptive model. It is therefore not so much "unrealistic" as incomplete, in the sense that it has no prescribed reaction mechanisms built into the system for incorrect forecasting. This, in the present state of knowledge, seems to be true of all models that involve optimization over time.

A system of shadow prices, or "duals" in linear programming terminology, can now be derived. These could be used for the Lange-Lerner type decentralization of decisions involving the implementation of the optimal plan. As the reader familiar with modern economic theory knows, these shadow prices are essentially marginal rates of substitution of various kinds between inputs and outputs.

In Figure 1 suppose that the point P on TT is the one at which it is necessary to produce in order to realize the plan. The slope of TT at P, or the marginal rate of transformation at that point between Textiles and Machines, is the shadow price-ratio of the two outputs for the first period. Since every point on TT corresponds to a point on the "efficiency locus" of the box diagram from which TT is derived, the shadow price-ratio of Labour and Capital is determined as the common marginal rate of substitution between these factors in producing the two outputs. P also determines the terms of trade between Corn and Textiles measured in Figure 1 by the slope of OR. This would not be a shadow price-ratio but an observable market price-ratio. Employment for the next period is determined as OS, so that the factor endowment for the next period can be specifically located as a point, labelled W, on the curve FF in Figure 2. The slope of FF at W measures the marginal rate at which the Capital and Labour that the current period bequeaths to the next can be transformed into each other. All price-ratio for the first period have therefore been determined.

Since the optimal point on T'T' is also known, the process described for ascertaining the shadow price-ratios in the first period could be repeated for the second and all subsequent periods. What, if any, is the connection between the shadow prices of the first period and those of later periods? As the reader might well suspect the shadow price-ratios for all periods are inextricably linked.

The connection is as follows. Since P is the optimal point on TT, W is the optimal point on FF, in the sense that it represents the optimal factor endowment for the optimum point on T'T'. Corresponding to W on FF will be the point W* on F*F* which determines the optimal amount of Capital and Labour to be used in producing the output which is to be maximized subject to some specified level of the other output. Which good is held fixed, and which is maximized, does not of course matter. The slope of F*F* at W*, which indicates the marginal rate of substitution between Labour and Capital in producing the good the output of which is to be maximized, must be equal to the slope of FF at W, which measures the rate at which Labour is substituted for Capital in producing the other good. But the slope of FF at W also measures the rate at which Capital can be transformed into Labour in the first period for use in the second. Thus the shadow factor-price ratio for the second period is connected with the corresponding ratio for the first period and hence with the shadow product-price ratio. The product-price ratio for the second period will be given by the one-to-one correspondence that exists between it and the factor-price ratio. In this way the whole system of shadow prices is linked over time. The crucial intertemporal efficiency condition that provides the link is the equality between the marginal rate of transformation between Capital and Labour in the tth period and the marginal rate of substitution between them in the (t - 1)th period.

IV

The analysis of the last two sections of the paper has determined, by an adaptation and extension of the Dorfman-Samuelson-Solow "turnpike" model to fit the circumstances considered, a development path that reconciles, for any particular objective specified, the two conflicting requirements of industrialization in a backward economy - the expansion of capital goods production and the transfer of manpower from the agricultural sector. This development path generates, for each period, the optimal choices for the terms of trade with the agricultural sector and the techniques of production to be adopted in the capital and consumer goods departments of the industrial sector.

The problem of choice of technique has been extensively studied in the development literature, the most recent and comprehensive contributions being those of Maurice Dobb[1] and A.K. Sen.[2] The advance made here over the work of these authors is to convert the industrial real wage and the terms of trade between agriculture and industry from parameters to variables determined simultaneously with the choice of both the pattern and techniques of production within a general equilibrium model. Choice of techniques is thus determined with reference to the entire process of planned development rather than analyzed in isolation as an aspect of that process. Another noteworthy difference with the work of Dobb and Sen is that techniques are not chosen once and for all over the entire horizon but are free to vary from period to period up to the horizon as efficiency dictates.

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1. Maurice Dobb, op. cit.

2. A.K.Sen, Choice of Techniques, Oxford, Basil Blackwell, 1960.

It is interesting to note that the recommendation by Arthur Lewis that the terms of trade should be kept stable over time would lead to a deviation from the optimum path.[1] The view of the terms of trade emerging from the present model seems rather to be that of the Soviet economist Preobrazhenski who, in the nineteen twenties, advocated setting the terms of trade with the rural sector so as to maximize "primitive socialist accumulation," whatever that may mean exactly.[2]

It has been seen that the conflict between investment in the consumer goods sector as against the capital goods sector, due to the specificity of the capital stock, is resolved by the Domar-Mahalanobis model, regardless of the desired composition of output at the horizon, increasingly in favour of the capital goods sector as the length of the horizon is extended. The present model has analyzed an entirely different ground of conflict. due to the necessity of consumer goods production for obtaining labour, which is ignored by these authors. It would therefore be interesting and important to see whether a similar resolution can be arrived at in the present case also.

In deriving the transformation curve <u>T'T'</u> from <u>TT</u> it was shown that all points on <u>T'T'</u> were generated only from points on the <u>XY</u> segment to <u>TT</u>. The possibility can therefore arise, as it cannot in the Domar - Mahalanobis model, of the production of Machines being excessive in the sense that less Machines and more Textiles in the current period could increase the output of both items in the next period. As the horizon is extended one period further into the future the segment of efficient points on <u>TT</u> would lie within <u>XY</u> itself since the choice of points on <u>T'T'</u> also would now be restricted to an intermediate segment, that necessary to generate <u>T" T"</u>. It can readily be seen that further extensions of the horizon would narrow the efficient segment on <u>TT</u> even more. Thus, as in the Domar-Mahalanobis model, the longer the horizon the less acute becomes the conflict between consumer goods and capital goods production. Unlike that model, however, the conflict is not always resolved in favour of capital goods production. This is because the effect of lengthening the horizon on the <u>XY</u> segment of <u>TT</u> is not for <u>X</u> to approach Y, which remains fixed, but for both points to be moved closer to each other.

Thus, suppose that the objective is maximization of the output of Machines at the horizon. Then, shifting the horizon from the

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1. See W . Arthur Lewis, <u>Theory of Economic Growth,</u> Irwin, Homewood, 1955, p.276.
2. See A. Erlich, "Preobrazhenski and the Economics of Soviet Industrialization", <u>Quarterly Journal of Economics</u>, February, 1950.

second to the third period would mean that the output-mix on T'T' would have to contain some Textiles instead of only Machines. If it is assumed, as was done earlier, that Textiles are relatively labour-intensive, the necessary adjustment on TT would be to produce less Machines and more Textiles. Further extensions of the horizon would reduce the desirable amount of Machine production still further, on TT as well as on T'T'. It can readily be seen that, whatever the original length of the horizon, extension by one more period reduces the optimal level of Machine production and hence raises that of Textiles. Consequently, in each of these periods, the terms of trade will be turned in favour of Corn, and both employment and the real wage in Industry will rise.

The effect on choice of techniques in each sector of Industry can also be determined, although with a little more difficulty. The movement along TT in the direction of more Textiles and less Machines corresponds to a similar movement along the "efficiency-locus" of the box diagram from which TT is derived. Since Textiles are the labour-intensive good it follows that the capital-labour ratio will be raised in both sectors. Thus the effect on choice of technique has been determined for the initial period.

Similar reasoning cannot be applied to T'T' and subsequent transformation curves since, unlike TT, each of them is not derived from a single box diagram. As Textiles are substituted for Machines along any one of these curves, however, the "as if" competitively imputed factor cost of a unit of Textiles will rise in terms of Machines, just as along TT, provided that these curves are convex. The convexity of T'T' can be shown to follow from that of FF, which has already been shown to be a consequence of the convexity of TT and OO. The convexity of the transformation curves for subsequent periods is likewise implied by the convexity of T'T', OO', OW' and F'F'.

The use of the diagram employed by Lerner[1] in his proof of the factor-price equalization theorem will indicate that, independently of the change in factor endowments, a rise in the cost of Textiles relative to Machines will raise the capital-labour ratio in both sectors if Textiles are the labour-intensive good. Thus the effect on choice of techniques in the subsequent periods will be the same as in the first period in spite of the rise in employment and fall in the stock of Capital which reduces the over-all capital-labour ratio.

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1. See Lerner, op.cit., p. 9, fig.4.

If the target at the horizon was maximization of the output of Textiles instead of the output of Machines the effects on all the variables of extending the horizon would be in the opposite direction.

In conclusion it may be pointed out that the rather paradoxical result, that producing more capital goods in the present could mean having less capital goods in the future would seem to indicate that the Soviet model of industrialization might perhaps be a false guide to countries planning their development under less extreme forms of socialism.

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THE ROBINSONIAN MODEL OF ACCUMULATION

by Dr. Ronald Findlay

The Robinsonian Model of Accumulation 1/

by RONALD FINDLAY

Mrs. Robinson's Accumulation of Capital 2/ seems to have won for her the position of the dragon in the fairy tales told by neo-classical capital theorists. Solow 3/ and Swan 4/ and also Green, 5/ have all responded only to the negative aspect of Mrs. Robinson's work, which is the critique of the concept of the production function and the marginal productivity theory of factor income shares associated with it. What has received surprisingly little attention is her own bedtime story of growth and distribution. This is no doubt partly due to the obscurity of Mrs. Robinson's literary presentation of what are fairly intricate quantitative relationships. The tortuous numerical examples do little to help.

In this article I shall attempt to elucidate Mrs. Robinson's ideas by a compact formulation, in simple mathematics, of what appears to me to be the essential framework of her model of accumulation. In the process some extension of her model is made possible, and there is some clarification of the relationship between her model and the Harrod-Domar 6/ type of model on the one hand and the Solow-Swan 7/ model on the other.

I

A first step in the formalisation of Mrs. Robinson's model has already been made by Kelvin Lancaster in his review article of her book 8/. It will be convenient to begin by reproducing the relevant part of Lancaster's article, and then to develop the model further in the necessary directions.

1/ The author is indebted to Professor R.M. Sundrum, of the Department of Statistics, University of Rangoon, for helpful discussions and comments.

2/ Joan Robinson, the Accumulation of Capital, 1956.

3/ Robert Solow, "The Production Function and the Theory of Capital", Review of Economic Studies, Vol.XXIII (1955-56).

4/ T.W. Swan, "Economic Growth and Capital Accumulation", Economic Record, vo.32 (1956).

5/ H.A.J. Green, "Growth Models, Capital and Stability", Economic Journal, vol. LXX (1960).

6/ R.F. Harrod, Towards a Dynamics, Macmillan, 1948, and E.D. Domar, Essays in the Theory of Economic Growth, 1958.

7/ Robert Solow, "A Contribution to the Theory of Economic Growth", Quarterly Journal of Economics, vol. 79 (1956), and Swan o.cit.

8/ Kelvin Lancaster, "Mrs. Robinson's Dynamics", Economica, vol. XXVII (1960).

There are two basic equations :

(1) $a_{11} \Delta M + a_{21} C = M$; and

(2) $wa_{12} \Delta M - (1 - wa_{22})C = 0$

The aij's represent fixed technical coefficients showing the amount of the jth input required per unit of the ith output. The numbering is such that 1,2 stand for producer goods and consumer goods, respectively, with reference to output, and for capital and labour, respectively, with reference to inputs. M stands for the initial stock of machines in the economy; these machines are assumed to be capable of being used in either the producer-goods or consumer-goods sector, and to be freely transferable from one sector to the other. It is convenient to assume that machines last forever with constant efficiency. ΔM stands for the flow output of new machines and C for output of consumer goods. w is the real wage expressed in units of C.

Equation (1) states that the machines used in each sector add up to the total available stock. Mrs. Robinson's saving assumption, that workers save nothing and capitalists everything that they earn, is expressed in (2), which states that all the profits earned in the consumer goods sector are paid out as wages in the producer-goods sector. Takinf w and the technical coefficients as data, the equations determine ΔM and C once M is specified.

Lancaster introduces a further equation

(3) $(1 - w.a_{22})/a_{21} = (P - w.a_{12})/a_{11}$

P being the price of a machine measured in terms of consumer goods, (3) states that the profit per machine-year in each of the sectors must be the same. From (3) it follows that

$$P = \frac{a_{11}}{a_{21}} - w.a_{12} \left(\frac{a_{11}a_{22}}{a_{21}a_{12}} - 1 \right)$$

Since w is given, P is thus determined. Lancaster remarks that the rate of profit on capital invested can now also be determined. He does not point out, however, that it is possible to determine the rate of profit from (1) and (2) alone, that is to say, independently of P. This can be done as follows.

We have the accounting identities :

(4) $Y = r\,PM + wL$

(5) $Y = PM + C$

Letting r stand for the rate of profit and L for employment, (4) states that national income (measured in terms of C) equals profits plus wages, and (5) that it equals the sum of the outputs of the consumer-goods and producer-goods sectors. Since we have wages equal to the output of the consumer-goods sector, by Mrs. Robinson's savings assumption, it follows that

$$rPM = P\Delta M, \text{ or } r = \Delta M/M.$$

which gives us the "von Neumann Theorem" that the rate of profit is equal to the rate of accumulation.

Lancaster leaves his formalisation of the Robinsonian model at the point where he determines ΔM, C and P. He does not display the model in dynamic or moving equilibrium. Thus he says nothing about "golden ages" and comparisons between them which are the main analytical contribution of the book. Instead, he presents a trenchant criticism of the inadequacy of Mrs. Robinson's approach in explaining the behaviour of the system in disequilibrium. The course chosen here will be to develop the simple model outlined above for analysis of growth and distribution in "golden ages".

II

Since Mrs. Robinson assumes that capitalists save everything they earn and workers nothing, (2) states, as we have seen, that the "surplus" in the consumer-goods sector is equal to the wages in the producer-goods sector, or, what is the same thing, that total wages in the two sectors are equal to the output of consumer goods. If we assume instead that capitalists consume a certain constant proportion, α, of their income while workers continue to save nothing, then (2) has to be replaced with

$$(2') \quad (1 - w.a_{22})C - \alpha[(1-w.a_{22})C + (P-w.a_{12})\Delta M] = w.a_{12}\Delta M$$

which states that the "surplus" in the consumer-goods sector equals consumption by capitalists plus wages in the producer-goods sector. The most important difference between (2') and (2) is that the former is a function of P. But, as a consequence of (3), we may eliminate P from (2'), so that we have instead

$$(1 - \alpha)(1-w.a_{22})C = \left[w.a_{12} + \alpha\left(\frac{a_{11}}{a_{21}} - w\,\frac{a_{11}a_{22}}{a21}\right)\right]\Delta M$$

From (1) it is possible to replace C by $(M - a_{11}\Delta M)/a_{21}$, whence, after some manipulation

$$\frac{\Delta M}{M} = \frac{(1-\alpha)(1 - w.a_{22})}{a_{11}+w(a_{12}a_{21}-a_{11}a_{22}}$$

which gives the rate of accumulation as a function of the wage-rate and the technical coefficients. This equation, which we may call the Accumulation Function, is of basic importance for the rest of the argument. It embodies (1) and (2) in a manner that is convenient for dynamic analysis. We thus have one equation and, taking the technical coefficients as given, two unknowns in the rate of accumulation and the wage-rate.

One way of closing the model would be to put

(6) $w = \bar{w}$

where $\bar{w}$ is a fixed "subsistence" wage, and a perfectly elastic supply curve of labour is postulated after the fashion of Marx's "industrial reserve army" or the "peasant hinterland" of Arthur Lewis and other modern writers on the economic development of backward countries.

Inserting $\bar{w}$ into the Accumulation Function, it becomes, if ΔM is taken as instantaneous flow output, a simple first-order linear differential equation which can be integrated to yield

$$M(t) = M_o e^{mt}$$

where M_o is some initial stock of machines, and m is the right-hand side of the Accumulation Function. Thus the stock of machines grows continuously at a constant relative rate which depends on $\bar{w}$, α and the four technical coefficients. The rate of growth in this model thus depends on six parameters instead of only two, as in the Harrod-Domar model, although both models have the same feature of steady exponential growth.

If the stock of machines is growing at a constant relative rate, it is easy to show that the outputs of both the consumer- and producer-goods sectors are growing at that same rate, so that there is a "balanced growth" equilibrium. This can be seen if (1) and (2) are written as a pair of linear simultaneous equations in ΔM and C, solution by Cramer's Rule revealing that both ΔM and C are proportional to M.

It remains to determine the rate of profit, which, because there is consumption out of profits by capitalists, will not be equal to the rate of accumulation. We have

(7) $rPM = P\Delta M + r\alpha PM$

which states that profits are equal to the value of the output of producer goods plus capitalist consumption. From (7) it follows that

$$r = \frac{1}{(1-\alpha)} \cdot \frac{\Delta M}{M}$$

The effect of variations in the capitalists' saving rate on the rate of accumulation and the rate of profit can be readily determined.

Inspection of the Accumulation Function reveals that the rate of accumulation is proportional to the capitalist propensity to save. From (7) it therefore follows that the rate of profit is independent of the thrift of the capitalists.

The influence of the capitalist propensity to save on the relative income shares is a little more difficult to determine. The share of profit, denoted Π, can be written as

$$(8)\quad \Pi = \frac{\frac{P\Delta M}{(1-\alpha)}}{P\Delta M + C} = \frac{1}{(1-\alpha)\left(1+\frac{1}{P}\cdot\frac{C}{\Delta M}\right)}$$

From (2') we obtain

$$\frac{C}{\Delta M} = \frac{\alpha\frac{a_{11}}{a_{21}} + \frac{w.\,a_{12}}{(1-w.\,a_{22})}}{(1-\alpha)}$$

so that

$$\Pi = \frac{1}{\alpha\left(\frac{a_{11}}{a_{21}}\cdot\frac{1}{p}-1\right)+\frac{w.\,a_{12}}{p(1-w.\,a_{22})}+1}$$

and

$$\frac{d\Pi}{d\alpha} = \frac{-\left(\frac{a_{11}}{a_{21}}\cdot\frac{1}{p}-1\right)}{D^2}$$

where D is the denominator of the expression for Π. The sign of $\frac{d\Pi}{d\alpha}$ clearly depends on whether $\frac{a_{11}}{a_{21}}\cdot\frac{1}{p} \gtrless 1$. Substituting for p from (3), it follows that $\frac{a_{11}}{a_{21}}\cdot\frac{1}{p} = \frac{a_{11}}{a_{11}-w(a_{11}a_{22}-a_{12}a_{21})}$ which is greater or less than unity depending on whether $\frac{a_{11}}{a_{12}} \gtrless \frac{a_{21}}{a_{22}}$.

Thus an increase in the thriftiness of the capitalists will raise the profit share if the consumer-goods sector is the more labour-intensive, and conversely.

The effect of changes in the wage-rate on the rates of accumulation and profit can be seen by differentiating the Accumulation Function with respect to the wage-rate, yielding

$$\frac{dm}{dw} = \frac{-(1-\alpha)\,a_{12}a_{21}}{\left[a_{11}+w(a_{12}a_{21}-a_{11}a_{22})\right]^2}$$

which is negative, so that the rate of accumulation varies inversely with the wage-rate. By virtue of (7), this means that the rate of profit also varies inversely with the wage-rate.

The profit share varies inversely with the wage-rate if the consumer-goods sector is more labour-intensive than the producer-goods sector. This follows from (8), since $C/\Delta M$ and P will move in opposite directions as a result of a change in the wage-rate. If the consumer-goods sector is the less labour-intensive sector, p and $C/\Delta M$ will shift in the same direction, so that the influence of the wage-rate on relative shares cannot be determined definitely in this case.

Examination of the Accumulation Function indicates that the rate of accumulation, and hence also the rate of profit, will be increased by a reduction in any one of the technical coefficients. The effect of technical progress on the profit share is difficult to determine since it turns out that $C/\Delta M$ and P are shifted in opposite directions, exxept in the case of a reduction in a_{22} which definitely increases the capitalist share of the national income.

III

The model has been developed so far with the wage-rate taken as given and the supply of labour regarded as perfectly elastic at this given wage-rate. An alternative assumption made by Mrs. Robinson is that the rate of population growth is an exogenously given constant, leaving the wage-rate to be determined by the system itself. This assumption is also made by the neo-classical growth theorists, Solow and Swan.

Thus, in addition to the Accumulation Function we have

$$(9)\quad \frac{1}{L}\cdot\frac{dL}{dt} = n.$$

The system is closed by an equation relating the behaviour of the wage-rate over time to the rates of accumulation and population growth :

$$(10)\quad dw/dt = f(m - n)$$

where $dw/dt - \gtreqless 0$ if $m - \gtreqless n$. Equation (10) states that the wage-rate will rise if the rate of accumulation exceeds the rate of population growth, fall in the opposite case, and remain constant if the two rates are equal.

A graphical solution of this alternative model is given in Figure 1. The wage-rate, w, is measured along the horizontal axis,

FIGURE 1

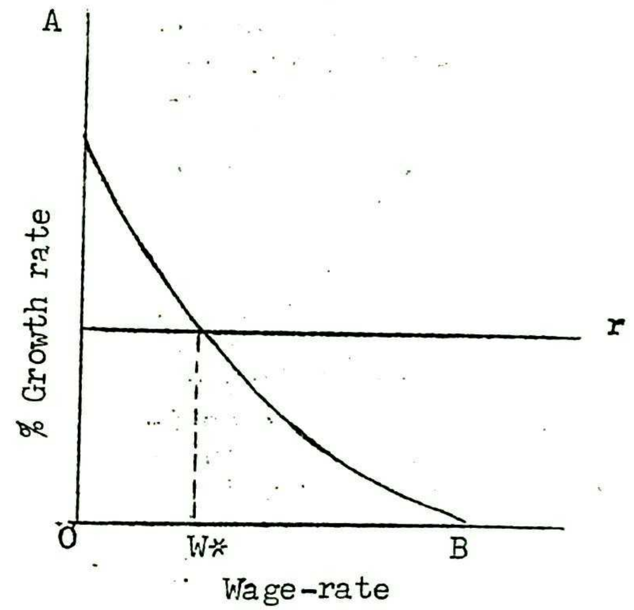
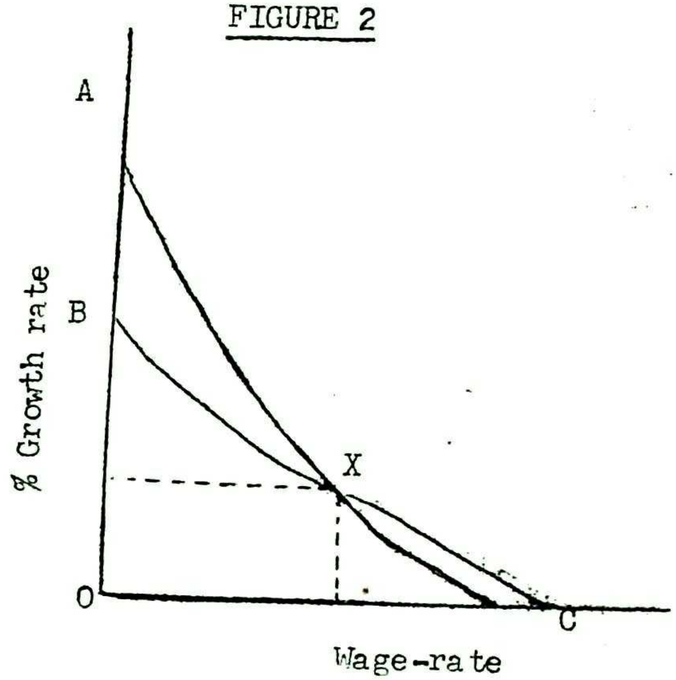


FIGURE 2



and relative rates of growth up the vertical axis. The AB curve shows the rate of accumulation, m, as a function of the wage-rate, the relationship being given by the Accumulation Function. When w is zero, it follows from the Accumulation Function that m is equal to $(1-\alpha)/a_{11}$. This gives the OA intercept on the vertical axis. If w is equal to $1/a_{22}$ it also follows from the Accumulation Function that m would be equal to zero. This gives the OB intercept on the horizontal axis. The negative slope has been proved earlier by differentiation of the Accumulation Function with respect to the wage-rate.

It can be shown that the sign of the second derivative depends on the relative factor-intensities of the two sectors, being negative if the producer-goods sector is the more labour-intensive, positive if it is the less labour-intensive, and zero if the factor-intensities are the same. In the diagram the AB curve has been drawn convex to the origin implying that the producer-goods sector is the less labour-intensive of the two. None of the subsequent results depends on this.

The horizontal line shows the constant rate of population growth, n. w^* is the wage-rate determined by the intersection of the m and n curves. If $w < w^*$, then $m > n$, so that by virtue of (10) w increases. If $w > w^*$, $m < n$, so that w falls. Thus w^* is the equilibrium wage-rate. In this version of the model everything adjusts to the constant rate of population growth, the wage-rate floating until the rate of accumulation is equated to the rate of population growth, producing a "golden age" situation where the stock of machines and hence outputs of both sectors grow at the same rate as population.

There is need for some further interpretation of the model when the two rates differ. One possible explanation is that when $m > n$, employment also grows at the rate m; this is made possible by assuming that the model refers only to the industrial sector of the economy, so that labour is being withdrawn from an agricultural sector to make up the difference between the rates of growth in industrial employment and population, a rising wage-rate in industry being necessary for this purpose. The rise in the wage-rate comes to an end as soon as balance is achieved between the rates of accumulation and population growth. What Mr. Kaldor calls the "three stages of capitalism"[1] can be illustrated in the present context by the subsistence-wage version of the model as the first stage, the situation where $m > n$ with a rising wage-rate as the second stage, and the "golden age" as the third.

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1. N.Kaldor, "Economic Growth and Capital Accumulation", in <u>The Theory of Capital</u>, 1961, F.A. Lutz and D.C.Hague (eds.)

The effects of shifts in the parameters on wages, profit and accumulation can be conveniently analysed in terms of figure 1. A rise in $(1 - \alpha)$ shifts the AB curve to the right, so that the effect would be to raise the wage-rate, leave the rate of accumulation unchanged and lower the rate of profit, since, by (7), the same rate of accumulation is divided by a larger capitalist propensity to save. The rate of accumulation is raised at the initial wage-rate, but the wage-rate rises until the rate of accumulation is ultimately restored to equality with the rate of population growth. Thus the economy moves from one "golden age" to another, with the same rate of growth but with a higher wage-rate and a lower profit-rate.

Nothing definite can be said about the effect on relative shares of cahnges in the capitalist propensity to save. In terms of (8), the effect on changes in $(1 - \alpha)$ on p depends on relative labour-intensities of the two sectors, while the effect on $C/\Delta M$ is wholly uncertain since the increase in $(1 - \alpha)$ and the associated increase in the wage-rate influence it in opposite directions.

It is readily seen from Figure 1 that the rate of accumulation and the rate of profit vary directly with the rate of population growth, and the wage-rate inversely. It is possible to determine the effect of different rates of population growth on the distributive shares if the producer-goods sector is less labour-intensive than the consumer-goods sector since in this case the induced change in the wage-rate changes $C/\Delta M$ and p in opposite directions, so that, from (8), the result follows that the share of profit is an increasing function of the rate of population growth. The effect on relative shares is uncertain in the opposite situation, since the effects on $C/\Delta M$ and P would then work against each other.

Reduction in any of the technical coefficients shifts the AB curve to thr right and thus increases the wage-rate, but ultimately leaves the rate of accumulation and the rate of profit unchanged. The effect on the relative shares of a change in any one of the technical coefficients is uncertain.

Mrs. Robinson devotes considerable attention in her book to what she calls "neutral" technical progress. This can be represented here as equi-proportionate reductions in a_{12} and a_{22}, which means that labour productivity increases at the same rate in both sectors. The effect of this is to shift the AB curve to the right and so to increase the equilibrium wage-rate, leaving the accumulation and profit-rates constant. If a_{11} and a_{21} remain unchanged, it follows from the Accumulation Function that, at the same rate of accumulation, the wage-rate will rise in the same proportion as labour productivity. As a result $C/\Delta M$ and P will be the same as before, and consequently so will the relative shares. It should be noted that in this case Mrs. Robinson's definition of neutrality involves a rise in the capital-labour ratios in both sectors and also for the economy as a whole, since $C/\Delta M$ remains unchanged.

Another possible example of what Mrs. Robinson calls "neutral" technical progress is provided by the case where a_{21} and a_{22} are reduced in the same proportion while a_{11} and a_{12} remain unchanged, so that technical progress takes place in the consumer-goods sector only, and is of such a nature as to leave the labour-intensity of that sector unchanged. In this case it can be shown that if a_{21} and a_{22} are reduced by a factor λ, less than unity, then w, $C/\Delta M$ and P will all be multiplied by $1/\lambda$. Since any technical progress leaves the rate of profit unchanged, it follows from (8) that the relative shares will be unchanged by technical progress of this type. This interpretation of Mrs. Robinson's criterion of neutrality is given by Solow[1]. Her criterion is that what she calls the "real-capital ratio", which can be represented by PM/wL in the present model, must remain unchanged after the technical progress. It can be seen that both cases described here have this property. In the former P remains constant whereas M/L rises, while in the latter it is the reverse.

IV

A possible variation that can be introduced into the model is to make the rate of population growth a function of the wage-rate. Here also the system will tend to a "golden age". Variations in the capitalist rate of saving and the technical parameters will now, however, change the growth rate of the "golden age" equilibrium. The Leibenstein-Nelson[2] "low level equilibrium trap" can be illustrated by drawing an appropriate population growth-rate curve.

So far a single set of input coefficients has been assumed throughout. Mrs. Robinson also discusses the case of a "spectrum" of techniques, which we may represent in the present model as alternative sets of a_{ij}'s the one being chosen depending upon profit maximisation by entre-preneurs. If there are x techniques in the

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1. Robert Solow, "A Wicksellian Model of the Distributive Shares", in Lutz and Hague, op. cit.

2. H. Leibenstein, Economic Backwardness and Economic Growth, 1957, and R.R. Nelson, "A Theory of the Low Level Equilibrium Trap in Underdeveloped Countries", American Economic Review, vol.46(1956).

consumer-goods sector and y techniques in the producer-goods sector, then there are xy possible choices of technique for the economy as a whole.

The case of alternative techniques will be examined here by an example involving only one technique in the consumer-goods sector but two possible techniques in the producer-goods sector. If seems that this simple case brings out the essential point in connection with mechanisation and economic growth, and, moreover, generalisation is not difficult.

The problem presented by multiplicity of techniques is essentially in the construction of the Accumulation Function, since the a_{ij}'s to be inserted have to be determined somehow by the model itself. The first step in the solution is to introduce a particular Accumulation Function for every possible technical position of the economy. In the example to be considered there is one technique for the consumer-goods and two for the producer-goods sector, so that there are two possible technical positions for the economy.

In Figure 2, AC and BC represent the two Accumulation Functions corresponding to the two technical choices for the economy. When the wage-rate is zero the rate of accumulation is equal to $(1-\alpha)/a_{11}$. Since there are two techniques in the producer-goods sector there will be two possible values for a_{11}, the one for the more "mechanised" technique being the larger of the two. Consequently, at the zero wage-rate, the rate of accumulation is greater for the less mechanised technique, this being represented in terms of Figure 2 by the excess of OA over OB. Thus AC and BC correspond to the less and the more mechanised techniques, respectively. At the other extreme, when the wage-rate is equal to $1/a_{22}$, the rate of accumulation would be zero for both techniques so that both curves meet at C on the horizontal axis where $w = 1/a_{22}$. Differentiating each of the Accumulation Functions with respect to the wage-rate and evaluating the derivatives at $w=1/a_{22}$, we obtain

$$\frac{dm}{dw} = \frac{-(1-\alpha)a_{22}}{a_{12}a_{21}}.$$

For both functions a_{21} and a_{22} are the same, but BC, being the more mechanised, has a smaller a_{12} to off-set its larger a_{11}, so that the absolute value of the negative slope of BC is greater than that of AC at C. Thus it is proved that BC does not lie entirely below AC but must intersect it at some point, represented in Figure 2 by X. At the wage-rate corresponding to X both techniques yield the same rate of accumulation and hence the same rate of profit so that entrepreneurs are indifferent between them. At lower wage-rates the less mechanised technique yields the higher rate of accumulation and profit, and at higher-wage-rates the more mechanised technique does this. Thus it is only the AX segment of AC and the XC segment of BC that would ever be operative, so that the outer frontier indicated by the darker line in Figure 2 represents the true Accumulation Function for the economy with techniques in the producer-goods sector switching as X is crossed. A greater number of techniques can be dealt with the same way by taking what is in effect the envelope of all the corresponding curves.

The relationship of mechanisation to the "second stage of capitalism" can now be illustrated. The excess of the rate of accumulation over the rate of population growth continues to drive up the wage-rate and reduce the rate of profit and the rate of accumulation itself; but increasing mechanisation slows down this process although it cannot reverse it.

V

The neo-classical growth model of Solow and Swan, employing a Cobb-Douglas production function, can be regarded from the present standpoint as a case where the two sectors have identical factor-intensities so that they can be integrated into a single sector, and where the "spectrum" of techniques is infinite. The other difference is that savings are a function of profits only in the present model, whereas they are a function of the whole national income in their model. The Accumulation Function in their model is

$$\frac{1}{K} \cdot \frac{dK}{dt} = s.\ \frac{Y}{K}(w)$$

where K is the capital stock, Y the national income, s the propensity to save (taken as a constant), and w the wage-rate. Together with (8) and (9), this equation also produces a "golden age" in the limit. As w rises, Y/K falls as a result of more capital-intensive techniques being adopted, and so the rate of accumulation falls. Thus the Accumulation Function for this neo-classical model also has a negative slope, the only difference being that it goes asymptotically to each of the axes. The wage-rate moves correspondingly with the marginal productivity of labour until the rates of growth of capital and of labour are equalised. The theorem that changes in thrift do not change the rate of growth but only influence factor prices also follows from the neo-classical model.

The Harrod-Domar model differs from the Robinsonian in not separating the producer-goods and consumer-goods sectors, and, what is more important, in assuming that savings are proportional to income and not to profits. In the Harrod-Domar model the rates of growth of capital and labour are entirely independent of each other. Substituting a neo-classical production function for the fixed coefficients, Solow proved that the capital growth rate would converge to the labour growth rate. He thus regarded fixed coefficients as the "crucial assumption" on with the independence of the two rates in the Harrod-Domar model rested. In the Robinsonian model, however, fixed coefficients are retained, but convergence to a "golden age" results in spite of this as a consequence of saving being a function of the distribution of income and not just of the level of income. Thus either variable technical coefficients or the Robinsonian savings function is sufficient to produce a "golden age" equilibrium. It is interesting to note that when the two are combined, as in the Robinsonian model with a spectrum of techniques, variable techniques actually slow up the approach to a "golden age".

A model that differs from the Harrod-Domar model only with respect to separating the producer-goods and consumer-goods sectors can readily be constructed. Equation (1) of the present model will be retained, but in place of (2) or (2') we must have

$$\alpha(C + P\Delta M) = C$$

$$P\Delta M - (1 - \alpha)C = 0$$

where α is now the common propensity to consume of both workers and capitalist. Substituting for C from (1) and P from(3), we obtain the Accumulation Function

$$\frac{\Delta M}{M} = \frac{(1 - \alpha)}{a_{11} - w(a_{11}a_{22} - a_{12}a_{21}) + (1 - \alpha)a_{11}}$$

From this expression it follows that the effect of changes in the wage-rate on the rate of accumulation depends on the sign of $(a_{11}a_{22} - a_{12}a_{21})$, that is, on the relative factor-intensities of the two sectors. If $a_{11}a_{22} < a_{12}a_{21}$ so that the consumer-goods sector is the more labour-intensive, increases in the wage-rate will reduce the denominator and so raise the rate of accumulation. If $a_{11}a_{22} > a_{12}a_{21}$, a rise in the wage rate will increase the denominator and lower the rate of accumulation. A "golden age" equilibrium can be consistent with either assumption, but only in the latter case, with a negatively inclined Accumulation Function, would the equilibrium be stable.

Another possible model is one where saving is a function of profits, but where the two sectors are not distinguished. In this case the Accumulation Function would be

$$\frac{\Delta M}{M} \quad \frac{(1 - \alpha)(1 - w \; a_2)}{a_1}$$

where α is the capitalist propensity to save, a_2 the labour input coefficient, and a_1 the capital input coefficient. It is easy to see that the rate of accumulation will vary inversely with the wage-rate, so that a "golden age" will be reached in this case also. An example of this type of model is provided by the earliest of Mr. Kaldor's successive contributions to the theories of growth and distribution.1

It has thus been demonstrated that fixed technical coefficients alone cannot be regarded as responsible for the Harrod-Domar result, since either the separation of the producer-goods and consumer-goods sectors or the circumstances that saving is a function of the distribution of income (as with Kaldor), or both (as with Mrs. Robinson), can produce a "golden age" equilibrium for a model with fixed coefficients.

It might be interesting, in conclusion, to reflect a little on the lineage of the growth models investigated here. Behind Harrod and Domar there is Keynes; beyond Solow and Swan there is Wicksell; and behind Mrs. Robinson there is Marx. It would appear that the Keynesian heritage, rooted as it is in the short period, is the least fruitful for the analysis of the long run where relative price movements can take the place of variations in income and employment in equilibrating savings and investment. But perhaps the last word has not been said, for a glaring deficiency of all the models considered here is the total neglect of monetary variables. If money is to be accorded its proper place in the theory of growth, it is almost certain that liquidity preference, and perhaps even the ideas of the "mysterious" Chapter 17 of the General Theory, will play a central role.

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1. N. Kaldor, "Alternative Theories of Distribution", Review of Economic Studies, vol.XXIII (1955-56).